Closed Matter-Current Constraints and Gaussian Physical Moduli

Bounded-particle exclusion and all-orders Fock structure in a machine-checked gauge-field model

Zed James

© 2026 Zed James. This work is licensed under the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.

Preprint DOI: 10.5281/zenodo.22818292

Abstract

We study a completed matter-current constraint in a finite-regulator gauge-field Fock model and find two complementary features of its exact kernel. First, every joint closed-current zero with uniformly bounded total-particle support vanishes. Any nonzero physical solution therefore has nonvanishing coefficients at arbitrarily large particle number. Second, the same current problem admits a three-parameter family of normalized Gaussian solutions.

The Gaussian family is generated by a positive adjoint-invariant quadratic form on the finite internal coordinate space. Three independent positive rescalings produce a precision family

$$G_\lambda, \qquad \lambda = (\lambda_1, \lambda_2, \lambda_3) \in (0, \infty)^3.$$

For every positive scale triple the corresponding Gaussian is square integrable, belongs to the domain of each minimal closed current row, and is annihilated by that operator. The normalized scale-to-state map is injective, so the exact closed kernel contains a genuine three-parameter family of states.

The particle-sector structure is equally rigid. The vacuum component is nonzero, every odd sector vanishes, the two-particle sector is nonzero, and nonzero even sectors occur above every finite particle cutoff. Gaussian–Hermite integration by parts gives a precision-weighted moment identity which, after inversion by the Gaussian covariance, yields an explicit raising/lowering recurrence for arbitrary Fock occupation. The same current constraint therefore fixes the support class of every nonzero solution while leaving a positive three-parameter family inside that completed class.

1 Introduction

Constraints in quantum field theory act at more than one level. There is an algebraic condition on a dense core, an analytic domain problem for the corresponding unbounded operator, and a completed Hilbert-space kernel in which physical states are ultimately represented. These levels are closely related but need not coincide automatically.

The present model makes this distinction concrete. At fixed finite regulator, the matter current is defined first on a dense finite-occupation core and then by its minimal closed realization on the completed Fock space. Earlier algebraic results show that no nonzero finite-support vector lies in the joint current kernel. The first question here is what survives after completion.

Because the one-particle mode set is finite, a uniform particle-number bound leaves only finitely many occupations. A completed vector supported below such a bound therefore lies exactly in the algebraic core. The finite-core exclusion then gives

$$\Psi \in \mathcal{K}, \qquad \operatorname{supp}\Psi \subseteq \{n : N(n) \leq B\} \quad \Longrightarrow \quad \Psi = 0, \tag{1}$$

where

$$\mathcal{K} = \bigcap_{\ell} \ker \overline{J}_{\ell}$$

is the joint kernel of the minimal closed current rows. Equivalently,

$$\Psi \in \mathcal{K}, \quad \Psi \neq 0 \quad \Longrightarrow \quad \forall B \in \mathbb{N}\ \exists n :\ N(n) > B,\ \Psi(n) \neq 0. \tag{2}$$

The infinite particle tail is therefore part of the exact support condition imposed by the current.[1]

The second question is whether the same closed constraint leaves any nontrivial state freedom. It does. The internal coordinate space carries a symmetric real pairing obtained from the model's involution and trace structure. The associated quadratic form is positive and invariant under the infinitesimal adjoint action. Three independent positive rescalings give

$$\lambda = (\lambda_1, \lambda_2, \lambda_3) \in (0,\infty)^3 \tag{3}$$

and a corresponding family of positive precision forms G_λ.[2]

Each G_λ defines a normalized Gaussian state Ψ_λ. For every strictly positive scale triple,

$$\Psi_\lambda \in \bigcap_{\ell} \operatorname{Dom}(\overline{J}_{\ell}), \qquad \overline{J}_{\ell}\Psi_\lambda = 0, \tag{4}$$

so

$$\Psi_\lambda \in \mathcal{K}. \tag{5}$$

The map

$$(0,\infty)^3 \longrightarrow \mathcal{K}, \qquad \lambda \longmapsto \Psi_\lambda \tag{6}$$

is injective.[3]

[1] *Formal counterpart:* `CI41.CanonicalCompletedMatterCurrentBoundedParticleDecision;CI118.CanonicalPhysicalGaussianParticleSectorQualification`.

