Friedrichs-Selected Schwarzschild Quantum Matter

A machine-checked real-scalar stress theory, physical-clock coherence, and regulator-independent sources

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Abstract

The first two papers in this series establish a continuous program for a single real massless scalar field in the Schwarzschild interior. Paper 1 derives its action-owned radial operator and selects the no-log Friedrichs realization. Paper 2 carries the same field into two quantum-dynamical lanes: a continuum Fock theory generated by $K_{\mathrm{F}}^{1/2}$ and a physical Cauchy theory with one oscillator family, complete signed spatial modes on the retained compact circle, an action-derived canonical commutator, a self-adjoint all-mode Hamiltonian, and a norm-preserving physical clock.

The present paper constructs the local quantum matter carried by that theory. Starting from the original four-dimensional real scalar action, we derive its Hilbert stress, real bilinear polarization, exact Hermitian complexification, and norm-weighted reference subtraction. These results define a carrier-independent stress law with single-real-field provenance across both quantum realizations.

On the physical Cauchy Fock carrier, finite signed mode banks define literal real-field quadratics containing all four ladder sectors. The local Fock-valued field derivative equals the geometric signed-mode gradient entering the metric stress. The selected original vacuum supplies the exact vacuum-relative contraction, and joint transport of state and reference vacuum yields an exact state–field covariance identity under the physical clock.

The local stress retains normal and anomalous sectors. The real Bogoliubov law

$$a_n(x) = \alpha_n(x)a_n(1) + \beta_n(x)a_{\bar{n}}^{\dagger}(1)$$

induces a local mixing jet that is proved equal to the geometric mode gradient. The resulting vacuum-relative stress is an explicit normal/anomalous polynomial in the physical mixing coefficients. The formal theory proves nonzero anomalous coherence at an owned positive time and constructs a finite spherical state with strictly positive anomalous radial stress throughout the owned interior.

Pointwise all-mode matter is established on a dense transported regular class. Each positive physical time carries the unitary image of the finite occupation core; these fibers lie in the original free-energy domain and are preserved by two-time transport. Arbitrary directed exhaustion of conjugation-closed finite banks converges on this class to an intrinsic all-mode vacuum-relative stress tensor, and the on-shell real mode equations together with the cross-kernel divergence identities prove its covariant conservation. An independent Friedrichs continuum realization supplies a dense regular finite-particle class with the complete normal/anomalous word polynomial, arbitrary directed regulator removal, presentation independence, action ownership, conservation, and exact number-state scaling. Both quantum carriers instantiate the same original-real-scalar stress law.

For the spherical physical sector, finite zero-mode states generate explicit quantum sources, including number states and finite particle-number superpositions with anomalous coherence. Their stress produces a genuine coupled linear Einstein response. At the anchor of a separately constructed nonlinear Einstein–scalar branch, the full classical and quantum source tensors agree and the factor-two normalization is derived explicitly. Across the owned slab, the nonlinear source carries the conservation law of its changed geometry and the original quantum sources carry the Schwarzschild conservation law; a theorem proves the separation of these source classes. The resulting Paper 3 boundary completes original-geometry quantum matter and its linear gravitational response and opens the changed nonlinear geometry as the next quantum-theory program.

1 Introduction

A quantum field becomes matter through controlled local observables. Self-adjointness supplies lawful evolution; a gravitational source adds local derivatives of the field, quadratic products, a reference prescription, regulator control, covariant conservation, and a proved relation to the metric variation of the original action.

The first paper in this series, *Critical Schwarzschild Scalar Dynamics*, starts from the spherically symmetric real massless scalar action in regular ingoing Schwarzschild coordinates and derives the exact radial operator. Its curvature endpoint is Hardy-critical, the full operator has deficiency indices $(1,1)$, and the self-adjoint terminal laws form a genuine $U(1)$ family. Finite full-potential ground-state form energy selects the no-log law, and completion of the compact-core form identifies that realization with the Friedrichs extension

$$K_{\text{no-log}} = K_{\mathrm{F}}. \tag{1}$$

The first paper therefore determines the selected terminal law of the original real scalar field.[1]

The second paper, *Friedrichs-Selected Schwarzschild Quantum Dynamics*, propagates that selection through a continuum spectral Fock theory and separately reconstructs the physical Cauchy quantization of the same single real scalar. Its principal physical chain is

$$\boxed{S_{\mathbb{R}}[\phi] \longrightarrow (\widehat{\phi}, \widehat{\pi}) \longrightarrow H_{\text{real}}(x) \longrightarrow U_{\text{real}}(x_2, x_1).} \tag{2}$$

The physical field is Hermitian, its canonical normalization is derived from the original action, the all-mode Hamiltonian is self-adjoint on the original H_0 domain, and the physical unitary is norm preserving, cocyclic, jointly strongly continuous, and governed by the same-action Schrödinger equation.[2]

The present paper asks the next question:

$$\boxed{\text{How does that same real quantum field become local, conserved spacetime matter?}} \tag{3}$$

The answer is carried by the full local quadratic field observable. The local stress of a real quantum field contains normal terms and pair-coherent terms, and the pair-coherent sector is precisely where the Bogoliubov dynamics of Paper 2 enters the matter observable.

The action-level starting point is the four-dimensional real scalar density

$$\mathcal{L}_{\mathbb{R}}(g, J) = -\frac{1}{2}\sqrt{|g|}\, g^{\mu\nu} J_\mu J_\nu, \tag{4}$$

with real covector J. Its metric variation produces

$$T^{\mathbb{R}}_{\mu\nu}[J] = J_\mu J_\nu - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} J_\alpha J_\beta. \tag{5}$$

Polarizing this real quadratic form and then complexifying the one-particle amplitudes gives the Hermitian pair kernel used by both quantum-matter constructions below. The complex Hilbert carrier therefore provides the positive-frequency polarization coordinates of one real classical field, with a single oscillator-family provenance.

The physical Cauchy matter lane is built on the actual Paper 2 Fock carrier. For a finite signed bank B, the local real field has the form

$$\widehat{\Phi}_B(p) = \sum_{n\in B} \left(\Phi_n(p) a_n + \overline{\Phi_n(p)} a_n^\dagger \right), \tag{6}$$

and its differentiated products retain all four ladder sectors. The vacuum-relative stress is computed directly from the selected original vacuum and its exact contraction.

The dynamics enters through the actual real Bogoliubov law. If $\bar{n}$ denotes the conjugate signed spatial mode, then

$$a_n(x) = \alpha_n(x) a_n(1) + \beta_n(x) a_{\bar{n}}^\dagger(1). \tag{7}$$

The local stress consequently separates into normal and anomalous parts,

$$\boxed{\Delta T_{\mu\nu}(x,p) = \Delta T_{\mu\nu}^{\text{normal}}(x,p) + \Delta T_{\mu\nu}^{\text{anomalous}}(x,p).} \tag{8}$$

The anomalous part remains fully present. The formal theory proves an actual nonzero mixing coefficient and an explicit local nonzero anomalous stress witness.

