Friedrichs-Selected Schwarzschild Quantum Dynamics

A machine-checked continuum Fock theory and real-scalar physical Cauchy evolution

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Abstract

A preceding analysis of the spherically symmetric real massless scalar field in the Schwarzschild interior classified the curvature endpoint of the action-derived radial operator, proved deficiency indices $(1, 1)$, and showed that the full Schwarzschild ground-state form selects the no-log realization. Completion of the original compact-core quadratic form identifies that realization with the Friedrichs extension K_{F}.

The present paper develops the quantum dynamics owned by that selection and constructs the physical Cauchy theory of the same single real scalar field. The nonnegative Friedrichs operator determines the positive frequency $\omega_{\mathrm{F}} = K_{\mathrm{F}}^{1/2}$, its continuum bosonic second quantization, a self-adjoint nonnegative Fock Hamiltonian, and a strongly continuous generated unitary evolution. A quantum-ownership theorem propagates the Paper 1 selector through this complete spectral chain.

The physical Cauchy construction begins again from the original real Schwarzschild scalar action. The remaining spacelike static coordinate is retained on an explicit compact circle and expanded in a complete signed Fourier carrier. The complex one-particle Hilbert space is the positive-frequency complexification of the real solution space; the physical field remains Hermitian and uses one oscillator family. The original action fixes the canonical normalization, including the $1/\sqrt{2}$ rescaling that gives $[\widehat{\phi}, \widehat{\pi}] = iI$. Finite signed banks carry the literal integrated action energy, retain the same-family pair terms, and satisfy the normalized initial law

$$H_k(1) = \omega_k \left(N_k + \frac{1}{2} I \right).$$

Their closed Hamiltonians are self-adjoint and nonnegative, occupation windows admit unitary Dyson evolution, the occupation cutoff is removed strongly, and the resulting dynamics satisfies the closed-Hamiltonian Schrödinger law on the owned positive interval.

On the complete signed carrier, the maximal free Hamiltonian is self-adjoint on its exact occupation-energy domain. The fixed-frame instantaneous pair kernel is square summable while its frequency-weighted square sum can diverge for a nonzero potential shift; the action-extracted evolved Bogoliubov mixing instead has an earned frequency-weighted square-summable kernel. The all-mode real canonical frame preserves the original free-energy domain and implements same-family squeezing.

An independently constructed full-action Hamiltonian closes from the real algebraic core, is self-adjoint for every owned positive time, satisfies $D(H_{\mathrm{real}}(x)) = D(H_0)$ and $H_{\mathrm{real}}(1) = H_0$, and generates a norm-preserving, jointly strongly continuous, cocyclic, interval-independent physical clock obeying

$$\frac{d}{dx} U_{\mathrm{real}}(x, x_0)\Psi = -iH_{\mathrm{real}}(x) U_{\mathrm{real}}(x, x_0)\Psi.$$

The same unitary transports the literal signed-Fourier field and momentum. A final authority theorem packages the original real action, Friedrichs selection, signed spatial completeness, canonical CCR, Hermitian field, integrated action energy, all-mode self-adjoint Hamiltonian, physical clock, weighted Bogoliubov mixing, and field/momentum transport in one dependency chain.

The pre-existing charged particle–antiparticle theory is retained as a separate complex-scalar extension whose action is proved to equal the sum of two real scalar actions. The theorem surface remains on the compact spatial circle and strictly positive physical time.

1 Introduction

The terminal law of a singular quantum field problem determines more than endpoint regularity. Once a self-adjoint realization is selected, its operator domain controls the frequency, the many-particle generator, the admissible quantum evolution, and the structures available for subsequent local observables.

The first paper in this series, *Critical Schwarzschild Scalar Dynamics*, begins with a spherically symmetric *real* massless scalar field in ingoing Schwarzschild coordinates and derives an exact half-line operator

$$\tau_M = -\partial_x^2 + \widetilde{V}_M(x), \tag{1}$$

with Hardy-critical curvature-end behavior

$$\widetilde{V}_M(x) = -\frac{1}{4x^2} + R_M(x), \qquad x^2 R_M(x) \longrightarrow 0 \quad (x \downarrow 0). \tag{2}$$

The minimal and maximal operators are independently constructed and satisfy

$$K_0^* = K_{\max}, \qquad K_{\min}^* = K_{\max}. \tag{3}$$

The horizon endpoint is limit-point and the curvature endpoint is limit-circle for the full Schwarzschild potential. The global deficiency indices are therefore

$$(n_+, n_-) = (1, 1), \tag{4}$$

so the self-adjoint terminal laws form a $U(1)$ family.

Every maximal-domain field has a unique terminal trace

$$u(x) = \sqrt{x}(A_u + B_u \log x) + o(\sqrt{x}), \tag{5}$$

and the full Schwarzschild ground-state form satisfies

$$\mathcal{Q}_M[u] < \infty \quad \Longleftrightarrow \quad B_u = 0. \tag{6}$$

The closure of the original compact-core quadratic form represents the same no-log operator, giving

$$\boxed{K_{\text{no-log}} = K_{\text{F}}.} \tag{7}$$

This theorem supplies the starting object of the present paper.[1]

The operator pedigree is therefore geometric and variational from the outset. The ingoing scalar action yields the Euler equation, the area field $\psi = r\phi$, the exact terminal coordinate, the natural radial Hilbert space, and the unitarily transported half-line operator. The quantum construction below inherits that operator rather than introducing an independent quantum-mechanical Hamiltonian.

Self-adjoint extension data continues to play an active role in contemporary high-energy theory. Recent work uses singular extension theory both in black-hole quantization and in string-motivated compactifications where the extension choice controls spectra and perturbative stability.[12, 11]

The quantum-dynamical problem separates into two carriers with distinct geometric meanings.

The first follows from spectral functional calculus:

$$\boxed{K_{\mathrm{F}} \longrightarrow \omega_{\mathrm{F}} = K_{\mathrm{F}}^{1/2} \longrightarrow \Gamma_s(\mathcal{H}) \longrightarrow d\Gamma(\omega_{\mathrm{F}}) \longrightarrow U_{\mathrm{F}}(t).} \tag{8}$$

It provides the continuum one-particle frequency, the bosonic Fock Hamiltonian, and the generated spectral-group evolution.

The second returns to the original real scalar action and its physical Cauchy variables:

$$\boxed{S_{\mathbb{R}}[\phi] \longrightarrow (\widehat{\phi}, \widehat{\pi}) \longrightarrow H_{\mathrm{real}}(x) \longrightarrow U_{\mathrm{real}}(x_2, x_1).} \tag{9}$$

The one-particle carrier is complex because a positive-frequency polarization of a real linear field is naturally complex. The physical field remains Hermitian, and the construction uses one oscillator family rather than independent particle and antiparticle species.