[2] *Formal counterpart:* `CI56.CanonicalFactorScaledGaugeMetric`.

[3] *Formal counterpart:* `CI134.CanonicalPositiveOrthantGaussianPhysicalFamily`.

The two results should be read together. The current kernel contains no nonzero bounded-particle state, yet after completion it contains an injective positive three-parameter Gaussian family. Constraint strength and residual state multiplicity therefore coexist in the same completed problem.

The particle anatomy of this family can be described exactly. The Gaussian is even under global position reversal. In the Hermite/Fock representation this gives

$$\Psi_\lambda^{(2m+1)} = 0, \qquad m \geq 0. \tag{7}$$

The vacuum and two-particle sectors are nonzero,

$$\Psi_\lambda^{(0)} \neq 0, \qquad \Psi_\lambda^{(2)} \neq 0, \tag{8}$$

and Eq. (2) then forces nonzero even support above every finite cutoff. Thus

$$\Psi_\lambda = \Psi_\lambda^{(0)} + \Psi_\lambda^{(2)} + \Psi_\lambda^{(4)} + \cdots \tag{9}$$

with an unbounded even tail.[4]

The same Gaussian geometry controls the coefficients at arbitrary occupation. Gaussian–Hermite integration by parts produces a linear precision balance. The corresponding covariance inverts the finite precision system and gives a pair kernel K_λ satisfying

$$c_s(n)\Psi_{\lambda,n+e_s} = \sum_r K_\lambda(s,r)a_r(n)\Psi_{\lambda,n-e_r}. \tag{10}$$

There is no degree cutoff. The recurrence continues through the same particle tower that the closed-current theorem requires.[5]

The discussion below follows this chain from the completed current kernel to the Gaussian family and then to its exact Fock structure. The model-specific internal labels used in the formal implementation are suppressed in the main text when they do not affect the mathematical statement; the appendix gives the corresponding Lean declarations.

1.1 Relation to nearby work

Multiparticle state spaces are becoming increasingly explicit in celestial holography, where one must keep track of complete Hilbert-space carriers rather than only one-particle data [2, 1, 3, 9]. Discrete celestial bases provide another example of the value of making the state-space carrier explicit [8]. The present construction reaches the multiparticle regime from a different direction: the closed current itself excludes every bounded-particle nonzero solution.

Asymptotic symmetry gives a related perspective. Ward identities and large gauge transformations organize scattering states through current structures, with infinite symmetry algebras providing a broader organization [5, 4, 7]. The current used here is not an asymptotic charge, but the shared structural point is that gauge constraints act on the state space and can remain informative after Hilbert completion.

[4] *Formal counterpart:* `CI135.CanonicalPositiveOrthantGaussianParticleParity;CI137.CanonicalPositiveOrthantGaussianParticleAnatomy`.

[5] *Formal counterpart:* `CI136.CanonicalPositiveOrthantGaussianPairRecurrence`.

The Gaussian side of the construction belongs to a familiar analytic setting. Gaussian states and covariance/precision descriptions are standard in bosonic quantum theory [10], while Gaussian measures and functional-integral methods provide the broader background for the position-space realization used here [11, 12]. Recent top-down flat-space constructions provide a complementary neighboring perspective [6]. The present analysis is operator-theoretic rather than entanglement-geometric, but it uses the same general strategy of keeping the regulated setting explicit enough that the completed problem can be stated exactly.

The proof assistant is most visible at the analytic interfaces. Finite-core exclusion must be transported to the completion; pointwise Gaussian cancellation must be promoted to membership in the domain of a closed current; and low-order Gaussian identities must be extended to arbitrary occupation. In particular, domain and adjoint questions for unbounded operators are treated as part of the statement rather than suppressed behind formal differential expressions [13, 14]. Each transition is a separate theorem in the formal development, rather than an implicit identification of algebraic and analytic statements [15].