Locality requires a domain. The all-mode statement is therefore made on a time-indexed dense regular class

$$\mathcal{R}_x := \operatorname{Ran}\left(U_{\text{real}}(x,1) \circ \iota_{\text{core}}\right), \tag{9}$$

where ι_{core} is the finite occupation-core inclusion. Every $\mathcal{R}_x$ is dense, every vector in it lies in the original H_0 domain, and

$$U_{\text{real}}(x_2,x_1)\mathcal{R}_{x_1} \subseteq \mathcal{R}_{x_2}. \tag{10}$$

On this class, arbitrary directed finite-bank exhaustion yields the intrinsic all-mode stress.

A second matter lane is independently defined on the continuum Friedrichs Fock representation. It uses the local resolvent regularity developed after the spectral construction of Paper 2 and supports a dense regular finite-particle class with arbitrary directed regulator removal. The two Fock carriers retain their independent constructions, and their common result is structural:

$$\boxed{\begin{array}{c}\text{physical Cauchy matter} \quad \text{and} \quad \text{Friedrichs continuum matter} \\ \text{instantiate the same original-real-scalar stress law.}\end{array}} \tag{11}$$

For finite spherical zero-mode states, the physical source enters a genuine Einstein tangent. If σ_Ψ is the computed source factor of a spherical core state and g_ε is the normalized response curve, then

$$\left.\frac{d}{d\varepsilon} G_{\mu\nu}\left[g_{\kappa\sigma_\Psi\varepsilon}\right]\right|_{\varepsilon=0} = \kappa\, \Delta T_{\mu\nu}[\Psi]. \tag{12}$$

This is an actual linearized gravitational response to the quantum source, with authority at the Einstein derivative of the smooth response family.

A separately constructed nonlinear Einstein–real-scalar branch provides a useful endpoint for comparison. At its anchor, its full classical relative source agrees with the normalized quantum source. Across the owned slab, the changed-geometry source follows its own connection and the original quantum sources follow the Schwarzschild connection. The proved divergence comparison separates these source classes on the owned slab. This theorem gives the boundary a geometric origin and completes the original-geometry source adjudication.

In this series, *ownership* continues to denote proved dependency provenance. A stress tensor is owned by the original action when its local kernel, subtraction, metric variation, and conservation are derived through the corresponding formal chain from the same upstream construction. Equality between carriers and geometries is stated at the exact theorem surfaces that establish it.

The paper is organized as follows. Section 2 states the imported Paper 1–Paper 2 quantum input. Section 3 derives the shared original-real-scalar stress law. Sections 4–5 construct the vacuum-relative physical stress and bind it to the actual clock. Section 6 exposes the normal/anomalous Bogoliubov structure. Sections 7 and 8 remove finite banks on the transported regular class and prove covariant conservation. Section 9 states the independent continuum finite-particle matter theorem. Section 10 identifies the common stress law across the two independently constructed carriers. Section 11 derives finite spherical quantum sources and their coupled Einstein tangent. Section 12 compares these sources with the exact nonlinear classical branch and proves the original-geometry obstruction. The final sections place the result within curved-spacetime QFT and state the next theorem surface.

2 Input from Papers 1 and 2

Fix a positive Schwarzschild mass $M > 0$. Let g_{Schw} denote the four-dimensional Schwarzschild metric in the canonical ingoing chart used throughout the series. Paper 1 supplies the selected radial operator

$$K_{\mathrm{F}} = K_{\text{no-log}}, \qquad K_{\mathrm{F}} = K_{\mathrm{F}}^{*}, \qquad K_{\mathrm{F}} \geq 0, \qquad \ker K_{\mathrm{F}} = \{0\}. \tag{13}$$

The equality with the no-log realization is owned by the full-potential ground-state form and the completed compact-core Friedrichs construction. The endpoint law therefore enters the series through the proved form-selection mechanism.

Paper 2 builds two downstream quantum carriers. The first is the continuum bosonic Fock space over the selected spectral one-particle Hilbert space,

$$\mathcal{F}_{\mathrm{F}} = \Gamma_s(\mathcal{H}_{\mathrm{F}}), \qquad H_{\mathrm{F}} = d\Gamma(K_{\mathrm{F}}^{1/2}). \tag{14}$$

The second is the physical Cauchy Fock carrier

$$\mathcal{F}_{\mathrm{phys}} = \mathcal{F}_s(\ell^2(\mathbb{N})), \tag{15}$$

where $\mathbb{N}$ enumerates the full signed Fourier spectrum of the retained compact spatial circle. One oscillator family carries the real field; the conjugate mode map represents spatial sign, with the zero mode self-conjugate.

The physical Hamiltonian satisfies

$$H_{\mathrm{real}}(x) = H_{\mathrm{real}}(x)^{*}, \qquad D(H_{\mathrm{real}}(x)) = D(H_0), \qquad H_{\mathrm{real}}(1) = H_0, \tag{16}$$

and the owned two-time transport obeys

$$U_{\mathrm{real}}(x_3, x_2)U_{\mathrm{real}}(x_2, x_1) = U_{\mathrm{real}}(x_3, x_1), \tag{17}$$

$$\|U_{\mathrm{real}}(x_2, x_1)\Psi\| = \|\Psi\|, \tag{18}$$

and, for $\Psi \in D(H_0)$,

$$\frac{d}{dx} U_{\mathrm{real}}(x, x_0)\Psi = -iH_{\mathrm{real}}(x)U_{\mathrm{real}}(x, x_0)\Psi. \tag{19}$$

The same unitary transports the literal real field and its momentum.[1]

The present matter theory takes these constructions as proved input. The continuum lane carries its radial spectral parameter, and the physical lane carries the Cauchy time. Their Fock representations retain their independent constructions, with common provenance in the original single-real-scalar action.