The compact spatial carrier is also explicit. The remaining spacelike static coordinate is placed on AddCircle(1) and expanded in a complete signed Fourier basis. A fixed enumeration

$$n \in \mathbb{N} \longleftrightarrow k_n \in \mathbb{Z} \tag{10}$$

retains zero and both signs, with

$$\lambda_n = (2\pi k_n)^2. \tag{11}$$

Finite-bank removal therefore means exhaustion of all Fourier modes on this compact circle. The spatial compactification itself remains part of the theorem surface.

In this paper, *ownership* denotes proved dependency provenance. A downstream object is owned by an upstream construction when the formal theory derives the relevant definition, equality, or domain from that construction without introducing an additional independent physical choice. Thus the finite-form selector owns the Friedrichs operator and its spectral quantum theory, while the original real action owns the physical Cauchy field, canonical normalization, Hamiltonian, and clock.

The main structural result is that these two downstream lanes now share the same original real scalar action without being identified with one another. The spectral parameter generated by $\sqrt{K_{\mathrm{F}}}$ remains distinct from the physical Cauchy clock. The final real-scalar authority theorem packages the common provenance while preserving this distinction.

The paper is organized as follows. Sections 2–4 construct the Friedrichs-selected continuum spectral theory. Sections 5–7 derive the real physical Cauchy carrier, its canonical normalization, Hermitian field, and finite-bank action Hamiltonian. Sections 8–11 remove the occupation window and construct the owned finite-bank physical clock. Sections 12–14 pass to the full signed spatial carrier, adjudicate the pair kernels, construct the all-mode canonical frame, and earn the independent full-action self-adjoint Hamiltonian. Section 18 states the original-action real-scalar physical quantum theorem. Section 19 records the complex charged theory as a separate two-real-field extension. Section 20 proves the downstream Friedrichs spectral ownership theorem. The concluding sections state the theorem surface and identify the next local-matter layer.

2 Input from the terminal-selection theorem

Fix a positive Schwarzschild mass $M > 0$. Let

$$\mathcal{H} = L^2((0, \infty), dx; \mathbb{C}) \tag{12}$$

be the terminal-coordinate one-particle Hilbert space, and let K_{F} denote the Friedrichs realization selected by the full Schwarzschild ground-state form.

The preceding paper supplies the identities

$$K_{\mathrm{F}} = K_{\text{no-log}}, \tag{13}$$

$$K_{\mathrm{F}} = K_{\mathrm{F}}^{*}, \qquad K_{\mathrm{F}} \geq 0, \tag{14}$$

and

$$\ker K_{\mathrm{F}} = \{0\}. \tag{15}$$

The selected domain is the actual no-log terminal domain. The result is owned simultaneously by the physical trace condition $B = 0$, the finite full-potential form, and the completed compact-core Friedrichs construction.[1]

This semibounded self-adjoint operator supports the positive square-root frequency

$$\omega_{\mathrm{F}} = K_{\mathrm{F}}^{1/2}. \tag{16}$$

Functional calculus supplies a self-adjoint nonnegative frequency whose square recovers the Friedrichs operator on the corresponding second domain.

The construction uses the operator itself as the spectral datum. An eigenbasis is unnecessary, and the full domain/action identities remain visible in the downstream theory.

3 The Friedrichs-selected continuum frequency

Let

$$\omega_{\mathrm{F}} = K_{\mathrm{F}}^{1/2}. \tag{17}$$

Its spectral group is

$$U_{\mathrm{F}}^{(1)}(t) = e^{-it\omega_{\mathrm{F}}}. \tag{18}$$

Strong continuity follows from the self-adjoint generator, and the group preserves the one-particle norm.

The frequency domain carries the continuum wave dynamics associated with the selected radial theory. The generated wave and momentum maps satisfy the actual continuum wave relation on their declared domains, and the same-action phase charge recovers the corresponding Klein–Gordon pairing.[2]

The parameter t in $U_{\mathrm{F}}^{(1)}(t)$ is the spectral-group parameter generated by ω_{F}. The physical Cauchy evolution is constructed independently in Sections 5–11; this preserves the geometric meaning of both structures.

[1] *Formal counterpart:* `QGC12.Operator.schwarzschildFriedrichs_eq_noLog`.
[2] *Formal counterpart:* `QGC12.Quantum.NoLogContinuumKGDynamics`.

4 Continuum bosonic second quantization

Let

$$\mathcal{F}_s(\mathcal{H}) = \bigoplus_{n=0}^{\infty} \mathcal{H}^{\widehat{\otimes}_s n} \tag{19}$$

be the symmetric bosonic Fock space over the selected one-particle carrier.

The many-particle Hamiltonian is the second quantization

$$H_{\mathrm{F}} = d\Gamma(\omega_{\mathrm{F}}). \tag{20}$$

On the n-particle sector, H_{F} acts through the symmetric sum of the same selected frequency in each slot. The resulting full Fock operator is self-adjoint:

$$\boxed{H_{\mathrm{F}} = H_{\mathrm{F}}^*.} \tag{21}$$

[3]

It is nonnegative on its maximal domain:

$$\langle \Psi, H_{\mathrm{F}}\Psi \rangle \geq 0, \qquad \Psi \in D(H_{\mathrm{F}}). \tag{22}$$

[4]

The domain is the actual square-summable sector-image domain. Sector Hamiltonians are self-adjoint, product-wave states form dense invariant cores, and the product-core restrictions close to the complete sector graphs. This gives an explicit operator realization of $d\Gamma(\omega_{\mathrm{F}})$ across all particle sectors rather than a finite-particle proxy.

The zero-particle vector

$$\Omega = (1, 0, 0, \ldots) \tag{23}$$

belongs to the Fock Hamiltonian domain and obeys

$$H_{\mathrm{F}}\Omega = 0. \tag{24}$$

It is the reference vacuum of the selected frequency representation.

The Fock evolution is

$$U_{\mathrm{F}}(t) = e^{-itH_{\mathrm{F}}}. \tag{25}$$

For every $\Psi \in D(H_{\mathrm{F}})$,

$$\frac{d}{dt} U_{\mathrm{F}}(t)\Psi = -iH_{\mathrm{F}} U_{\mathrm{F}}(t)\Psi. \tag{26}$$

[5]

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[3] *Formal counterpart:* `QGC12.Quantum.physicalNoLogContinuumFockHamiltonian_isSelfAdjoint`.
[4] *Formal counterpart:* `QGC12.Quantum.physicalNoLogContinuumFockHamiltonian_nonnegative`.
[5] *Formal counterpart:* `QGC12.Quantum.physicalNoLogContinuumFockHamiltonian_hasDerivAt`.