2 The completed current problem

Let $\mathcal{C} \subset \mathcal{F}$ be the finite-occupation core of the regulated Fock space. An occupation n contains bosonic multiplicities together with fermionic occupation data, and $N(n)$ denotes the total particle number.

For each current label ℓ,

$$J_\ell^{\mathrm{core}} : \mathcal{C} \longrightarrow \mathcal{C} \tag{11}$$

is densely defined. Its physical realization is the minimal closed operator $\overline{J}_\ell$ on $\mathcal{F}$, and

$$\mathcal{K} = \bigcap_\ell \ker \overline{J}_\ell. \tag{12}$$

For $B \in \mathbb{N}$, let

$$\Omega_B = \{n : N(n) \le B\}. \tag{13}$$

Finite regulator and finite particle number imply that Ω_B is finite. If a completed vector Ψ vanishes outside Ω_B, then

$$\Psi = \sum_{n \in \Omega_B} \Psi(n)|n\rangle \in \mathcal{C}. \tag{14}$$

Hence the finite-core exclusion theorem gives

$$\Psi \in \mathcal{K}, \qquad \operatorname{supp}\Psi \subseteq \Omega_B \quad \Longrightarrow \quad \Psi = 0. \tag{15}$$

Taking the contrapositive yields Eq. (2).[6]

This is the first structural fact we will use. It concerns every nonzero element of the completed joint kernel and is independent of the Gaussian family constructed below.

[6] *Formal counterpart:* `CI41.CanonicalCompletedMatterCurrentBoundedParticleDecision`.

3 Invariant quadratic geometry

Let the finite internal coordinate space split into three orthogonal sectors,

$$\mathfrak{g}_{\text{coord}} = \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_3. \tag{16}$$

The model carries a trace-type pairing and an involution. Composing one leg of the pairing with that involution gives a real symmetric form G_0 which is positive definite on the coordinate space.

If B_γ denotes an infinitesimal adjoint-action matrix, the form satisfies

$$B_\gamma G_0 + G_0 B_\gamma^T = 0. \tag{17}$$

The associated quadratic

$$Q_0(x) = x^T G_0 x$$

is therefore invariant under the infinitesimal adjoint action.

Independent positive scaling of the three sectors gives

$$G_\lambda = \lambda_1 G_1 \oplus \lambda_2 G_2 \oplus \lambda_3 G_3, \qquad \lambda_i > 0. \tag{18}$$

Every G_λ remains symmetric, positive definite, and adjoint invariant. We write

$$Q_\lambda(x) = x^T G_\lambda x. \tag{19}$$

The theorem domain established here is the full positive orthant $(0, \infty)^3$. No maximal-domain claim outside this positive region is needed.[7]

The finite formal instantiation contains a specific block decomposition and one nontrivial mixed two-dimensional block. Those coordinates are useful for machine verification and for the nonzero two-particle witness below, but are not needed to state the invariant geometry in the manuscript.

4 The Gaussian family

Let $\|x\|_0^2$ denote the reference quadratic carried by the ambient Gaussian measure. Define

$$C_\lambda(x) = Q_\lambda(x) - \|x\|_0^2 \tag{20}$$

and

$$A_\lambda(x) = \exp\left[-\frac{1}{4} C_\lambda(x)\right]. \tag{21}$$

The corresponding position-space state is placed in the fermion-vacuum fiber,

$$\psi_\lambda(x) = A_\lambda(x) e_\varnothing. \tag{22}$$

Multiplying $|A_\lambda|^2$ by the reference Gaussian measure cancels the reference quadratic and leaves

$$|\psi_\lambda(x)|^2 \, d\mu_0(x) \propto \exp\left[-\frac{1}{2} Q_\lambda(x)\right] dx. \tag{23}$$

[7] *Formal counterpart:* `CI56.CanonicalFactorScaledGaugeMetric`.

Since G_λ is positive definite, the norm is finite and nonzero throughout $(0,\infty)^3$.