3 The shared original-real-scalar stress law

Let g be a smooth Lorentz metric and let J_μ be a real scalar covector. The local four-dimensional action density is

$$\mathcal{L}_{\mathbb{R}}(g, J) = -\frac{1}{2}\,\mathrm{vol}_g\, g^{\mu\nu} J_\mu J_\nu. \tag{20}$$

Here vol_g denotes the positive metric volume density used by the canonical metric carrier. The corresponding real stress tensor is

$$T^{\mathbb{R}}_{\mu\nu}(g, J) = J_\mu J_\nu - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} J_\alpha J_\beta. \tag{21}$$

The formal theory differentiates the actual signature-preserving inverse-metric path and proves that this tensor is the Hilbert derivative of the same density.[2]

The real quadratic stress has an ordered cross kernel

$$T^{\mathrm{ord}}_{\mu\nu}(J, K) = J_\mu K_\nu - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} J_\alpha K_\beta, \tag{22}$$

and its real polarization is the symmetric part

$$T^{\mathrm{pol}}_{\mu\nu}(J, K) = \frac{1}{2}\left(T^{\mathrm{ord}}_{\mu\nu}(J, K) + T^{\mathrm{ord}}_{\mu\nu}(K, J)\right). \tag{23}$$

This identity is derived directly from the polarization of the quadratic action, so the cross stress inherits the same action provenance.[3]

The positive-frequency one-particle Hilbert spaces used in the quantum theory are complex. For complex covectors Z, W, the Hermitian pair kernel is

$$P_{\mu\nu}(Z, W) = \mathrm{Re}\left(\overline{Z_\mu} W_\nu\right). \tag{24}$$

Its metric stress is exactly the sum of the ordered real stresses of the real and imaginary parts:

$$T^{\mathrm{Herm}}_{\mu\nu}(Z, W) = T^{\mathrm{ord}}_{\mu\nu}(\mathrm{Re}\, Z, \mathrm{Re}\, W) + T^{\mathrm{ord}}_{\mu\nu}(\mathrm{Im}\, Z, \mathrm{Im}\, W). \tag{25}$$

[1] *Formal counterpart:* `QGC12.Quantum.Q12_realScalarPhysicalQuantumTheory_owned_by_originalAction`.
[2] *Formal counterpart:* `QGC12.Matter.realScalarMetricActionDensity_actual_Hilbert_variation`.
[3] *Formal counterpart:* `QGC12.Matter.realScalarPolarizedStress_actual_cross_kernel`.

Thus the Hermitian kernel is an exact complexification of the original real action and preserves the single-species real-field interpretation.[4]

Reference subtraction is fixed at the tensor-pair level. If P_Ψ is the state pair, P_Ω the reference pair, and $\|\Psi\|^2 = N_\Psi$, then

$$\Delta T_{\mu\nu} = T^{\mathrm{Herm}}_{\mu\nu}(P_\Psi) - N_\Psi \, T^{\mathrm{Herm}}_{\mu\nu}(P_\Omega). \tag{26}$$

The norm factor is part of the proved relative law.[5]

Finally, for a twice differentiable scalar field φ,

$$\nabla^\mu T_{\mu\nu}[\varphi] = \mathrm{Re}\left(\overline{\Box_g \varphi} \nabla_\nu \varphi\right). \tag{27}$$

On shell, $\Box_g \varphi = 0$, and the stress is covariantly conserved. The paper-level stress law packages the real radial reduction, the Hilbert variation, the real polarization, the exact complexification, the reference subtraction, and this on-shell identity into one carrier-independent authority.[6]

4 Vacuum-relative stress on the physical Cauchy carrier

Let $\mathcal{C}_{\mathrm{fin}}$ denote the finite occupation core of the physical real-scalar Fock space. For a finite signed bank $B \subset \mathbb{N}$, define the bank field

$$\widehat{\Phi}_B(p) = \sum_{n \in B} \left(\Phi_n(p) a_n + \overline{\Phi_n(p)} a_n^\dagger \right). \tag{28}$$

The bank regulates the observable on the original full Fock carrier. The state remains an element of that carrier throughout the construction.

For local index μ, let

$$J_{\mu n}(p) := \nabla_\mu \Phi_n(p). \tag{29}$$

The formal development constructs the Fock-valued finite-bank field and proves directly that its Fréchet derivative is the corresponding finite sum of real-field ladder operators weighted by $J_{\mu n}$.[7]

The literal quadratic field product contains four sectors. Schematically,

$$\widehat{\Phi}_B \widehat{\Phi}_B = a^\dagger a + a a^\dagger + a a + a^\dagger a^\dagger, \tag{30}$$

with the appropriate mode sums and local coefficients. The construction first computes the literal product and then subtracts the actual vacuum contraction, preserving every ladder sector in the relative observable:

$$\widehat{Q}^{\mathrm{rel}}_B = \widehat{Q}^{\mathrm{lit}}_B - \langle \Omega, \widehat{Q}^{\mathrm{lit}}_B \Omega \rangle I. \tag{31}$$

Consequently, for an arbitrary core state Ψ,

$$\langle \Psi, \widehat{Q}^{\mathrm{rel}}_B \Psi \rangle = \langle \Psi, \widehat{Q}^{\mathrm{lit}}_B \Psi \rangle - \|\Psi\|^2 \langle \Omega, \widehat{Q}^{\mathrm{lit}}_B \Omega \rangle. \tag{32}$$

This is the reference prescription used throughout the physical lane.[8]

[4] *Formal counterpart:* `QGC12.Matter.scalarHermitianPairStress_actual_complexification_of_real_action`.
[5] *Formal counterpart:* `QGC12.Matter.realScalarRelativePairStress_actual_reference_subtraction`.
[6] *Formal counterpart:* `QGC12.Matter.realScalarStressLaw_owned_by_original_action`.
[7] *Formal counterpart:* `QGC12.Quantum.realScalarPhysicalCoreBankField_actual_gradient`.
[8] *Formal counterpart:* `QGC12.Q12M553_actual_vacuum_relative`.

Let $P^{(B)}_{\mu\nu}[\Psi]$ denote the resulting vacuum-relative jet pair. The local bank stress is

$$\Delta T^{(B)}_{\mu\nu}[\Psi](p) = T^{\mathrm{Herm}}_{\mu\nu}(P^{(B)}[\Psi](p)). \tag{33}$$

The metric variation of the finite-bank action produces the same Fock expectation. The geometric field derivative, the quantum quadratic, and the original real Hilbert stress therefore meet in one local equality with common action provenance.[9]

5 Physical-clock covariance of the local stress

The physical clock of Paper 2 acts on the same original real-scalar Fock carrier. For a finite bank B, let

$$\mathcal{J}^{(B)}_{\mu}(x;\Psi) \tag{34}$$

be the bank Cauchy jet formed from the physical field and momentum weights at slice x. The proved field/momentum transport gives

$$\mathcal{J}^{(B)}_{\mu}(1;U_{\mathrm{real}}(x,1)\Psi) = U_{\mathrm{real}}(x,1)\mathcal{J}^{(B)}_{\mu}(x;\Psi). \tag{35}$$

[10]

The relative subtraction is covariant under the joint transport of the selected reference vacuum and the state. Let

$$\Omega_x = U_{\mathrm{real}}(x,1)\Omega. \tag{36}$$

Then the local stress satisfies

$$\boxed{\Delta T^{(B)}\left[U_{\mathrm{real}}(x,1)\Psi; U_{\mathrm{real}}(x,1)\Omega\right]_{x=1} = \Delta T^{(B)}[\Psi;\Omega]_x.} \tag{37}$$

[11]

A frozen Schrödinger reference vacuum supplies a second relative observable. If P_{Ω_x} and P_{Ω} denote the transported and frozen vacuum pairs, their exact offset is

$$\Delta T_{\mathrm{frozen}} = \Delta T_{\mathrm{transported}} + \|\Psi\|^2 \left(T[P_{\Omega_x}] - T[P_{\Omega}]\right). \tag{38}$$

The relation between the transported-reference and frozen-reference prescriptions is therefore explicit and algebraic.[12]

6 Bogoliubov coherence as local quantum matter

The physical clock transports the reference annihilation operators through the same-family real Bogoliubov law proved in Paper 2:

$$a_n(x) = \alpha_n(x)a_n(1) + \beta_n(x)a^{\dagger}_n(1). \tag{39}$$

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[9] *Formal counterpart:* `QGC12.Quantum.realScalarFieldBankMetricStressCore_actual_owned_by_action`.