4.1 Bosonic field algebra

On the dense finite-particle core, smeared creation and annihilation operators satisfy the canonical commutation relations

$$[a(f), a(g)] = 0, \tag{27}$$

$$[a^\dagger(f), a^\dagger(g)] = 0, \tag{28}$$

and

$$[a(f), a^\dagger(g)] = \langle f, g\rangle I. \tag{29}$$

The annihilation and creation structures transform covariantly under the selected one-particle evolution, so the sector dynamics and bosonic algebra share the same one-particle owner.[6]

This completes the continuum spectral quantization lane:

$$K_{\mathrm{F}} \longrightarrow \omega_{\mathrm{F}} \longrightarrow H_{\mathrm{F}} \longrightarrow U_{\mathrm{F}}(t). \tag{30}$$

5 The real physical Cauchy carrier

The spectral construction establishes the continuum quantum theory canonically owned by the terminal realization selected in Paper 1. The physical Cauchy construction asks a different question: whether the same original real Schwarzschild scalar action supports a genuine interior-time quantum evolution on its physical field variables.

The geometric interior supplies a timelike terminal coordinate and a spacelike static coordinate. The physical action therefore owns a temporal Legendre momentum, a Cauchy symplectic form, and a Hamiltonian energy density. The reduced area-field action differs from the original real geometric density by an earned boundary derivative, so the physical Cauchy theory and the radial endpoint theory remain separate uses of the same action.

The spacelike static coordinate is retained on the compact circle

$$S^1 = \mathrm{AddCircle}(1). \tag{31}$$

The real physical carrier enumerates the complete Fourier basis by a bijection

$$n \mapsto k_n \in \mathbb{Z}, \qquad 0, -1, +1, -2, +2, \ldots, \tag{32}$$

with Fourier characters

$$e_n(s) = e^{2\pi i k_n s} \tag{33}$$

and spectral values

$$\lambda_n = (2\pi k_n)^2. \tag{34}$$

The family is orthonormal and complete:

$$\overline{\mathrm{span}}\{e_n : n \in \mathbb{N}\} = L^2(S^1). \tag{35}$$

[7]

[6] *Formal counterpart:* `QGC12.Quantum.NoLogContinuumSmearedCCR`.

[7] *Formal counterpart:* `QGC12.Quantum.realTemporalSpatialFourier_complete`.

The opposite spatial label is an involution $n \mapsto \bar{n}$ with

$$k_{\bar{n}} = -k_n, \qquad \lambda_{\bar{n}} = \lambda_n. \tag{36}$$

The zero mode is its own opposite. This signed carrier contains spatial orientation within one real scalar theory; it carries no independent charge species.

The positive-frequency one-particle Hilbert space is complex, as expected for a chosen complex structure on a real classical solution space. The physical field operator remains Hermitian and is constructed from one oscillator family. In the signed-character convention its mode components pair annihilation at n with creation at the opposite spatial label $\bar{n}$.

6 The original real action and canonical normalization

Let q denote the real area-field Cauchy coordinate and let p denote the Legendre momentum derived from the original real scalar action. The formal construction begins with real geometric jets and derives the reduced interior action, its boundary completion, the real symplectic form, and the one-half Hamiltonian energy directly from that action.[8]

The historical unit-flux mode convention carries a factor two relative to the canonical normalization of one real oscillator. The new physical lane computes that factor explicitly. Rescaling the positive-frequency mode by

$$u_n^{\mathrm{real}}(x) = \frac{1}{\sqrt{2}} u_n^{\mathrm{unit\,flux}}(x) \tag{37}$$

earns the canonical real-field commutator

$$\boxed{[\widehat{\phi}_n(x), \widehat{\pi}_n(x)] = iI.} \tag{38}$$

[9]

The mode field is Hermitian in the actual Fock pairing. On the algebraic core one may write schematically

$$\widehat{\phi}_n(x) = u_n(x)a_n + \overline{u_n(x)}a_{\bar{n}}^\dagger, \tag{39}$$

with the corresponding momentum obtained from the action-derived first Cauchy datum. Summing the signed spatial characters gives the literal real field on the compact Cauchy slice. Hermiticity is proved on the same one-family carrier rather than inferred from a charged adjoint relation.[10]

The full-potential temporal modes retain the same scalar equation as the original physical Cauchy construction. The finite signed-bank Klein–Gordon pairing reproduces the positive one-particle Hilbert pairing after the real-action normalization.

7 Real finite-bank action energy

For a conjugation-closed signed bank of bandwidth K, the carrier contains

$$2K + 1 \tag{40}$$

[8] *Formal counterpart:* `QGC12.Matter.realInteriorTemporalActionDensity_actual_original_action`.

[9] *Formal counterpart:* `QGC12.Quantum.realTemporalFieldMomentum_actual_action_CCR`.

[10] *Formal counterpart:* `QGC12.Quantum.realTemporalActualFreeFourierTestBankValue_actual_Hermitian`.

spatial labels: the zero mode and both signs for every nonzero frequency. The literal spatial field and its derivative are reconstructed from these modes, and Parseval's identity computes the integrated action energy on the normalized Haar measure of the circle.

The one-oscillator quadratic retains the complete same-family pair structure. For each mode its energy has the schematic form

$$H_n(x) = A_n(x)a_n^\dagger a_n + B_n(x)a_n^\dagger a_{\bar{n}}^\dagger + \overline{B_n(x)}a_{\bar{n}}a_n + E_{0,n}(x)I. \tag{41}$$

The coefficients are derived from the real action, and the corresponding core energy pairing is symmetric and nonnegative.

At the frequency-normalized initial slice the pair coefficients vanish and the literal oscillator energy becomes

$$\boxed{H_n(1) = \omega_n\left(N_n + \frac{1}{2}I\right).} \tag{42}$$

The half-quantum is the computed vacuum action of the single real oscillator.[11]

The signed-bank Hamiltonian is the actual spatial integral of this field energy. Its commutators with the bank field and momentum reproduce the corresponding finite physical Heisenberg equations, including the self-partner zero mode.[12]

The finite-bank quadratic form closes to a self-adjoint, nonnegative operator

$$H_K^{\mathrm{real}}(x), \tag{43}$$

which extends the literal algebraic-core action.[13]

8 Finite occupation windows

The finite-bank Hamiltonian remains unbounded in total occupation. Let P_N denote the projection onto a finite total-occupation window. The bounded compression

$$H_{K,N}^{\mathrm{real}}(x) = P_N H_{K,\mathrm{core}}^{\mathrm{real}}(x) P_N \tag{44}$$

is the actual fixed-Cauchy energy compression, with polarization fixed at the initial Cauchy slice and the full Schwarzschild potential evaluated at physical time x.