Equivalently, choose the positive diagonal scale map S_λ satisfying

$$Q_1(S_\lambda x) = Q_\lambda(x). \tag{24}$$

Then

$$\|\psi_\lambda\|^2 = (\det S_\lambda)^{-1} Z_0, \qquad Z_0 > 0. \tag{25}$$

The exact exponent of each scale in $\det S_\lambda$ depends on the finite multiplicity of the three internal sectors. In the formal model these multiplicities sum to sixty active coordinates; the main argument uses only positivity of the determinant.[8]

Normalize by

$$\widehat{\psi}_\lambda = \frac{\psi_\lambda}{\|\psi_\lambda\|}. \tag{26}$$

The established Hermite unitary

$$U : \mathcal{F} \xrightarrow{\sim} \mathcal{H}_{\text{position}} \tag{27}$$

gives the Fock state

$$\Psi_\lambda = U^{-1}\widehat{\psi}_\lambda, \qquad \|\Psi_\lambda\| = 1. \tag{28}$$

On the earlier restricted scale domain, this agrees exactly with the pre-existing Gaussian construction; the positive-orthant theorem extends that family without replacing it.[9]

5 Exact current closure

The matter current separates into fermionic, scalar, and bosonic-coordinate contributions,

$$J_\ell^{\text{core}} = J_{\ell,F} + J_{\ell,S} + J_{\ell,B}. \tag{29}$$

The Gaussian closes each term for a different reason.

The state lies in the fermion-vacuum fiber, so

$$J_{\ell,F}\psi_\lambda = 0. \tag{30}$$

Its amplitude depends only on the bosonic internal coordinates, and the scalar part closes by the real-form scalar identity,

$$J_{\ell,S}\psi_\lambda = 0. \tag{31}$$

For a selected bosonic coordinate s,

$$\partial_s A_\lambda(x) = \frac{A_\lambda(x)}{2}\left(x_s - \sum_r (G_\lambda)_{sr} x_r\right). \tag{32}$$

[8] *Formal counterpart:* `CI133.CanonicalPositiveOrthantGaussianNormalizability`.

[9] *Formal counterpart:* `CI57.CanonicalFactorScaledInvariantGaussian;CI134.CanonicalPositiveOrthantGaussianPhysicalFamily`.

The corrected Gaussian derivative removes the reference term and leaves the precision contribution. Inserting it into the bosonic current reduces the remaining contraction to the infinitesimal variation of Q_λ. Equation (17) therefore gives

$$J_{\ell,B}\psi_\lambda = 0. \tag{33}$$

Thus

$$J_\ell^{\mathrm{diff}}\psi_\lambda(x) = 0 \qquad \forall \lambda, \ell, x. \tag{34}$$

The entire positive orthant survives the literal differential current equation.[10]

5.1 From the differential equation to the closed operator

Pointwise cancellation does not by itself imply that Ψ_λ lies in the domain of the physical closed current. The required bridge is the maximal-adjoint characterization.

Let $\ell^\dagger$ denote the adjoint current label. The core currents satisfy formal adjunction, and the physical current obeys

$$\overline{J}_\ell = (J_{\ell^\dagger}^{\mathrm{core}})^*. \tag{35}$$

This is an equality of closed operators, including domains and values.[11]

Hence, if $u, v \in \mathcal{F}$ satisfy

$$\langle v, \phi \rangle = \langle u, J_{\ell^\dagger}^{\mathrm{core}}\phi \rangle \qquad \forall \phi \in \mathcal{C}, \tag{36}$$

then

$$u \in \mathrm{Dom}(\overline{J}_\ell), \qquad \overline{J}_\ell u = v. \tag{37}$$

For the Gaussian family, normalizability and one-coordinate Gaussian integration by parts give

$$\left\langle \Psi_\lambda, J_{\ell^\dagger}^{\mathrm{core}}\phi \right\rangle = 0 \qquad \forall \phi \in \mathcal{C}. \tag{38}$$

Therefore

$$\Psi_\lambda \in \mathrm{Dom}(\overline{J}_\ell), \qquad \overline{J}_\ell \Psi_\lambda = 0, \tag{39}$$

and

$$\Psi_\lambda \in \mathcal{K} \qquad \forall \lambda \in (0,\infty)^3. \tag{40}$$

[12]

This is the operator-theoretic center of the paper: the Gaussian is not only a pointwise differential zero, but an actual vector in the domain and kernel of the minimal closed physical current.