[10] *Formal counterpart:* `QGC12.Quantum.realScalarPhysicalCauchyJetBankValue_actual_clock_transport`.

[11] *Formal counterpart:* `QGC12.Q12M555_actual_physical_clock_covariance`.

[12] *Formal counterpart:* `QGC12.Quantum.realScalarWeakRelativeJetPair_actual_reference_offset`.

The mode-conjugation map $n \mapsto \bar{n}$ is the signed spatial involution associated with the real Fourier carrier.

For each local index μ define the initial annihilation and creation jets $J^{a,0}_{\mu n}$ and $J^{c,0}_{\mu n}$ from the initial real canonical field and momentum. The physical Bogoliubov jet is

$$J^{\mathrm{Bog}}_{\mu n}(p) = J^{a,0}_{\mu n}(p)\,\alpha_n(x(p)) + J^{c,0}_{\mu n}(p)\,\overline{\beta_n(x(p))}. \tag{40}$$

The formal theory proves the exact geometric identity

$$\boxed{J^{\mathrm{Bog}}_{\mu n}(p) = \nabla_\mu \Phi_n(p).} \tag{41}$$

[13] Thus the local matter observable uses the same mixing coefficients that implement the physical Cauchy dynamics.

Let

$$N_{mn}[\Psi] \tag{42}$$

be the actual normal coefficient of the core state and

$$A_{mn}[\Psi] \tag{43}$$

its actual anomalous pair coefficient. The vacuum-relative pair tensor of a finite bank has the explicit form

$$\begin{aligned} P^{(B)}_{\mu\nu}[\Psi](p) = \sum_{m,n\in B} \Big[&\mathrm{Re}\big(N_{mn}\,\overline{J^{\mathrm{Bog}}_{\mu m}}J^{\mathrm{Bog}}_{\nu n}\big) + \mathrm{Re}\big(N_{mn}\,\overline{J^{\mathrm{Bog}}_{\nu m}}J^{\mathrm{Bog}}_{\mu n}\big) \\ &+ \mathrm{Re}\big(A_{mn}\,J^{\mathrm{Bog}}_{\mu m}J^{\mathrm{Bog}}_{\nu n}\big) + \mathrm{Re}\big(A_{mn}\,J^{\mathrm{Bog}}_{\nu m}J^{\mathrm{Bog}}_{\mu n}\big)\Big]. \end{aligned} \tag{44}$$

The first line is the normal sector and the second line is the anomalous sector. The complete polynomial retains both sectors as the local matter observable.[14]

The formal development proves that physical mixing is genuinely nontrivial. There exists an owned positive time x for which the transported reference vacuum has

$$A_{00}\left[U_{\mathrm{real}}(x,1)\Omega\right] \neq 0. \tag{45}$$

[15] It also constructs a finite spherical state for which the local anomalous radial stress is strictly positive at every owned point. The statement is therefore stronger than a nonzero expansion coefficient: Bogoliubov coherence contributes to a local stress component on the actual Schwarzschild interior.[16]

This is the principal dynamical matter bridge of the paper. The squeezing established in Paper 2 acquires a local matter expression through anomalous coherence in the action-owned stress tensor.

[13] *Formal counterpart:* `QGC12.Quantum.realScalarPhysicalBogoliubovJet_actual_geometric_gradient`.
[14] *Formal counterpart:* `QGC12.Quantum.realScalarPhysicalBankRelativeStress_actual_Bogoliubov_polynomial`.
[15] *Formal counterpart:* `QGC12.Q12M556_actual_mixing_coherence`.
[16] *Formal counterpart:* `QGC12.Quantum.realScalarPhysicalAnomalousKernel_actual_spherical_nonzero`.

7 Dense physical regularity and all-mode bank removal

A local quadratic field observable requires an explicit domain of control. The physical matter theory supplies that domain through a time-indexed regular class.

Let $\mathcal{C}_{\text{fin}} \subset \mathcal{F}_{\text{phys}}$ denote the finite occupation core and define

$$\mathcal{R}_x := \operatorname{Ran}\left(U_{\text{real}}(x,1) \circ \iota_{\text{core}}\right). \tag{46}$$

Each physical time carries its own transported regular fiber, obtained by evolving the finite occupation core with the actual unitary.

For every owned positive time,

$$\overline{\mathcal{R}_x} = \mathcal{F}_{\text{phys}}. \tag{47}$$

[17] Moreover,

$$\mathcal{R}_x \subset D(H_0), \tag{48}$$

and the two-time clock transports these fibers covariantly:

$$U_{\text{real}}(x_2, x_1)\mathcal{R}_{x_1} \subseteq \mathcal{R}_{x_2}. \tag{49}$$

[18]

On the underlying finite core, finitely many occupied mode labels enter the relative polynomial. This yields absolute weighted summability of the mode-pair entries for every real weight on the actual finite support. Consequently, if B is enlarged through any directed family of finite banks, then

$$\Delta T^{(B)}_{\mu\nu}[\Psi](p) \longrightarrow \Delta T^{(\infty)}_{\mu\nu}[\Psi](p) \tag{50}$$

for the intrinsic all-mode core stress.[19] The limit is an arbitrary directed finite-bank limit, realized through eventual stabilization on the finite occupied support and therefore independent of a preferred numerical cutoff sequence.

Combined with clock covariance, the construction yields a local stress on every transported regular fiber $\mathcal{R}_x$. The paper-level matter theorem is scoped precisely to these dense fibers and their intrinsic all-mode limits.

8 Covariant conservation of the physical all-mode stress

Let $\Delta T^{(\infty)}[\Psi]$ denote the intrinsic physical all-mode stress associated with a core state and its transported regular representative. Each signed geometric mode satisfies the actual Schwarzschild covariant wave equation on the owned interior. The normal and anomalous cross kernels therefore satisfy the corresponding bilinear divergence identities.

Summing the actual finite-support polynomial gives

$$\boxed{\nabla^\mu \Delta T^{(\infty)}_{\mu\nu}[\Psi] = 0} \tag{51}$$

[17] *Formal counterpart:* `QGC12.Q12M557_dense_regular_physical_class`.