On each compact positive interval $[\delta, 1]$, coefficient continuity gives a unitary Dyson propagator

$$U_{K,N}(x_2, x_1). \tag{45}$$

It satisfies the literal bounded Schrödinger equation

$$\frac{\partial}{\partial x_2}U_{K,N}(x_2, x_1)\Psi = -iH_{K,N}^{\mathrm{real}}(x_2)U_{K,N}(x_2, x_1)\Psi. \tag{46}$$

[14]

[11] *Formal counterpart:* `QGC12.Quantum.realFrequencyNormalizedTemporalHamiltonian_actual_initial_diagonal`.

[12] *Formal counterpart:* `QGC12.Quantum.realTemporalFourierBankHamiltonian_actual_integrated_action_energy`.

[13] *Formal counterpart:* `QGC12.Quantum.realTemporalClosedHamiltonian_isSelfAdjoint`.

[14] *Formal counterpart:* `QGC12.Quantum.realTemporalOccupationPropagator_actual_Schrodinger`.

9 Removing the occupation cutoff

The occupation-window removal is earned from the actual one-family ladder grade, explicit number growth, and finite-order Dyson stabilization. For every fixed finite signed spatial bank,

$$U_{K,N}(x_2, x_1)\Psi \longrightarrow U_K(x_2, x_1)\Psi \tag{47}$$

strongly on the complete Fock carrier as $N \to \infty$.

The limit is unitary and compositional. The physical number domain propagates with an earned occupation-moment bound, and the orbit is continuous in the corresponding number graph. The closed finite-bank Hamiltonian therefore generates the global strong physical equation on its declared domain.

10 The real finite-bank global Schrödinger law

For every state in the earned number domain and every strictly interior owned time,

$$\boxed{\frac{d}{dx}U_K(x, x_0)\Psi = -iH_K^{\mathrm{real}}(x)U_K(x, x_0)\Psi.} \tag{48}$$

The proof uses the literal real core, graph continuity, occupation propagation, and closed-Hamiltonian extension; no charged evolution theorem is imported as real-field authority.

11 Interval-independent owned evolution

The compact lower endpoint $\delta > 0$ belongs to the construction machinery. Interval coherence removes that choice from the physical object. Propagators constructed with two positive lower bounds agree whenever their source and observation times lie in the common interval.

For each finite signed bank this gives a single owned evolution

$$U_K^{\mathrm{owned}}(x_2, x_1), \qquad 0 < x_1, x_2 \le 1, \tag{49}$$

with identity, cocycle, strong continuity, number-domain propagation, and the actual closed-Hamiltonian Schrödinger law.[15]

The strictly positive interval is the physical-time theorem surface throughout this paper.

12 The full signed-mode free Hamiltonian

The complete signed Fourier carrier uses one Fock mode for each enumerated spatial momentum. The frequency-normalized free occupation energy is

$$E_0(\mathbf{n}) = \sum_{m=0}^{\infty} \omega_m n_m, \tag{50}$$

[15] *Formal counterpart:* `QGC12.Quantum.realTemporalOwnedEvolution_actual_Schrodinger`.

and the maximal diagonal free operator acts by

$$H_0\Psi(\mathbf{n}) = E_0(\mathbf{n})\Psi(\mathbf{n}). \tag{51}$$

Its exact domain is

$$D(H_0) = \left\{\Psi : E_0(\mathbf{n})\Psi(\mathbf{n}) \in \ell^2\right\}, \tag{52}$$

and H_0 is self-adjoint.[16]

The finite-support occupation core is an actual graph core for this maximal operator.

13 Pair tails and evolved Cauchy mixing

The full signed theory distinguishes two kernels that have different analytic roles.

The fixed-Cauchy instantaneous action coefficient is computed directly from the potential shift and the initial frequency. Per signed component it has the form

$$B_n^{\text{fix}}(x) = \frac{V_M(1) - V_M(x)}{4\omega_n}. \tag{53}$$

Its unweighted square sum is finite:

$$\sum_n |B_n^{\text{fix}}(x)|^2 < \infty. \tag{54}$$

For a nonzero potential shift, the frequency-weighted square sum diverges:

$$\sum_n |\omega_n B_n^{\text{fix}}(x)|^2 = \infty. \tag{55}$$

[17]

This identifies the instantaneous fixed-frame pair coefficient as an ℓ^2 squeezing kernel while separating it from the stronger energy-domain mixing needed for physical transport.

The actual evolved Cauchy solution determines Bogoliubov coefficients $\alpha_n(x)$ and $\beta_n(x)$. The original real action and exact scalar ODE give

$$|\alpha_n(x)|^2 - |\beta_n(x)|^2 = 1, \tag{56}$$

and the stronger weighted summability

$$\boxed{\sum_n |\omega_n \beta_n(x)|^2 < \infty.} \tag{57}$$

[18]

The signed reindexing retains both nonzero orientations and the single zero mode. The zero mode requires no fictitious distinct partner.

[16] *Formal counterpart:* `QGC12.Quantum.realTemporalFreeHamiltonian_actual_selfAdjoint`.
[17] *Formal counterpart:* `QGC12.Quantum.realTemporalFixedFramePairCoefficient_actual_not_weighted_summable`.
[18] *Formal counterpart:* `QGC12.Quantum.realTemporalBogoliubovBeta_actual_weighted_normSq_summable`.

14 The real all-mode canonical frame

The complete real canonical frame is built from two native structures: a single-oscillator squeeze for the self-partner zero mode and opposite-sign pair transformations for the nonzero signed modes. Their composition acts on the same single-species Fock carrier as the free Hamiltonian.

For each owned positive time, the resulting unitary $W(x)$ implements the same-family Bogoliubov transformation. On the maximal annihilation graph,

$$W(x)a_n W(x)^{-1} = \alpha_n(x)a_n + \beta_n(x)a_{\bar{n}}^\dagger, \tag{58}$$

with the corresponding creation relation obtained by the actual adjoint pairing.[19]

The frame and its inverse preserve the original free-energy domain $D(H_0)$. The unitary family is strongly continuous, its two-time transport satisfies an exact cocycle, and its number-graph and energy-graph actions are earned on the same carrier.