6 Gaussian physical moduli

The normalized family defines

$$\Phi : (0,\infty)^3 \longrightarrow \mathcal{K}, \qquad \Phi(\lambda) = \Psi_\lambda. \tag{41}$$

[10] *Formal counterpart:* `CI65.CanonicalFactorScaledGaussianMatterCurrentClosure`.

[11] *Formal counterpart:* `CanonicalW5CorrectedHilbertPhysicalMatterCurrentAdjoint`.

[12] *Formal counterpart:* `CI69.CanonicalFactorScaledGaussianClosedGraph;CI69.CanonicalFactorScaledGaussianFullAdjoint;CI134.CanonicalPositiveOrthantGaussianPhysicalFamily`.

This map is injective,

$$\Phi(\lambda) = \Phi(\lambda') \implies \lambda = \lambda'. \tag{42}$$

Equality of normalized Fock states is transported to equality of their position representatives; the normalization is fixed at the origin, and suitable sector probes recover the three scale factors independently.[13]

We use the phrase *Gaussian physical moduli* for this state-level injective family inside the closed kernel. No quotient by gauge orbit, symmetry orbit, or observational equivalence is imposed in this paper.

The regulated carrier also admits an isometric embedding

$$\iota : \mathcal{F} \longrightarrow \mathcal{F}_{\mathrm{cont}} \tag{43}$$

into a completed continuum-momentum representation, compatible with the closed current graph. Setting

$$\Psi_\lambda^{\mathrm{cont}} = \iota(\Psi_\lambda),$$

the transported family remains normalized, nonzero, current-closed, and injective. This is exact transport of the regulated family, not regulator removal.[14]

The finite formal implementation includes explicit subunit regression points inside the positive orthant. They confirm that normalizability, current closure, injectivity, parity, and the recurrence do not depend on the earlier lower-bound-one scale restriction.[15]

7 Particle-sector anatomy

Every Ψ_λ lies in the scope of Eq. (2), so

$$\forall B \in \mathbb{N}\ \exists n : \ N(n) > B, \quad \Psi_\lambda(n) \neq 0. \tag{44}$$

The Gaussian amplitude is even,

$$A_\lambda(-x) = A_\lambda(x). \tag{45}$$

Odd total-degree Hermite functions change sign under the same reversal, while the reference Gaussian measure is invariant. Hence

$$\Psi_\lambda^{(2m+1)} = 0 \qquad \forall m \geq 0. \tag{46}$$

The vacuum coefficient is nonzero,

$$\Psi_\lambda^{(0)} \neq 0. \tag{47}$$

[16]

The two-particle sector is also nonzero. One finite internal block of G_λ has a nonvanishing off-diagonal entry throughout the positive orthant. If the pair kernel defined below vanished identically, the covariance–precision identity would force that entry to vanish, a contradiction. Hence some

[13] *Formal counterpart:* `CI134.CanonicalPositiveOrthantGaussianPhysicalFamily`.
[14] *Formal counterpart:* `CI134.CanonicalPositiveOrthantGaussianPhysicalFamily`.
[15] *Formal counterpart:* `CI138.CanonicalPositiveOrthantGaussianSubunitRegressions`.
[16] *Formal counterpart:* `CI135.CanonicalPositiveOrthantGaussianParticleParity`.

pair-kernel entry is nonzero; the one-step recurrence then propagates the nonzero vacuum coefficient into degree two:

$$\Psi_\lambda^{(2)} \neq 0. \tag{48}$$

The exact model-specific witness for this off-diagonal block is recorded in the formal theorem map.[17]

Combining unbounded support with odd-sector vanishing,

$$\forall B \in \mathbb{N}\ \exists n :\ N(n) > B, \quad N(n) \text{ even}, \quad \Psi_\lambda(n) \neq 0. \tag{49}$$

Thus

$$\Psi_\lambda = \Psi_\lambda^{(0)} + \Psi_\lambda^{(2)} + \Psi_\lambda^{(4)} + \cdots \tag{50}$$

with nonzero vacuum, nonzero two-particle content, and unbounded even support.