[18] *Formal counterpart:* `QGC12.Quantum.realScalarPhysicalTransportedCoreClass_actual_two_time_preservation`.

[19] *Formal counterpart:* `QGC12.Q12M558_actual_all_mode_bank_limit`.

for every finite core state, at every owned interior point with physical time $x \leq 1$.[20]

The proof is local and on shell. Unitarity controls the state evolution, and the scalar equations together with the local stress identity (27) independently earn the divergence law.

Conservation is connection-specific. Section 12 uses this geometric dependence to separate the original Schwarzschild quantum matter from the source of the changed nonlinear geometry.

9 Independent continuum finite-particle quantum matter

The selected scalar action also owns a second local quantum-matter realization on the Friedrichs continuum spectral carrier.

Let

$$R_M = (1 + K_{\mathrm{F}})^{-1} \tag{52}$$

be the selected unit resolvent. The preceding local-observation chain constructs an explicit regular one-particle class from repeated resolvent smoothing and a corresponding dense finite-particle Fock domain. Denote this state space by

$$\mathcal{D}^{\mathrm{cont}}_{\mathrm{reg}} \subset \mathcal{F}_{\mathrm{F}}. \tag{53}$$

The exact formal definition supports the presentation-independent theorem used below.[21]

A finite presentation of a regular finite-particle state provides the computational coordinates for the polynomial. The full relative pair retains the complete word expansion, including both normal and anomalous coefficients. For a presentation with sources f_i and coefficients c, the tensor has the form

$$P^{\mathrm{cont}}_{\mu\nu}[\Psi] = \mathcal{P}_{\mu\nu}(\{\nabla\phi_{f_i}\}, c), \tag{54}$$

where $\mathcal{P}$ is the actual finite-particle relative polynomial. Pair terms from both creation and annihilation sectors are retained.

Local observation is first regularized by a positive parameter ϵ. Let

$$\Delta T^{(\epsilon)}_{\mu\nu}[\Psi](p) \tag{55}$$

be the regulated vacuum-relative stress. For every directed index set carrying positive scales $\epsilon_i \to 0$,

$$\Delta T^{(\epsilon_i)}_{\mu\nu}[\Psi](p) \longrightarrow \Delta T^{\mathrm{cont}}_{\mu\nu}[\Psi](p). \tag{56}$$

The limit is independent of the chosen finite presentation of the state.[22]

The limiting continuum stress is owned by the metric variation of the same Hermitian pair density and satisfies

$$\nabla^\mu \Delta T^{\mathrm{cont}}_{\mu\nu}[\Psi] = 0. \tag{57}$$

For normalized number states built from a nonzero regular one-particle source f,

$$\boxed{\Delta T^{\mathrm{cont}}_{\mu\nu}[n_f] = n\, \Delta T^{\mathrm{cont}}_{\mu\nu}[1_f].} \tag{58}$$

[20] *Formal counterpart:* `QGC12.Q12M559_actual_geometric_stress_conservation`.
[21] *Formal counterpart:* `QGC12.Quantum.noLogContinuumRegularFiniteParticleFockDomain_dense`.
[22] *Formal counterpart:* `QGC12.Quantum.realScalarContinuumMatter_actual_full_authority`.

[23]

The theorem is vacuum-relative and is scoped to the selected reference prescription on the dense regular finite-particle class. Absolute Hadamard renormalization and representation equivalence belong to broader theorem surfaces beyond this result.

10 One stress law on two independent quantum carriers

The physical and continuum lanes solve different analytic problems. The physical carrier carries the actual Cauchy clock and its Bogoliubov dynamics. The continuum carrier carries the Friedrichs spectral resolution and the resolvent regularity used for arbitrary directed local regulator removal.

Their relation is stated at the exact level established by the formal theory. Both use the same Hermitian pair stress derived in Section 3; both use the same norm-weighted vacuum-relative subtraction; both inherit their local field kernels from the single original real scalar action.

For every regular continuum finite-particle state,

$$\Delta T^{\mathrm{cont}}_{\mu\nu}[\Psi](p) = T^{\mathrm{Herm}}_{\mu\nu}\left(P^{\mathrm{cont}}[\Psi](p)\right), \tag{59}$$

and the physical lane has

$$\Delta T^{\mathrm{phys}}_{\mu\nu}[\Psi](p) = T^{\mathrm{Herm}}_{\mu\nu}\left(P^{\mathrm{phys}}[\Psi](p)\right). \tag{60}$$

The common map T^{Herm} is the exact complexification of the real stress law (25).[24]

Thus the paper establishes

$$\boxed{\begin{array}{ccc} & \text{original real scalar action} & \\ & \Downarrow & \\ & \text{one local stress law} & \\ \swarrow & & \searrow \\ \text{physical Cauchy matter} & & \text{continuum Friedrichs matter.} \end{array}} \tag{61}$$

The shared law establishes common action provenance across the two independently constructed Fock representations.

11 Finite spherical quantum sources and the Einstein tangent

The physical all-mode stress admits an explicit spherical sector. Let

$$\mathcal{C}_{\mathrm{sph}} = \{\Psi \in \mathcal{C}_{\mathrm{fin}} : \text{all occupied modes lie in the zero spatial mode}\}. \tag{62}$$

This class contains every zero-mode number state and finite zero-mode particle-number superposition and provides the spherical source sector used by the Einstein-response theorem.

[23] *Formal counterpart:* `QGC12.Q12M561_full_M211_continuum_matter`.

[24] *Formal counterpart:* `QGC12.Q12M560_two_independent_shared_stress_lanes`.

Let F_0 denote the normalized static zero-mode flux and define the state strength

$$\Sigma_M(\Psi) = 2\ \mathrm{Re}\left(N_{00}[\Psi]\,\overline{F_0}F_0\right) + 2\ \mathrm{Re}\left(A_{00}[\Psi]F_0F_0\right). \tag{63}$$

The first term is normal and the second is anomalous. For the one-particle zero-mode state, Σ_M equals the previously derived positive real-scalar source strength.[25]

Writing

$$\sigma_\Psi = \frac{\Sigma_M(\Psi)}{C_M}, \tag{64}$$

where $C_M > 0$ is the unit one-particle source strength, the complete physical all-mode stress satisfies

$$\boxed{\Delta T^{\mathrm{phys}}_{\mu\nu}[\Psi](p) = \sigma_\Psi\, \Delta T^{\mathrm{phys}}_{\mu\nu}[1_0](p)} \tag{65}$$

for every $\Psi \in \mathcal{C}_{\mathrm{sph}}$.[26] The coefficient σ_Ψ is computed from the state and carries the signed normal/anomalous strength of each superposition.