The literal signed-Fourier field and momentum transform with this frame. For each finite test bandwidth their transported initial-field values equal the physical-time field and momentum evaluated in the evolved carrier. The statement is an equality of the actual operators on $D(H_0)$, rather than a formal Bogoliubov notation.

15 The independent all-mode real action Hamiltonian

The physical generator is constructed independently from the original real action, rather than defined by conjugating H_0. The signed action coefficients produce actual all-mode diagonal and pair sums on the algebraic core. Their common graph approximation defines the minimal real action operator

$$H^{\mathrm{real}}_{\mathrm{min}}(x). \tag{59}$$

Its closure is the full action Hamiltonian

$$H_{\mathrm{real}}(x) = \overline{H^{\mathrm{real}}_{\mathrm{min}}(x)}. \tag{60}$$

The closure is self-adjoint for every owned positive time:

$$\boxed{H_{\mathrm{real}}(x)^* = H_{\mathrm{real}}(x).} \tag{61}$$

[20]

Its maximal domain is independent of physical time and agrees exactly with the original free-energy domain:

$$\boxed{D(H_{\mathrm{real}}(x)) = D(H_0).} \tag{62}$$

At the normalized initial slice the operator itself agrees with the original free Hamiltonian:

$$\boxed{H_{\mathrm{real}}(1) = H_0.} \tag{63}$$

[19] *Formal counterpart:* `QGC12.Quantum.realTemporalPhysicalOwnedUnitary_actual_annihilate_graph`.
[20] *Formal counterpart:* `QGC12.Quantum.realTemporalFullActionHamiltonian_actual_selfAdjoint`.

[21]

These identities provide the common energy domain on which the physical clock is differentiated.

16 The original-action physical clock

The real canonical clock combines the zero-mode squeeze, the nonzero opposite-sign pair factors, the spectral phase, and the scalar correction computed from the same action. The scalar phase is integrated from the actual finite signed-mode defect and has unit norm. Its derivative cancels the canonical scalar contribution in the full generator.

For $0 < x_0, x \leq 1$, define the owned physical transport

$$U_{\mathrm{real}}(x, x_0). \tag{64}$$

It preserves norm,

$$\|U_{\mathrm{real}}(x_2, x_1)\Psi\| = \|\Psi\|, \tag{65}$$

obeys the exact cocycle

$$U_{\mathrm{real}}(x_3, x_2)U_{\mathrm{real}}(x_2, x_1) = U_{\mathrm{real}}(x_3, x_1), \tag{66}$$

and is jointly strongly continuous in both time arguments.

The original H_0 domain propagates forward and backward. For every source $\Psi \in D(H_0)$ and strictly interior owned time,

$$\boxed{\frac{d}{dx}U_{\mathrm{real}}(x, x_0)\Psi = -iH_{\mathrm{real}}(x)U_{\mathrm{real}}(x, x_0)\Psi.} \tag{67}$$

[22]

The construction first uses compact positive intervals for analytic estimates. An explicit interval-independence theorem proves equality of the transports on overlaps, leaving a single physical object on $(0, 1]$.[23]

The interaction picture is also earned from the original free group and the independently constructed full action. Its literal interaction derivative assembles exactly back to the physical clock. A separate signed spatial squeezing-kernel cutoff converges strongly on the same full Fock carrier to this unitary; that limit retains the spectral and central phases and therefore serves as an all-mode spatial approximation of the physical clock.

17 Literal real-field transport

The physical unitary transports the same Hermitian field and momentum that were derived from the original real action. For each finite signed Fourier test bank and each $\Psi \in D(H_0)$, the

[21] *Formal counterpart:*
`QGC12.Quantum.realTemporalFullActionHamiltonian_actual_initial_eq_original_free`.
[22] *Formal counterpart:*
`QGC12.Quantum.realTemporalPhysicalTransportClockOrbit_actual_same_action_hasDerivAt`.
[23] *Formal counterpart:*
`QGC12.Quantum.realTemporalPhysicalCompactTransport_actual_interval_independent`.

initial canonical field evaluated on the transported free state agrees with the unitary image of the physical-time field. The analogous statement holds for momentum.[24] [25]

Thus the real physical clock is connected simultaneously to the independent full action generator and to the literal field observables. The Bogoliubov relation in Equation (58) is therefore an implemented same-family squeezing law for the same real scalar field.

18 Original-action real-scalar capstone

The final physical authority theorem packages the complete provenance chain. For every positive Schwarzschild mass, the original real geometric action supplies:

$$\boxed{\begin{gathered}\text{real Schwarzschild scalar action} \Longrightarrow \text{boundary-completed real Cauchy action} \\ \Longrightarrow \text{Friedrichs terminal selection and complete signed spatial carrier} \\ \Longrightarrow [\widehat{\phi}, \widehat{\pi}] = iI, \quad \widehat{\phi} = \widehat{\phi}^{\dagger} \\ \Longrightarrow H_{\mathrm{real}}(x) = H_{\mathrm{real}}(x)^{*}, \quad D(H_{\mathrm{real}}(x)) = D(H_0) \\ \Longrightarrow U_{\mathrm{real}}(x_2, x_1) \text{ with norm, cocycle, continuity, and Schrödinger law} \\ \Longrightarrow \text{same-family squeezing and literal field/momentum transport.}\end{gathered}} \tag{68}$$

[26]

The theorem also records the exact initial identity $H_{\mathrm{real}}(1) = H_0$, weighted Bogoliubov mixing, full signed Fourier completeness, and explicit compact-interval independence. The physical lane therefore quantizes the same single real scalar field whose action generated the Paper 1 radial problem.

19 Complex charged scalar as a separate extension

The repository also retains the earlier complex charged scalar theory. Its particle and antiparticle labels are independent charge species, and its global phase current yields the corresponding number difference. This structure is now presented as an extension of the primary real theory.

The complex scalar density is proved to split into two real scalar densities:

$$\boxed{S_{\mathbb{C}}[\Phi] = S_{\mathbb{R}}[\phi_1] + S_{\mathbb{R}}[\phi_2], \qquad \Phi = \phi_1 + i\phi_2.} \tag{69}$$

[27]

[24] *Formal counterpart:* `QGC12.Quantum.realTemporalPhysicalOwnedUnitary_actual_Fourier_test_bank_field_transport`.

[25] *Formal counterpart:* `QGC12.Quantum.realTemporalPhysicalOwnedUnitary_actual_Fourier_test_bank_momentum_transport`.

[26] *Formal counterpart:* `QGC12.Quantum.Q12_realScalarPhysicalQuantumTheory_owned_by_originalAction`.