8 All-orders Gaussian–Hermite recurrence

Let $H_n(x)$ be the bosonic Hermite amplitude for occupation n, and define

$$T_{\lambda,n}(x) = A_\lambda(x) H_n(x). \tag{51}$$

Multiplication by coordinate x_s obeys

$$x_s T_{\lambda,n} = c_s(n) T_{\lambda,n+e_s} + a_s(n) T_{\lambda,n-e_s}, \tag{52}$$

where $c_s(n)$ and $a_s(n)$ are the ordinary creation and annihilation amplitudes.

Differentiation gives

$$\partial_s T_{\lambda,n} = a_s(n) T_{\lambda,n-e_s} + \frac{1}{2}\left[x_s T_{\lambda,n} - \sum_r (G_\lambda)_{sr} x_r T_{\lambda,n} \right]. \tag{53}$$

Define

$$M_n = \int T_{\lambda,n}(x)\, d\mu_0(x), \qquad X_{s,n} = \int x_s T_{\lambda,n}(x)\, d\mu_0(x). \tag{54}$$

Gaussian integration by parts yields

$$X_{s,n} + \sum_r (G_\lambda)_{sr} X_{r,n} = 2 a_s(n) M_{n-e_s}. \tag{55}$$

[18]

After normalization and Hermite/Fock transport, set

$$L_{s,n} = c_s(n) \Psi_{\lambda,n+e_s} + a_s(n) \Psi_{\lambda,n-e_s}. \tag{56}$$

Then

$$L_{s,n} + \sum_r (G_\lambda)_{sr} L_{r,n} = 2 a_s(n) \Psi_{\lambda,n-e_s}. \tag{57}$$

[17] *Formal counterpart:* `CI137.CanonicalPositiveOrthantGaussianParticleAnatomy`.
[18] *Formal counterpart:* `CanonicalCH11GaussianHermiteMomentBalance`.

The covariance extracted from the one-excitation moments satisfies

$$\Sigma_\lambda(I + G_\lambda) = 2I. \tag{58}$$

Write

$$\Sigma_\lambda = I + K_\lambda. \tag{59}$$

Solving Eq. (57) with Eq. (58) gives

$$c_s(n)\Psi_{\lambda,n+e_s} = \sum_r K_\lambda(s,r)a_r(n)\Psi_{\lambda,n-e_r}. \tag{60}$$

Since $c_s(n) \neq 0$,

$$\Psi_{\lambda,n+e_s} = \frac{\sum_r K_\lambda(s,r)a_r(n)\Psi_{\lambda,n-e_r}}{c_s(n)}. \tag{61}$$

These identities hold at arbitrary occupation and have no degree cutoff.[19]

The same finite geometry has therefore appeared in three forms: as the invariant precision G_λ, as the covariance Σ_λ, and as the pair kernel K_λ which propagates the Fock coefficients. This is the structure taken up in the next paper, where the recurrence is promoted to a closed quadratic pair generator.

9 Discussion

The completed current problem has a simple structural form. Every bounded-particle joint zero vanishes, while the positive scale orthant embeds injectively into the normalized joint kernel:

$$\left.\begin{array}{c}\operatorname{supp}\Psi \subseteq \{N \leq B\} \\ \Psi \in \mathcal{K}\end{array}\right\} \Longrightarrow \Psi = 0, \qquad (0,\infty)^3 \hookrightarrow \mathcal{K}. \tag{62}$$

The first statement fixes the support class of every nonzero solution. The second records three-parameter state freedom which survives exact current closure.

The Gaussian family provides an explicit inhabitant of the required completed class. Its vacuum is nonzero, odd sectors vanish, the two-particle sector is nonzero, and even support continues above every finite cutoff. The infinite tail is not inferred from Gaussian folklore; it follows first from the closed-current theorem and is then organized by parity and the all-orders recurrence.

The invariant form G_λ plays two roles without being identified by notation. It is a positive internal quadratic preserved by the adjoint action, and after the Gaussian is constructed it becomes the precision of the fluctuation measure. Integration by parts transports this same matrix into the Fock recurrence. The internal geometry and the completed state geometry are related by explicit maps rather than by a change of vocabulary.