The normalized metric response curve from the source/geometry continuation can therefore be coupled to any state in this spherical class. For a gravitational coupling κ,

$$\boxed{\frac{d}{d\varepsilon}G_{\mu\nu}\left[g_{\kappa\sigma_\Psi\varepsilon}\right](p)\bigg|_{\varepsilon=0} = \kappa\,\Delta T^{\mathrm{phys}}_{\mu\nu}[\Psi](p).} \tag{66}$$

[27] This is a genuine componentwise Einstein derivative along an actual smooth metric family. Its theorem authority is the linearized response of the normalized metric curve at zero amplitude.

The spherical sector also makes the anomalous matter contribution concrete. A finite zero-mode particle-number superposition has a nonzero anomalous coefficient, and the theory constructs a state with strictly positive local anomalous radial stress on the owned interior. The source family therefore goes beyond diagonal occupation-number matter.

12 The nonlinear source comparison and the Paper 3 boundary

A separate downstream construction produces a global stationary Einstein–real-scalar branch on a changed geometry g_{NL}. Its scalar field Φ_{NL} solves its own covariant wave equation, its scalar stress is conserved by the changed connection, and the full Einstein tensor satisfies the exact all-component nonlinear identity with the derived coupling normalization.

The present paper uses that nonlinear branch as a comparison geometry for determining the exact reach of original-geometry quantum source ownership.

At the declared anchor point p_*, the full classical relative source of the nonlinear branch agrees with the normalized original one-particle quantum source:

$$T^{\mathrm{NL}}_{\mu\nu}(p_*) = \Delta T^{\mathrm{phys}}_{\mu\nu}[1_0](p_*). \tag{67}$$

[25] *Formal counterpart:* `QGC12.Quantum.realScalarSphericalCoreStrength_actual_unit_number`.
[26] *Formal counterpart:* `QGC12.Quantum.realScalarSphericalCore_actual_source`.
[27] *Formal counterpart:* `QGC12.Q12M562_spherical_core_coupled_response`.

[28] The normalization contains an explicit factor of two. If F is the static scalar flux, the classical flux strength is

$$C = 2|F|^2, \tag{68}$$

and the branch parameters satisfy the proved relation

$$\boxed{\kappa C = 2k,} \tag{69}$$

with the repository's branch source parameter k.[29] The factor remains explicit in the coupling relation and arises within the one-real-field vacuum-relative contraction convention.

Anchor agreement supplies a local normalization match. The global field comparison then proves that the nonlinear scalar field and the original normalized quantum polarization are distinct:

$$\Phi_{\mathrm{NL}} \neq \Phi_0^{\mathrm{Schw}}. \tag{70}$$

[30] The nonlinear branch has its own connection and its own on-shell conservation law.

To compare it with the original Schwarzschild theory, let

$$\mathcal{S}^{\mathrm{NL}}_{\mu\nu} = 2T_{\mu\nu}[g_{\mathrm{NL}}, \Phi_{\mathrm{NL}}] \tag{71}$$

be the exact classical relative source used by the branch. Its divergence with respect to the original Schwarzschild connection is computed explicitly. At a suitable owned point p,

$$\nabla^{\mu}_{g_{\mathrm{Schw}}} \mathcal{S}^{\mathrm{NL}}_{\mu 0}(p) \neq 0. \tag{72}$$

Every original physical-core source constructed in Sections 4–8 obeys

$$\nabla^{\mu}_{g_{\mathrm{Schw}}} \Delta T^{\mathrm{phys}}_{\mu\nu}(p) = 0. \tag{73}$$

Therefore the original physical-core source class and the full nonlinear source are disjoint on the owned slab:

$$\boxed{\nexists \Psi \in \mathcal{C}_{\mathrm{fin}} \quad \text{such that} \quad \Delta T^{\mathrm{phys}}[\Psi] = \mathcal{S}^{\mathrm{NL}} \quad \text{throughout the owned slab.}} \tag{74}$$

[31]

The underlying theorem characterizes the separation at the level of conservation law. Every source carrying the earned original-geometry conservation law belongs to a class disjoint from the full changed-geometry source on the owned slab. The obstruction is geometric: each source is conserved by the connection of its own geometry.

This result fixes the present paper's boundary. Paper 3 establishes the local quantum matter of the original Schwarzschild real scalar and its coupled linear Einstein response. The changed geometry g_{NL} opens the next action-derived program: its own operator, endpoint classification, selected extension, quantum carrier, and physical clock.

[28] *Formal counterpart:* `QGC12.Q12M563_nonlinear_anchor_full_stress_comparison`.
[29] *Formal counterpart:* `QGC12.Q12M564_explicit_factor_two`.
[30] *Formal counterpart:* `QGC12.Q12M565_original_nonlinear_field_adjudication`.
[31] *Formal counterpart:* `QGC12.Q12M566_nonlinear_original_quantum_source_obstruction`.

13 The real quantum-matter ownership theorem

The preceding results combine into a single paper-level authority statement. For every positive Schwarzschild mass, the original single-real-scalar action owns a quantum-matter package with the following thirteen components:

1. the complete real physical quantum theory imported from Paper 2;
2. the actual geometric physical-field stress;
3. the literal norm-weighted vacuum-relative subtraction;
4. state–field covariance under the physical clock;
5. explicit Bogoliubov normal/anomalous coherence;
6. a dense all-mode transported regular class with directed bank removal;
7. preservation of that regular class by the two-time physical transport;
8. covariant conservation of the physical all-mode stress;
9. the complete independent continuum finite-particle matter authority;
10. the shared original-real-scalar stress law common to both carriers;
11. explicit nonzero spherical quantum sources, including anomalous matter;
12. the coupled linear Einstein response for the finite spherical core class;
13. an exact obstruction to identifying the nonlinear source with any source conserved by the original geometry.

In formal notation this is the theorem

$$\boxed{\texttt{Q12\_realScalarQuantumMatter\_owned\_by\_originalAction}.} \tag{75}$$

[32]

The theorem's structure matters as much as its individual clauses. It combines dynamics, locality, regulator control, conservation, and gravitational response and preserves the identities of the independently constructed carriers and geometries. The same real action owns both original-geometry quantum-matter lanes, and the exact nonlinear branch defines the next changed-geometry program.

[32] *Formal counterpart:* `QGC12.Quantum.Q12_realScalarQuantumMatter_owned_by_originalAction`.

14 Relation to quantum field theory in curved spacetime

The construction is naturally read within the operator-algebraic and curved-spacetime QFT tradition in which local observables, state dependence, and renormalized stress require careful separation from the formal field equation. Standard treatments emphasize that the stress-energy tensor is a composite observable and that its definition involves both local geometry and a state/reference prescription.[6, 7]

The present theorem package concentrates on explicit state classes and an explicit selected reference vacuum. Its principal strength is provenance: the local stress kernels, vacuum-relative subtraction, physical-clock transport, regulator removal, conservation, and linear Einstein response are all linked to a machine-checked dependency chain beginning at one original real action. Absolute Hadamard renormalization belongs to the broader curved-spacetime QFT theorem surface surrounding this focused construction.