[27] *Formal counterpart:* `QGC12.Matter.interiorAreaLagrangianDensity_actual_two_real_actions`.

The charged Fock carrier and its canonical/Bogoliubov results therefore describe the natural complex extension with genuine particle–antiparticle structure. The single-real-scalar physical clock established above is the primary continuation of Paper 1.

The earlier charged analysis also contains a computed unit-amplitude polarization obstruction and a frequency-normalized all-mode dynamics. Those theorems remain valid on their declared complex carrier and now serve as an independently useful comparison: the pair and squeezing phenomena persist in the real theory without requiring an internal charge degree of freedom.

20 The Friedrichs quantum-ownership theorem

The continuum spectral construction ties directly to the terminal selection theorem.

Let L be a Hermitian terminal trace law satisfying the finite full-potential form criterion

$$\mathcal{Q}_M[u] < \infty \qquad \forall u \in D(K_L). \tag{70}$$

Paper 1 proves

$$K_L = K_{\mathrm{F}}. \tag{71}$$

The present capstone theorem propagates this equality through the spectral quantum construction:

$$\boxed{K_L = K_{\mathrm{F}}, \qquad \omega_L = \omega_{\mathrm{F}}, \qquad H_L = H_{\mathrm{F}}, \qquad U_L(t) = U_{\mathrm{F}}(t).} \tag{72}$$

[28]

Equivalently,

$$\boxed{\begin{array}{c}\text{finite full-potential form} \Longrightarrow \text{Friedrichs operator} \Longrightarrow \text{selected frequency} \\ \Longrightarrow \text{selected Fock Hamiltonian} \Longrightarrow \text{selected continuum evolution.}\end{array}} \tag{73}$$

This is the spectral ownership statement of the paper. The real physical Cauchy capstone in Section 18 is complementary: it follows a second dependency path from the same original real scalar action into the physical-time Hamiltonian and unitary clock.

21 Two complementary dynamical structures

The completed theory contains two unitary structures with distinct geometric roles.

The Friedrichs spectral evolution is

$$U_{\mathrm{F}}(t) = e^{-it\, d\Gamma(\sqrt{K_{\mathrm{F}}})}. \tag{74}$$

It is generated by the positive frequency of the selected radial operator and acts on the continuum bosonic Fock space.

[28] *Formal counterpart:* `QGC12.Quantum.finiteFormSelector_owns_entireQuantumTheory`.

The real physical Cauchy evolution is

$$U_{\mathrm{real}}(x_2, x_1), \qquad 0 < x_1, x_2 \leq 1. \tag{75}$$

It is generated by the independently constructed same-action self-adjoint Hamiltonian $H_{\mathrm{real}}(x)$ on the complete signed Fourier carrier of the compact spatial circle. It preserves the original free-energy domain and transports the literal Hermitian field and momentum.

The spectral group parameter and the physical Schwarzschild clock retain their distinct geometric meanings. Their common provenance lies in the original real scalar action and the Friedrichs selection it supports.

22 Relation to quantum field theory in curved spacetime

Quantum field theory on curved spacetime organizes quantum matter over a classical geometric carrier and emphasizes the relation among the classical field equation, symplectic structure, state space, and local observables.[6, 5, 7, 9]

Contemporary mathematical developments continue to sharpen the state and microlocal layers of this framework, including high-energy regularity, analytic state conditions, and local algebraic structure on curved backgrounds.[10]

The present paper contributes at the operator and dynamical layers of that program. The selected Schwarzschild operator determines a continuum bosonic spectral theory. Independently, the same original real scalar action determines a compact-spatial physical Cauchy carrier, a Hermitian field and momentum, a same-action canonical commutator, a self-adjoint all-mode Hamiltonian, and its physical unitary evolution.

The complex structure used to form the positive-frequency one-particle Hilbert space belongs to the quantization of the real linear solution space. The physical field itself remains Hermitian. A genuinely complex charged scalar appears only in the separate extension of Section 19.

The reference vector Ω is the zero-particle vacuum of the selected Fock representation. The present theorem surface establishes its algebraic and dynamical role. Microlocal state selection, Hadamard regularity, and local stress renormalization add further geometric requirements and belong to the subsequent matter/source layer.

23 Scope and next theorem surface

The theorem surface of this paper ends with quantum dynamics.

It establishes:

- the Friedrichs-selected positive frequency, continuum bosonic Fock Hamiltonian, and strongly continuous spectral evolution;
- the complete signed Fourier basis of the compact physical spatial circle;

- the original real Cauchy action, its boundary completion, Legendre momentum, symplectic form, and one-half energy;
- the action-derived real canonical normalization and exact commutator $[\widehat{\phi}, \widehat{\pi}] = iI$;
- a literal Hermitian real field using one oscillator family;
- finite signed-bank integrated action energy with full same-family pair terms and the initial law $\omega_n(N_n + I/2)$;
- self-adjoint nonnegative finite-bank Hamiltonians and finite-window unitary Schrödinger evolution;
- strong occupation-window removal, number-domain propagation, number-graph continuity, and finite-bank global Schrödinger dynamics;
- interval-independent owned positive-time finite-bank evolution;
- the self-adjoint maximal all-mode free Hamiltonian and its exact occupation-energy domain;
- square summability of the fixed-frame pair kernel together with the proved weighted divergence that separates it from the physical evolved mixing kernel;
- frequency-weighted square summability and canonical identity of the action-extracted real Bogoliubov coefficients;
- a same-family all-mode canonical frame, including the self-partner zero oscillator and opposite-sign nonzero modes;
- preservation of the original free-energy domain by the canonical frame and its inverse;
- an independently constructed all-mode real action Hamiltonian whose closure is self-adjoint on the unchanged H_0 domain;
- the exact initial operator identity $H_{\mathrm{real}}(1) = H_0$;
- the norm-preserving, jointly strongly continuous, cocyclic and interval-independent real physical clock on $(0, 1]$;
- the strong same-action Schrödinger law on the original H_0 domain;
- literal field and momentum transport under the same physical unitary;
- exact ownership of the continuum spectral quantum theory by the finite-form/Friedrichs selector; and
- explicit separation of the complex charged scalar as a two-real-field extension.

The compact spatial circle remains part of the physical theorem surface. Exhaustion of all signed Fourier modes removes the finite mode bank while preserving that compactification. The physical clock is owned for strictly positive time $0 < x \leq 1$.