The distinction between pointwise current cancellation and closed-current membership is equally important. Equation (34) lives at the differential level; Eq. (40) is a domain statement for an unbounded operator. The maximal-adjoint characterization provides the analytic bridge between them.

[19] *Formal counterpart:* `CI136.CanonicalPositiveOrthantGaussianPairRecurrence;CanonicalCH11GaussianPairCoefficientRecurrence`.

The theorem domain here is the full positive orthant. The finite-to-continuum-momentum statement is exact transport of this regulated family into a closed completed carrier, not a regulator-removal theorem. Questions about maximal scale domain, observational quotient, multiparticle representation, and continuum stability are treated separately.

10 Conclusion

The completed matter-current constraint considered here imposes a rigid support condition:

$$\Psi \in \mathcal{K}, \quad \Psi \neq 0 \quad \Longrightarrow \quad \forall B \in \mathbb{N}\, \exists n : \; N(n) > B, \quad \Psi(n) \neq 0. \tag{63}$$

No nonzero bounded-particle state survives the exact joint current.

At the same time, the positive adjoint-invariant internal geometry produces a normalized Gaussian family

$$(0, \infty)^3 \hookrightarrow \mathcal{K}. \tag{64}$$

Each state belongs to the domain of every minimal closed current row and is annihilated there. The family is injective at the state-vector level.

Its Fock anatomy is exact:

$$\Psi_\lambda = \Psi_\lambda^{(0)} + \Psi_\lambda^{(2)} + \Psi_\lambda^{(4)} + \cdots, \qquad \Psi_\lambda^{(0)} \neq 0, \qquad \Psi_\lambda^{(2)} \neq 0, \tag{65}$$

with nonzero even support above every finite cutoff. The Gaussian covariance inverts the precision system and yields the arbitrary-occupation recurrence

$$c_s(n)\Psi_{\lambda,n+e_s} = \sum_r K_\lambda(s,r) a_r(n) \Psi_{\lambda,n-e_r}. \tag{66}$$

The current constraint therefore does two things at once: it excludes every bounded-particle nonzero solution and preserves a three-parameter family of exact states after completion. The same family carries an all-orders pair recurrence, which supplies the input for the closed pair-generation theorem of the next paper.

A Lean theorem map

The main text intentionally uses invariant operator and representation language rather than project-specific carrier names. The table below gives the exact Lean counterparts. The Paper 1 theorem surface is pinned to repository commit

`c91f73117df75a0c7d159b54431af73a80b06bbb`,

the commit that closes the CI133–CI138 positive-orthant Gaussian family used throughout this manuscript.

Mathematical result	Formal module
Bounded-particle exclusion	`CI41.CanonicalCompletedMatterCurrentBoundedParticleDecision`
Positive adjoint-invariant internal metric	`CI56.CanonicalFactorScaledGaugeMetric`
Scaled Gaussian construction and compatibility subdomain	`CI57.CanonicalFactorScaledInvariantGaussian`
Positive-orthant L^2 normalizability	`CI133.CanonicalPositiveOrthantGaussianNormalizability`
Physical matter-current assembly	`CanonicalW5CorrectedHilbertMatterGaugeCurrent`
Pointwise matter-current closure	`CI65.CanonicalFactorScaledGaussianMatterCurrentClosure`
Maximal-adjoint current characterization	`CanonicalW5CorrectedHilbertPhysicalMatterCurrentAdjoint`
Gaussian cutoff/closed-graph adjunction	`CI69.CanonicalFactorScaledGaussianClosedGraph`
Full formal-adjoint bridge	`CI69.CanonicalFactorScaledGaussianFullAdjoint`
Positive-orthant closed-current family and injectivity	`CI134.CanonicalPositiveOrthantGaussianPhysicalFamily`
Parity, vacuum, and unbounded even support	`CI135.CanonicalPositiveOrthantGaussianParticleParity`
Covariance and all-orders pair recurrence	`CI136.CanonicalPositiveOrthantGaussianPairRecurrence`
Nonzero two-particle sector and particle anatomy	`CI137.CanonicalPositiveOrthantGaussianParticleAnatomy`
Subunit regression controls and old-domain compatibility	`CI138.CanonicalPositiveOrthantGaussianSubunitRegressions`
Earlier bounded-support joint-kernel consequence	`CI118.CanonicalPhysicalGaussianParticleSectorQualification`
Analytic Gaussian normal form	`CI95.CanonicalFactorScaledGaussianAnalyticNormalForm`
Arbitrary-occupation Gaussian–Hermite moment balance	`CanonicalCH11GaussianHermiteMomentBalance`
Earlier closed-subdomain pair recurrence	`CanonicalCH11GaussianPairCoefficientRecurrence`

