Gaussian Pair Geometry and Physical Radiative Response

Quadratic-sector variational stress, physical-helicity response, and exact information compression in a machine-checked gauge-field model

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Abstract

The preceding papers in this series construct a three-parameter family of completed Gaussian current-zero states, identify the faithful quadratic pair geometry that generates their all-orders Fock structure, and establish the Bose/Fock, analytic and Poincaré structure of the generated degree-two sector. The present paper determines exactly which Gaussian parameters remain distinguishable after variational and radiative maps are applied.

For every positive scale triple $a = (a_Y, a_W, a_C)$, the covariance

$$\Sigma_a = \mathbf{1} + K_a, \qquad \Sigma_a = \Sigma_b \iff a = b,$$

is faithful to all three Gaussian moduli. For a general finite action observable, we evaluate the connection-quadratic Taylor symbol against this covariance. The resulting quadratic-sector density is independent of the ambient metric, and its variational stress therefore satisfies

$$T^{(2)}_{a,S,\sigma,g} = 0.$$

A separate radiative branch probes the hypercharge diagonal,

$$s(a) = \operatorname{Re}\omega_a(e_Y, e_Y) = \frac{2}{1 + a_Y},$$

and weights a Hilbert-space realization of the nonzero action residual. The physical-helicity adjoint bridge sends this source to a nonzero radiative one-particle source, with

$$J_a^{\mathrm{rad}} = \frac{2}{1 + a_Y} J_0^{\mathrm{rad}} \neq 0.$$

The equality kernels give a complete information-compression hierarchy. The covariance retains all three positive moduli; the quadratic-sector stress has a total equality kernel at zero; the unnormalized hypercharge response retains exactly a_Y; and normalization removes the remaining scale dependence. The resulting normalized repeated-source Bose pair is a nonzero unit vector with Bose symmetry, diagonal Poincaré covariance and exact degree-two Fock support. Paper 4 uses an established one-particle celestial-coordinate realization, while Paper 5 establishes the two-particle multiplicity-free principal-series decomposition.

1 Introduction

The first paper in this series studies the completed kernel of a closed matter-current constraint. Every nonzero current-zero state has support above every finite particle-number cutoff [1]:

$$\Psi \neq 0, \quad \Psi \in \mathcal{K} \quad \Longrightarrow \quad \forall B \in \mathbb{N}\ \exists n:\ N(n) > B,\ \Psi(n) \neq 0. \tag{1}$$

The same kernel contains an injective three-parameter Gaussian family $a \mapsto \Psi_a$, so completion and continuous physical moduli coexist in the same exact constraint problem.

Paper 2 isolates the finite relational datum generating that completed state [2]. For every $a \in (0,\infty)^3$,

$$P_{2n}\Psi_a = \frac{V_a}{n!}(\overline{A}_a^\dagger)^n\Omega, \qquad P_{2n+1}\Psi_a = 0, \tag{2}$$

and the normalized degree-two seed

$$\Xi_a = V_a^{-1}P_2\Psi_a = \overline{A}_a^\dagger\Omega \tag{3}$$

recovers the pair kernel K_a and is faithful:

$$\Xi_a = \Xi_b \Longrightarrow K_a = K_b \Longrightarrow a = b. \tag{4}$$

Thus the nontruncatable completed state is governed by finite pair geometry.

Paper 3 qualifies the representation carried by that pair [3]. The generated degree-two sector has a completed Bose realization, an exact Fock-degree realization, a transported Poincaré action and a nonzero analytic realization of the complete finite coordinate bank. Its semantic boundary is expressed by the sequence

$$\text{Fock realization} \longrightarrow \text{analytic Hilbert-space realization} \longrightarrow \text{full asymptotic particle data}, \tag{5}$$

where the current direct Gaussian-to-asymptotic junction is excluded by the exact mismatch between the native kinetic generator and the null-asymptotic translation generator. Paper 3 closed by posing the next dynamical problem in a deliberately schematic form: faithful Gaussian covariance might pass through an expectation-valued stress and then into a radiative response. The present paper treats that prospective route as a question and tests it directly. The calculation resolves the problem into two separately derived branches: a quadratic-sector variational branch and a hypercharge-selected radiative branch.