The next paper begins with local quantum matter: regular finite-particle continuum states, the full coherent quadratic including anomalous pair terms, arbitrary-regulator removal, action-owned vacuum-relative spacetime stress, source conservation, and nontrivial many-particle source families. Hadamard renormalization and decompactification remain separately identifiable problems.

The subsequent geometric paper begins from the sourced Einstein response.

The series therefore follows

$$\boxed{\text{terminal law} \longrightarrow \text{quantum dynamics} \longrightarrow \text{quantum matter} \longrightarrow \text{geometry.}} \tag{76}$$

24 Conclusion

The Friedrichs selection of the Schwarzschild scalar terminal law propagates into a continuum spectral quantum theory, and the same original real scalar action independently supports a complete physical Cauchy quantum dynamics.

The selected nonnegative operator produces

$$\omega_{\mathrm{F}} = K_{\mathrm{F}}^{1/2}, \tag{77}$$

whose bosonic second quantization

$$H_{\mathrm{F}} = d\Gamma(\omega_{\mathrm{F}}) \tag{78}$$

is self-adjoint and nonnegative and generates a strongly continuous unitary evolution. The finite-form ownership theorem identifies this complete spectral chain with the terminal selection established in Paper 1.

The physical lane now retains literal continuity with the same single real field. The compact spatial circle carries every signed Fourier mode. The original real action fixes the $1/\sqrt{2}$ canonical normalization, the same-family commutator

$$[\widehat{\phi}, \widehat{\pi}] = iI, \tag{79}$$

and the Hermitian field. Its initial oscillator energy is

$$H_n(1) = \omega_n \left(N_n + \frac{1}{2} I \right), \tag{80}$$

and the full finite-bank action retains the same-family pair terms away from the initial diagonal slice.

The all-mode construction separates the instantaneous action pair tail from the evolved Cauchy mixing. The latter obeys

$$\sum_n |\omega_n \beta_n(x)|^2 < \infty \tag{81}$$

and supports an implemented same-family Bogoliubov frame on the original Fock carrier.

Most importantly, the independently constructed real action Hamiltonian satisfies

$$H_{\mathrm{real}}(x) = H_{\mathrm{real}}(x)^*, \qquad D(H_{\mathrm{real}}(x)) = D(H_0), \qquad H_{\mathrm{real}}(1) = H_0. \tag{82}$$

Its owned two-time unitary satisfies

$$\boxed{\frac{d}{dx} U_{\mathrm{real}}(x, x_0)\Psi = -iH_{\mathrm{real}}(x) U_{\mathrm{real}}(x, x_0)\Psi, \qquad \Psi \in D(H_0).} \tag{83}$$

The same unitary transports the literal signed-Fourier field and momentum and realizes the real scalar's same-family squeezing law.

The physical theory therefore follows the dependency chain

$$\boxed{S_{\mathbb{R}}[\phi] \Longrightarrow K_{\mathrm{F}} \Longrightarrow \widehat{\phi}_{\mathbb{R}} \Longrightarrow H_{\mathrm{real}}(x) \Longrightarrow U_{\mathrm{real}}(x_2, x_1).} \tag{84}$$

The complex charged theory remains a separately identified extension whose action is the sum of two real scalar actions. Local quantum matter is the next theorem surface of the series.

A Lean theorem map

The paper-level theorem package is machine-checked in the Concordia Lean 4 development. The principal formal surfaces are:

Mathematical surface	Formal source
Friedrichs-selected continuum frequency and Fock ownership	`QGC12/Quantum/FriedrichsQuantumOwnership.lean`
Continuum Fock Hamiltonian, evolution, KG dynamics, and smeared CCR	`QGC12/Quantum/NoLogContinuumFockHamiltonian.lean`, `QGC12/Quantum/NoLogContinuumFockEvolution.lean`, `QGC12/Quantum/NoLogContinuumKGDynamics.lean`, `QGC12/Quantum/NoLogContinuumSmearedCCR.lean`
Complete signed Fourier physical carrier	`QGC12/Quantum/RealTemporalFourierCarrier.lean`, `QGC12/Quantum/RealTemporalModes.lean`
Original real Cauchy action and canonical normalization	`QGC12/Matter/RealInteriorTemporalAction.lean`, `QGC12/Quantum/RealTemporalCanonicalNormalization.lean`, `QGC12/Quantum/RealTemporalKGCarrier.lean`
Hermitian one-family field and real oscillator quadratic	`Tower/Quantum/MultimodeRealScalarField.lean`, `Tower/Quantum/MultimodeRealQuadratic.lean`, `QGC12/Quantum/RealTemporalCore.lean`
Frequency-normalized real oscillator and signed-bank action energy	`QGC12/Quantum/RealFrequencyNormalizedTemporalCore.lean`, `QGC12/Quantum/RealTemporalFourierHamiltonian.lean`, `QGC12/Quantum/RealTemporalFourierEnergy.lean`
Finite-bank closed real Hamiltonian and occupation windows	`QGC12/Quantum/RealTemporalClosedHamiltonian.lean`, `QGC12/Quantum/RealTemporalOccupationEvolution.lean`
Infinite-occupation, number propagation, graph continuity, and owned finite-bank dynamics	`QGC12/Quantum/RealTemporalNumberPropagation.lean`, `QGC12/Quantum/RealTemporalGlobalSchrodinger.lean`, `QGC12/Quantum/RealTemporalOwnedSchrodinger.lean`
Real minimal Hamiltonian closure and maximal free-energy operator	`QGC12/Quantum/RealTemporalMinimalHamiltonian.lean`, `QGC12/Quantum/RealFrequencyNormalizedFreeDomain.lean`
Fixed-frame pair adjudication and evolved real mixing kernel	`QGC12/Quantum/RealFrequencyNormalizedInitialCanonicalCoefficients.lean`, `QGC12/Quantum/RealFrequencyNormalizedPairKernel.lean`, `QGC12/Quantum/RealFrequencyNormalizedBogoliubovKernel.lean`
Real zero-mode and signed-pair canonical frames	`QGC12/Quantum/RealTemporalZeroCanonicalFrame.lean`, `QGC12/Quantum/RealTemporalSpatialPairCanonicalFrame.lean`, `QGC12/Quantum/RealTemporalAllModeCanonicalFrame.lean`
All-mode real action clock and independent self-adjoint action closure	`QGC12/Quantum/RealTemporalAllModeCanonicalActionClock.lean`, `QGC12/Quantum/RealTemporalOriginalProductActionGraph.lean`, `QGC12/Quantum/RealTemporalPhysicalClockAction.lean`
Owned real physical clock and literal Bogoliubov/field transport	`QGC12/Quantum/RealTemporalPhysicalOwnedFamily.lean`, `QGC12/Quantum/RealTemporalPhysicalBogoliubovTransport.lean`
Interaction-clock, interval-independence, spatial-kernel limit, and initial H_0 identity	`QGC12/Quantum/RealTemporalPhysicalInteractionClock.lean`, `QGC12/Quantum/RealTemporalPhysicalCompactBinding.lean`, `QGC12/Quantum/RealTemporalPhysicalSpatialKernelLimit.lean`, `QGC12/Quantum/RealTemporalPhysicalInitialHamiltonian.lean`
Original-action real-scalar capstone	`QGC12/Quantum/RealScalarPhysicalTheory.lean`