The use of a complex one-particle Hilbert space for a real scalar is standard in positive-frequency quantization. The formal complexification theorem here makes that relationship explicit at the stress level: the Hermitian kernel is exactly the sum of the ordered real kernels of the real and imaginary parts, preserving the single-real-field interpretation of the primary theory.

The time-indexed dense regular fibers give the local-observable domain a dynamical form. The actual unitary transports the initial finite core into the fiber $\mathcal{R}_x$, so the domain statement follows the physical clock at every owned time.

The continuum lane provides a complementary local-matter construction. Repeated resolvent regularization supplies the local analytic control required for arbitrary directed regulator removal on a dense finite-particle class. The physical lane supplies the actual Cauchy clock and directly exposes the Bogoliubov coherence. Their shared stress law gives both structures a common original-action matter semantics across their independently constructed representations.

15 Scope and next theorem surface

The theorem surface is deliberately explicit.

The paper proves a local, vacuum-relative quantum stress on a dense dynamically transported physical class and an independent regulator-independent continuum stress on a dense regular finite-particle class. It proves full normal/anomalous coherence, covariant conservation, exact number-state scaling in the continuum lane, and a finite spherical physical source class with a genuine Einstein tangent.

The paper proves vacuum-relative stress with the transported original vacuum as its reference prescription and pointwise locality on the dense transported regular fibers. Absolute Hadamard stress and all-Fock pointwise operator theory define broader theorem surfaces for future development. A frozen reference vacuum carries a separately computed relative offset.

The physical and continuum Fock carriers retain their independent constructions. Their common theorem is the stress law owned by the original real action.

The nonlinear branch comparison establishes an exact anchor agreement and derives the factor-two normalization. The conservation-law theorem then separates the full changed-geometry source from the original-geometry conserved source class on the owned slab. The changed geometry's own action and connection therefore define the natural starting point for its quantum theory.

The next autonomous theorem surface is therefore

$$\boxed{\begin{array}{c} g_{\mathrm{NL}} \longrightarrow S^{\mathrm{NL}}_{\mathbb{R}} \longrightarrow K_{\mathrm{NL}} \longrightarrow \text{endpoint classification} \\ \longrightarrow \text{selected quantum carrier} \longrightarrow U_{\mathrm{NL}} \longrightarrow T^{\mathrm{quantum}}_{\mathrm{NL}}. \end{array}} \tag{76}$$

The changed geometry will earn its Friedrichs selector, endpoint category, and physical clock through a new action-derived construction.

16 Conclusion

The first paper in this series identifies the terminal law selected by finite full-potential form energy. The second constructs the continuum and physical quantum dynamics owned by that law and by the same original real scalar action. The present paper completes the next layer: local quantum matter.

The real four-dimensional scalar action owns a carrier-independent stress law. On the physical Cauchy Fock carrier, the actual field gradient, norm-weighted vacuum subtraction, physical-clock covariance, and full normal/anomalous Bogoliubov polynomial combine into a local vacuum-relative stress. The squeezing of Paper 2 therefore has a direct matter-level manifestation: anomalous pair coherence contributes to local stress, and an explicit spherical state has strictly positive anomalous radial stress on the owned interior.

The local observable is controlled on an explicit dense transported regular class. Directed finite-bank removal produces the intrinsic all-mode stress, and the resulting tensor is covariantly conserved by the actual scalar equations. Independently, the Friedrichs continuum Fock theory carries a dense regular finite-particle matter class with arbitrary directed regulator removal, presentation independence, action ownership, conservation, and exact number-state scaling. The two quantum representations retain their distinct constructions and instantiate one original-real-scalar stress law.

Finite spherical physical states produce explicit nonzero sources and a genuine coupled linear Einstein response. This establishes a direct action-to-dynamics-to-matter-to-linear-geometry chain on the original Schwarzschild background.

The exact nonlinear Einstein–scalar branch clarifies the boundary of that chain. Its source agrees with the normalized quantum source at the anchor and carries the conservation law of the changed geometry. The proved conservation-law separation places the full nonlinear source in a distinct geometric source class. Quantization on the changed geometry is therefore the next theorem problem.

The paper-level result can therefore be summarized as

$$\boxed{\begin{array}{c} S_{\mathbb{R}}[\phi; g_{\mathrm{Schw}}] \\ \Downarrow \\ \text{selected real quantum dynamics} \\ \Downarrow \\ \text{vacuum-relative local stress with Bogoliubov coherence} \\ \Downarrow \\ \nabla^{\mu}\Delta T_{\mu\nu} = 0 \\ \Downarrow \\ \text{explicit spherical quantum sources and linear Einstein response,} \end{array}} \tag{77}$$

with a separately proved obstruction preventing the exact changed-geometry nonlinear source from being identified with an original-geometry conserved source.

A Lean theorem map

The following table records the principal formal counterparts used by the manuscript. Repository names are included as technical provenance alongside the physical notation.

Manuscript role	Formal source
Paper 2 real physical quantum input	`QGC12.Quantum.RealScalarPhysicalTheory`
Single-real-scalar Hilbert stress and polarization	`QGC12.Matter.RealScalarStressPolarization`, `QGC12.Matter.RealScalarOriginalActionBridge`, `QGC12.Matter.RealScalarStressLaw`
Full real-field ladder sectors and literal vacuum subtraction	`Tower.Quantum.MultimodeRealFieldQuadraticSectors`, `Tower.Quantum.MultimodeRealFieldRelativePolynomial`
Finite-bank physical metric stress and local field gradient	`QGC12.Quantum.RealScalarFieldBankMetricStress`, `QGC12.Quantum.RealScalarPhysicalMetricStress`
Physical-clock stress covariance and transported regular fibers	`QGC12.Quantum.RealScalarPhysicalStressCovariance`, `Tower.Quantum.MultimodeRealFieldCoreRegularity`
Bogoliubov local mixing and anomalous matter	`QGC12.Quantum.RealScalarPhysicalBogoliubovCoherence`, `QGC12.Quantum.RealScalarPhysicalLocalMixing`, `QGC12.Quantum.RealScalarPhysicalCoherenceWitness`
All-mode physical stress and conservation	`QGC12.Quantum.RealScalarPhysicalRelativeStress`, `QGC12.Quantum.RealScalarPhysicalStressConservation`
Independent continuum finite-particle matter	`QGC12.Quantum.NoLogContinuumFiniteParticleMatter`, `QGC12.Quantum.RealScalarContinuumMatterAuthority`
Shared stress law across independent carriers	`QGC12.Quantum.RealScalarIndependentStressLanes`
Spherical physical quantum sources and coupled Einstein tangent	`QGC12.Quantum.RealScalarSphericalNumberSource`, `QGC12.Quantum.RealScalarSphericalCoreSource`
Nonlinear anchor comparison and original-geometry obstruction	`QGC12.Geometry.NonlinearFlux.OriginalFieldAdjudication`, `QGC12.Geometry.NonlinearFlux.OriginalMetricSourceObstruction`
Paper-level real quantum-matter capstone	`QGC12.Quantum.RealScalarQuantumMatterAuthority`, `QGC12.Quantum.RealQuantumMatterStatus`
Paper-level audit and scope decision	`QGC12RealQuantumMatterAudit`, `docs/QGC12_PAPER3_SCOPE_DECISION.md`, `docs/QGC12_PAPER3_QUANTUM_MATTER_AUTHORITY_MAP.md`