The common upstream datum is

$$\boxed{\Sigma_a = \mathbf{1} + K_a, \qquad \Sigma_a = \Sigma_b \Longleftrightarrow a = b.} \tag{6}$$

We determine exactly which Gaussian parameters remain distinguishable after each map is applied:

$$\Sigma_a = \Sigma_b \Longleftrightarrow a = b, \tag{7}$$
$$T^{(2)}_{a,S,\sigma,g} = 0, \tag{8}$$
$$s(a) = s(b) \Longleftrightarrow a_Y = b_Y, \tag{9}$$
$$J_a^{\mathrm{grav}} = J_b^{\mathrm{grav}} \Longleftrightarrow a_Y = b_Y, \tag{10}$$
$$J_a^{\mathrm{rad}} = J_b^{\mathrm{rad}} \Longleftrightarrow a_Y = b_Y, \tag{11}$$
$$r_a = r_b, \qquad \Phi_a^{(2)} = \Phi_b^{(2)} \qquad \forall a, b. \tag{12}$$

These equality kernels organize the rest of the paper.

The variational branch starts from the physical realization of a finite action observable O. Its connection-quadratic Taylor symbol is

$$Q_2(O)_{rs} = \frac{1}{2}\frac{\partial^2 O}{\partial A_r \partial A_s}\bigg|_{A=0}, \tag{13}$$

and the Gaussian covariance functional is

$$\boxed{\mathfrak{Q}_a[O] = \operatorname{Re}\sum_{r,s} Q_2(O)_{rs}\Sigma_a(r,s).} \tag{14}$$

For genuinely quadratic observables, this covariance contraction is the exact Gaussian expectation; Section 3 gives the precise distinction between that case and the quadratic-Taylor functional for a general finite observable. Using Eq. (14) in the metric-variation framework gives a constant quadratic-sector density $L^{(2)}_{a,S,\sigma}$. Hence

$$D_g L^{(2)}_{a,S,\sigma}[h] = 0, \qquad \boxed{T^{(2)}_{a,S,\sigma,g} = 0}. \tag{15}$$

The pointwise and admissible-path variation identities remain exact at this zero value.

The radiative branch begins with the distinguished hypercharge coordinate used in this construction. Let

$$e_Y = e_{(\text{electroweak},\text{hypercharge})}. \tag{16}$$

Then

$$\boxed{s(a) = \operatorname{Re}\omega_a(e_Y, e_Y) = \operatorname{Re}\Sigma_a(Y,Y) = \frac{2}{1+a_Y} > 0.} \tag{17}$$

The action supplies the six-component residual coefficient vector R_0, while its completed metric-source realization uses a chosen nonzero mass-shell profile q:

$$J_0^{\text{grav}}(m) = R_0(m)q. \tag{18}$$

The profile q is constructed below from the positive-helicity polarization witness; the action-derived datum is the coefficient vector R_0. The hypercharge coefficient then gives

$$J_a^{\text{grav}} = s(a)J_0^{\text{grav}}. \tag{19}$$

The physical-helicity adjoint bridge factors as

$$\boxed{\mathcal{E}_{\text{rad}} = U^{-1}_{\text{mom}\to\text{cel}} \circ \iota^*_{\text{pol}},} \tag{20}$$

Here ι^*_{pol} is the Hilbert adjoint of the injective physical-polarization embedding, and $U^{-1}_{\text{mom}\to\text{cel}}$ is the inverse one-particle momentum/celestial unitary.

Sections 7–8 derive the positive-helicity witness and show that

$$\boxed{J_a^{\text{rad}} = \frac{2}{1+a_Y}J_0^{\text{rad}} \neq 0.} \tag{21}$$

Normalization then removes the remaining positive amplitude, and the resulting repeated-source Bose pair is independent of the Gaussian scale triple.

The next paper begins from the generic radiative two-particle Hilbert space containing $\Phi_a^{(2)}$ and develops its celestial principal-series decomposition independently of the Gaussian provenance recorded here.