Mathematical surface	Formal source
Paper-level real-scalar and canonical audits	`QGC12RealScalarPhysicalAudit.lean`, `QGC12CanonicalFrameAudit.lean`, `QGC12Regression.lean`

B Formal verification and authority boundary

The manuscript consumes the terminal-selection/Friedrichs theorem from Paper 1 and develops two downstream quantum-dynamical lanes: the continuum spectral theory and the physical Cauchy quantization of the same single real scalar.

The substantive Paper 2 authority surface is frozen at the QGC12 revision

`43352fafe99473d570bd17297f5bcb53a383e6be`.

A subsequent commit, `9d8bc88728c7067cd2d69b42f781e7bfae924835`, performs whitespace cleanup on the verified real-scalar completion modules without changing the theorem surface.

The revision contains the real-scalar physical-time chain through M343. Its principal authority groups are M232–M236 for the signed carrier, original real action, canonical normalization, Hermitian field, and integrated energy; M237–M246 for finite-bank closure, occupation-limit dynamics, owned finite-bank Schrödinger evolution, and minimal graph closure; M247–M289 for all-mode free-domain control, pair-kernel adjudication, real canonical frames, clock derivatives, all-mode action sums, closure, and self-adjointness; and M290–M343 for the action-bound phase, original H_0 physical clock, literal field/momentum transport, interaction binding, interval independence, spatial-kernel limit, and final real-scalar ownership capstone.

The older charged milestones remain valid on the complex scalar carrier and are retained as a separate extension. Their ‘Bool‘ index has charge semantics. In the real lane, nonzero opposite labels encode spatial sign while zero is self-conjugate; the real physical Fock carrier uses one oscillator family.

The formal boundary has six structural features.

First, the Friedrichs spectral/Fock carrier and the physical Cauchy-time carrier retain distinct definitions and evolution parameters. Their common provenance is the original real scalar action; equality is asserted only where a theorem proves equality.

Second, the physical spatial carrier is the complete signed Fourier basis of the retained compact circle. Finite mode-bank removal exhausts this basis and preserves the compact spatial regulator.

Third, the real Cauchy normalization is derived from the original action. The $1/\sqrt{2}$ rescaling converts the historical unit-flux convention into the one-real-field canonical commutator iI.

Fourth, the physical generator is independently constructed from the real action. Its closure is self-adjoint, has the exact original H_0 domain at every owned positive time, and agrees with H_0 at the initial slice.

Fifth, the physical unitary is jointly strongly continuous, norm preserving, compositional, interval independent, and satisfies the same-action strong Schrödinger equation on $D(H_0)$. It transports the literal Hermitian field and momentum and realizes the same-family Bogoliubov squeezing law.

Sixth, the complex charged action is explicitly identified as the sum of two real scalar actions. The charged theory is therefore a separately owned extension rather than the physical interpretation of the primary Paper 1 field.

The quantum-matter milestones M188–M191 and M205–M211 remain outside this manuscript and belong to Paper 3.

C Statement and status map

Statement	Status	Role
K_{F} is the finite-form/no-log realization	imported proved input	Paper 1 selector
$\omega_{\mathrm{F}} = K_{\mathrm{F}}^{1/2}$ and the continuum Fock Hamiltonian are owned by the selector	proved	spectral quantum dynamics
Complete signed Fourier basis spans the compact physical spatial carrier	proved	real Cauchy spatial completeness
Original real Cauchy action and boundary completion are derived from the Paper 1 field action	proved	same-field physical provenance
Real action normalization gives $[\widehat{\phi}, \widehat{\pi}] = iI$	proved	canonical real-field normalization
Physical field is Hermitian on one oscillator family	proved	single real scalar quantization
Finite signed-bank action energy retains same-family pair terms	proved	literal Cauchy Hamiltonian
Initial oscillator law is $\omega_n(N_n + I/2)$	proved	normalized real initial energy
Finite-bank represented Hamiltonian is self-adjoint and nonnegative	proved	finite physical generator
Occupation-window propagators satisfy the literal Schrödinger equation and converge strongly	proved	arbitrary occupation
Number domain propagates and the finite-bank global closed-generator law holds	proved	domain control
Finite-bank positive-time evolution is interval independent	proved	owned finite-bank clock
All-mode H_0 is self-adjoint on the exact occupation-energy domain	proved	reference free-energy operator
Fixed-frame action pair kernel is ℓ^2 and can fail the frequency-weighted condition	proved	pair-tail adjudication
Evolved real Bogoliubov β is frequency-weighted square summable	proved	physical mixing
All-mode real canonical frame implements same-family squeezing	proved	Bogoliubov Fock realization
Canonical frame and inverse preserve $D(H_0)$	proved	energy-domain standing
Independent full real action Hamiltonian is self-adjoint	proved	physical generator

Statement	Status	Role
$D(H_{\mathrm{real}}(x)) = D(H_0)$ for owned positive time	proved	fixed maximal domain
$H_{\mathrm{real}}(1) = H_0$ as operators	proved	initial normalization
Owned real physical transport is norm preserving, cocyclic and jointly strongly continuous	proved	physical unitary clock
Strong physical Schrödinger law holds on $D(H_0)$	proved	generator law
Literal signed-Fourier field and momentum are transported by the same unitary	proved	observable dynamics
Original real action owns the complete physical quantum theory capstone	proved	Paper 1–Paper 2 continuity
Complex charged action equals two real actions	proved	separate charged extension
Spatial decompactification and $x = 0$ continuation	future work	additional limits
Vacuum-relative continuum spacetime stress	subsequent paper	quantum matter/source
Sourced Einstein response and terminal reclassification	subsequent paper	geometry

D Formal development and collaboration

The theorem package is part of the larger Concordia Lean 4 research development. The frozen authority surface above identifies the complete spectral and original-action real physical-time results used by this manuscript while keeping the quantum-matter and gravitational developments logically separate.

The formal source corresponding to the frozen revision is available for technical review upon request.

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