Internal names such as `Tower`, `Witness`, `W5`, `R3`, and the explicit gauge-sector labels record the provenance of the formal implementation. They are suppressed in the manuscript where they do not alter the mathematical statement. In particular, the concrete $5 + 15 + 40$ coordinate multiplicities and the explicit mixed internal block remain available in the formal development without being required as manuscript-level notation.

B Formal development and collaboration

The results are supported by a Lean 4 formal development maintained in a private research repository. The theorem map above records the formal chain from finite-core exclusion through Gaussian normalizability, closed-current membership, injective positive-orthant moduli, particle-sector anatomy, and the all-orders recurrence. The pinned Paper 1 snapshot uses Lean 4 `v4.28.0-rc1` and mathlib revision

`5352afccd6866369be9de43f5b7ec47203555f44`.

The full repository is not publicly accessible. Paper-scoped source review, theorem-level verification, and access to relevant portions of the formal development may be arranged in the context of professional collaboration, technical review, or institutional partnership.

References

[1] S. Pasterski, "A Chapter on Celestial Holography," arXiv:2310.04932 [hep-th].

[2] S. Pasterski, "Lectures on Celestial Amplitudes," Eur. Phys. J. C **81** (2021) 1062, arXiv:2108.04801 [hep-th].

[3] S. Pasterski, M. Pate and A.-M. Raclariu, "Celestial Holography," in *Snowmass 2021*, arXiv:2111.11392 [hep-th].

[4] A. Strominger, *Lectures on the Infrared Structure of Gravity and Gauge Theory*, Princeton University Press (2018), arXiv:1703.05448 [hep-th].

[5] A. Strominger, "Asymptotic Symmetries of Yang–Mills Theory," JHEP **07** (2014) 151, arXiv:1308.0589 [hep-th].

[6] E. Jørstad, R. C. Myers and S. Pasterski, "Flat Space Entanglement: A Coulomb Branch Perspective," arXiv:2606.13889 [hep-th].

[7] L. Freidel, D. Pranzetti and A.-M. Raclariu, "On Infinite Symmetry Algebras in Yang–Mills Theory," JHEP **12** (2023) 009, arXiv:2306.02373 [hep-th].

[8] L. Freidel, D. Pranzetti and A.-M. Raclariu, "A Discrete Basis for Celestial Holography," JHEP **02** (2024) 176, arXiv:2212.12469 [hep-th].

[9] J. Kulp and S. Pasterski, "Multiparticle States for the Flat Hologram," arXiv:2501.00462 [hep-th].

[10] C. Weedbrook, S. Pirandola, R. García-Patrón, N. J. Cerf, T. C. Ralph, J. H. Shapiro and S. Lloyd, "Gaussian Quantum Information," Rev. Mod. Phys. **84** (2012) 621–669, arXiv:1110.3234 [quant-ph].

[11] J. Glimm and A. Jaffe, *Quantum Physics: A Functional Integral Point of View*, 2nd ed., Springer (1987).

[12] B. Simon, *Functional Integration and Quantum Physics*, 2nd ed., AMS Chelsea Publishing (2005).

[13] M. Reed and B. Simon, *Methods of Modern Mathematical Physics, Vol. I: Functional Analysis*, Academic Press (1980).

[14] M. Reed and B. Simon, *Methods of Modern Mathematical Physics, Vol. II: Fourier Analysis, Self-Adjointness*, Academic Press (1975).

[15] M. R. Douglas, S. Hoback, A. Mei and R. Nissim, "Formalization of QFT," arXiv:2603.15770 [hep-th].