1.1 Relation to nearby work

Particle content in relativistic quantum theory is organized by unitary Poincaré representations and little-group data [4, 5]. The one-particle radiative Hilbert space used below has the physical helicity labels ± 2 and an established Wigner/Poincaré action. The pair construction uses its completed symmetric tensor square. The Lorentz group acts as the global conformal group on the celestial sphere, and conformal-primary bases provide momentum-to-celestial transforms for massless fields, including fields with spin [9, 10].

Metric variation supplies the variational tensor through the pointwise identity [6]

$$D_g L[h] = -\frac{1}{2}\,\mathrm{vol}_g\, T : h. \tag{22}$$

The present result applies this identity to the explicitly realized quadratic-sector density $L^{(2)}_{a,S,\sigma}$, whose ambient metric dependence is constant. The resulting zero stress is therefore an exact statement about this density within the present metric-variation framework.

The radiative bridge uses the Hilbert adjoint of the physical-polarization embedding. The identity

$$\langle \iota_{\mathrm{pol}}\psi, J\rangle_{\mathrm{met}} = \langle \psi, \iota^{*}_{\mathrm{pol}} J\rangle_{\mathrm{hel}} \tag{23}$$

places the six-component metric response and the two-helicity radiative Hilbert space in one completed Hilbert-space relation. The subsequent celestial paper reorganizes such massless states into principal-series channels, where multiparticle spaces and composite channels become central [11, 12, 13].

2 Pair geometry and covariance

Let

$$a = (a_Y, a_W, a_C) \in (0,\infty)^3 \tag{24}$$

denote the positive Gaussian scale triple inherited from the preceding papers. The normalized generated pair Ξ_a recovers the symmetric pair kernel K_a. The corresponding covariance is

$$\Sigma_a = \mathbf{1} + K_a. \tag{25}$$

The same geometry obeys the finite precision relation

$$\Sigma_a(\mathbf{1} + G_a) = 2\mathbf{1}, \tag{26}$$

where G_a is the positive precision displacement in the conventions of the Gaussian family.[1]

The pair kernel is faithful to the scale triple, so

$$\boxed{\Sigma_a = \Sigma_b \quad\Longleftrightarrow\quad a = b.} \tag{27}$$

Thus Σ_a is a faithful finite coordinate on the positive Gaussian family before the action and radiative maps are applied.[2]

[1] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.GaussianPairDynamicalPredecessor`.

[2] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

2.1 The positive two-point functional

Let $\mathcal{H}_G$ denote the finite complex coordinate Hilbert space associated with the Gaussian connection bank. Define

$$\omega_a(f, g) = \langle f, \Sigma_a g \rangle_{\mathcal{H}_G}. \tag{28}$$

This covariance/two-point organization is compatible with the classical quasifree CCR framework, where covariance data determine quasifree states and their Fock-type realizations [7]. The present construction is finite-dimensional and fixed by the Gaussian family developed in the preceding papers. Self-adjointness and the strict contraction bound

$$\|K_a\| < 1 \tag{29}$$

give

$$\operatorname{Re} \omega_a(f, f) \geq (1 - \|K_a\|)\|f\|^2, \tag{30}$$

and therefore

$$f \neq 0 \Longrightarrow \operatorname{Re} \omega_a(f, f) > 0. \tag{31}$$

The functional is Hermitian,

$$\omega_a(f, g) = \overline{\omega_a(g, f)}, \tag{32}$$

and its basis moments are exactly

$$\omega_a(e_r, e_s) = \Sigma_a(r, s). \tag{33}$$

[3]

For an actual quadratic observable represented by a matrix Q, define

$$\boxed{\mathbb{E}_a[Q] = \sum_{r,s} Q_{rs} \Sigma_a(r, s).} \tag{34}$$

This is the exact Gaussian expectation used below whenever the observable is already quadratic in the native coordinate space.[4]

3 Action-derived quadratic Taylor symbols

Let O be a finite observable in the action algebra, and let A_r denote the physical connection coordinate associated with the Gaussian label r. Define

$$H_O(r, s) = \left. \frac{\partial^2 O}{\partial A_r \, \partial A_s} \right|_{A=0}. \tag{35}$$

The physical realization and even-derivative algebra give

$$H_{O_1+O_2} = H_{O_1} + H_{O_2}, \qquad H_{qO} = qH_O, \qquad H_O(r, s) = H_O(s, r). \tag{36}$$

[5]

[3] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.GaussianTwoPointFunctional`.
[4] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.
[5] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.GaussianMatterDensityRealization`.