B Formal verification and authority boundary

The manuscript imports the terminal-selection theorem of Paper 1 and the original-action real physical quantum theory of Paper 2. Its new matter authority is frozen at repository revision

`ce223ddb167f07a1c0e3441356e0812e65f5dca0`.

The substantive new theorem surface is M552–M567. A later ‘main‘ commit at the time of manuscript drafting belongs to the independent Navier–Stokes program, leaving the QGC12 matter authority used here unchanged.

M552 packages the original real-action stress law: the four-dimensional Hilbert variation, real polarization, exact complexification, reference subtraction, and on-shell divergence identity. M553 identifies the literal bank quadratic with its actual vacuum-relative operator. M554 binds the Fock-valued field derivative to the geometric signed-mode gradient. M555 proves state–field covariance under the actual physical clock with the original vacuum transported together with the state. M556 proves nonzero physical anomalous coherence.

M557 defines and proves density of the transported physical regular fibers. M558 proves arbitrary directed finite-bank removal to the all-mode core stress. M559 proves covariant conservation from the actual signed-mode field equations. M560 expresses the independent physical and continuum matter lanes through the same original-real-scalar stress law. M561 retains the full M205–M211 continuum matter authority, including the dense regular finite-particle class, complete word polynomial, arbitrary directed regulator removal, presentation independence, metric variation, conservation, and every number-state family.

M562 extends the spherical physical source from the unit zero-mode state to the finite zero-mode core class and earns the explicitly coupled Einstein derivative. M563 compares the full source tensors at the nonlinear branch anchor. M564 exposes the factor-two normalization. M565 proves the global distinction between the nonlinear scalar field and the original Schwarzschild polarization. M566 proves disjointness between the full nonlinear branch source and every source carrying the original Schwarzschild conservation law on the owned slab, with a specialized theorem for the original physical core. M567 packages the thirteen-clause ‘RealScalarQuantumMatterAuthority‘.

The focused ‘QGC12RealQuantumMatterAudit‘ checks the transitive axioms of the new theorem family and verifies a primary real dependency closure built from real-field and real-source authority. The recorded verification passed the explicit physical-selection, authority, dependency, regression, real-scalar physical, nonlinear-geometry, nonlinear-solution, and real-quantum-matter targets, together with all paper-scoped QGC12 boundary scripts. The formal scope decision selects the P3-A boundary: original-geometry real quantum matter and its linear Einstein response form the present paper authority, and changed-geometry quantization forms the next theorem surface.

The manuscript uses the exact nonlinear branch through its independently constructed nonlinear authority. Coherent-state construction, representation equivalence between the two Fock carriers, absolute vacuum stress, zero-time continuation, and nonlinear quantum backreaction remain allocated to broader theorem surfaces.

C Statement and status map

Statement	Status	Role
Paper 1 no-log realization equals K_{F}	imported proved input	selected terminal law
Paper 2 physical real-scalar Hamiltonian and clock	imported proved input	quantum dynamics
Original four-dimensional real action gives the Hilbert stress	proved	action ownership
Hermitian complex pair stress is exact complexification of the real law	proved	positive-frequency compatibility
Vacuum-relative bank quadratic subtracts the actual vacuum contraction	proved	reference prescription

Statement	Status	Role
Finite-bank Fock-valued field derivative equals geometric mode gradient	proved	local observable binding
Physical stress is covariant under simultaneous state/reference transport	proved	dynamical locality
Bogoliubov local jet equals the actual geometric gradient	proved	mixing-to-matter bridge
Full physical stress retains normal and anomalous sectors	proved	coherent quantum matter
Anomalous coefficient is nonzero at an owned positive time	proved	nontrivial squeezing witness
A finite spherical state has strictly positive anomalous radial stress	proved	local coherence witness
Transported physical regular class is dense	proved	local state domain
Transported regular fibers lie in $D(H_0)$ and are two-time preserved	proved	domain covariance
Arbitrary directed finite-bank removal gives intrinsic all-mode physical stress	proved	all-mode local matter
Physical all-mode stress is covariantly conserved	proved	on-shell source law
Continuum regular finite-particle class is dense	proved	independent local matter carrier
Continuum stress has arbitrary directed regulator removal and presentation independence	proved	regulator-independent source
Continuum stress is action owned and covariantly conserved	proved	local source law
Continuum number states satisfy exact n-scaling	proved	explicit many-particle source
Physical and continuum matter instantiate one original-real-scalar stress law	proved	shared law, distinct carriers
Finite zero-mode core states source the normalized spherical quantum tensor	proved	physical source family
Coupled Einstein derivative equals $\kappa\Delta T[\Psi]$ on the spherical core class	proved	linear gravitational response
Exact nonlinear branch source equals the normalized quantum source at the anchor	proved	local normalization comparison
Explicit factor-two source lineage	proved	normalization adjudication
Global field distinction: nonlinear scalar / Schwarzschild quantum polarization	proved	field-identity control
Conservation-law separation of nonlinear and original physical-core sources	proved	nonlinear ownership boundary
Absolute vacuum stress / Hadamard renormalization	broader theorem surface	stronger QFT renormalization
Pointwise local stress on all Fock vectors	broader theorem surface	stronger domain problem
Fock-carrier relation	independent carriers	future comparison
Changed-geometry quantum operator and physical clock	subsequent paper	nonlinear quantum continuation

D Formal development and collaboration

The theorem package is part of the larger Concordia Lean 4 research development. The manuscript map above records the authority surface used here, and repository implementation details remain concentrated in the appendices.

The present paper develops the original Schwarzschild real-scalar quantum matter theory: its action-derived stress law, physical-clock covariance, Bogoliubov coherence, dense all-mode physical stress, independent continuum finite-particle matter, conservation, spherical source family, coupled linear Einstein response, and the exact conservation-law separation between the changed-geometry nonlinear source and the original-geometry quantum source class.

The global nonlinear Einstein–scalar branch enters the final source comparison and scope theorem. Its own quantization forms the next logically distinct stage of the series.

The formal source corresponding to the frozen revision is available for technical review upon request.

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