The connection-quadratic Taylor symbol is

$$\boxed{Q_2(O)_{rs} = \frac{1}{2}H_O(r,s).} \tag{37}$$

Its Hermiticity follows from rational reality and Hessian symmetry:

$$\overline{Q_2(O)_{sr}} = Q_2(O)_{rs}. \tag{38}$$

The Gaussian covariance functional associated with this Taylor sector is

$$\boxed{\mathfrak{Q}_a[O] = \mathrm{Re}\sum_{r,s} Q_2(O)_{rs}\Sigma_a(r,s).} \tag{39}$$

It is rational linear:

$$\mathfrak{Q}_a[O_1+O_2] = \mathfrak{Q}_a[O_1]+\mathfrak{Q}_a[O_2], \qquad \mathfrak{Q}_a[qO] = q\,\mathfrak{Q}_a[O]. \tag{40}$$

The lineage is therefore

$$O \longmapsto H_O \longmapsto Q_2(O) \longmapsto \mathfrak{Q}_a[O]. \tag{41}$$

For a native quadratic observable Q, Eq. (34) supplies its exact Gaussian expectation. For a general finite observable, Eq. (39) gives the rigorously established quadratic-Taylor covariance functional.[6]

4 Quadratic-sector density and variational stress

The finite master used here depends on finite algebraic data S and an independently supplied discrete sector scale σ. Write

$$M_{S,\sigma} = \sum_k c_k(S,\sigma)O_k. \tag{42}$$

The Gaussian modulus $a \in (0,\infty)^3$ and the discrete sector scale σ are independent arguments of the construction.

Define the quadratic-sector density realization by

$$\boxed{\rho_a^{(2)}(O)(x,g) = \mathfrak{Q}_a[O].} \tag{43}$$

Then

$$L^{(2)}_{a,S,\sigma}(x,g) := \rho_a^{(2)}(M_{S,\sigma})(x,g) = \sum_k c_k(S,\sigma)\mathfrak{Q}_a[O_k]. \tag{44}$$

The right-hand side is independent of x and of the ambient metric matrix g in this realization. Hence

$$D_g L^{(2)}_{a,S,\sigma}(x,g)[h] = 0. \tag{45}$$

[7]

[6] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.
[7] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

Let $T^{(2)}_{a,S,\sigma,g}$ denote the symmetric variational stress of this quadratic-sector density. For every symmetric perturbation h,

$$D_g L^{(2)}_{a,S,\sigma}[h] = -\frac{1}{2}\sqrt{|g|}\, T^{(2)}_{a,S,\sigma,g} : h. \tag{46}$$

Combining Eqs. (45) and (46) gives

$$\boxed{T^{(2)}_{a,S,\sigma,g} = 0} \qquad \forall a, S, \sigma, g. \tag{47}$$

The admissible-path derivative is likewise exact:

$$\frac{d}{dt}\Big|_{t=0} L^{(2)}_{a,S,\sigma}(x, g_t) = -\frac{1}{2}\sqrt{|g|}\, T^{(2)}_{a,S,\sigma,g} : \dot{g}_0 = 0. \tag{48}$$

The theorem therefore classifies the metric response of the realized connection-quadratic Taylor density. Its equality kernel is total:

$$T^{(2)}_{a,S,\sigma,g} = T^{(2)}_{b,S,\tau,g} = 0 \qquad \forall a, b, \sigma, \tau. \tag{49}$$

5 Hypercharge covariance channel

Let

$$e_Y = e_{(\text{electroweak},\text{hypercharge})} \in \mathcal{H}_G \tag{50}$$

be the distinguished coordinate used by the response construction. The exact two-point identity is

$$\boxed{s(a) = \operatorname{Re}\omega_a(e_Y, e_Y) = \operatorname{Re}\Sigma_a(Y, Y) = \frac{2}{1 + a_Y}.} \tag{51}$$

Hence

$$s(a) > 0 \tag{52}$$

and

$$\boxed{s(a) = s(b) \iff a_Y = b_Y.} \tag{53}$$

[8]

Thus this branch probes only the hypercharge diagonal of the full covariance; the weak and color directions are invisible to it.

6 Action-derived residual and profile realization

At the zero principal-matter configuration, the non-gravitational action-variation channels satisfy

$$R^{\text{ferm}}_0(m) = R^{\text{scal}}_0(m) = R^{\text{gauge}}_0(m) = R^{\text{bdry}}_0(m) = R^{\text{defect}}_0(m) = 0 \qquad \forall m. \tag{54}$$

[8] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

The surviving six-component response is the gravitational residual vector

$$R_0(m) = \begin{cases} -\dfrac{233}{115200}, & m = 0, \\ 0, & m \neq 0. \end{cases} \tag{55}$$

with

$$R_0 \neq 0. \tag{56}$$

[9]

A completed metric-source vector additionally requires a mass-shell profile. Let q denote the nonzero physical profile constructed in Eq. (77) from the positive-helicity witness. The Hilbert-space realization used in this paper is

$$\boxed{J_0^{\mathrm{grav}}(m) = R_0(m)q.} \tag{57}$$

Thus the action determines R_0, while the physical-helicity construction supplies the source profile q. The action does not presently select this profile uniquely; no action-level canonicality of q is asserted. Since $R_0(0) \neq 0$ and $q \neq 0$,

$$J_0^{\mathrm{grav}} \neq 0. \tag{58}$$

The hypercharge covariance coefficient weights this realized source:

$$\boxed{J_a^{\mathrm{grav}} = s(a)J_0^{\mathrm{grav}}.} \tag{59}$$

Positivity of $s(a)$ gives

$$J_a^{\mathrm{grav}} \neq 0 \qquad \forall a \in (0,\infty)^3, \tag{60}$$

and nonvanishing of J_0^{grav} gives the exact equality kernel

$$\boxed{J_a^{\mathrm{grav}} = J_b^{\mathrm{grav}} \quad \Longleftrightarrow \quad a_Y = b_Y.} \tag{61}$$

The two Paper 4 descendants can now be displayed on their respective spaces:

$$\Sigma_a \longrightarrow \begin{cases} \rho_a^{(2)} \longrightarrow T^{(2)}_{a,S,\sigma,g} = 0, \\ s(a) \longrightarrow J_a^{\mathrm{grav}} \neq 0. \end{cases} \tag{62}$$

The current framework does not yet define a map from the smooth variational stress to the completed metric source. Equation (62) therefore keeps the two branches separate and records the provenance of each construction.

7 Physical-helicity adjoint bridge

Let

$$\mathcal{H}^{(6)}_{\mathrm{met}} \tag{63}$$

[9] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.GaussianActionStressSemanticAudit`.

denote the completed six-component metric-source Hilbert space and let

$$\mathcal{H}^{\mathrm{cel}}_{\mathrm{hel}} \tag{64}$$

denote the completed celestial-coordinate one-particle Hilbert space with helicity labels

$$\lambda \in \{+2, -2\}. \tag{65}$$

The physical polarization tensor defines an injective continuous linear map

$$\iota_{\mathrm{pol}} : \mathcal{H}^{\mathrm{cel}}_{\mathrm{hel}} \hookrightarrow \mathcal{H}^{(6)}_{\mathrm{met}}. \tag{66}$$

Its Hilbert adjoint is

$$\mathcal{A}^{\mathrm{cel}}_{\mathrm{hel}} := \iota^{*}_{\mathrm{pol}} : \mathcal{H}^{(6)}_{\mathrm{met}} \longrightarrow \mathcal{H}^{\mathrm{cel}}_{\mathrm{hel}}, \tag{67}$$

and satisfies

$$\boxed{\langle \iota_{\mathrm{pol}}\psi, J\rangle_{\mathrm{met}} = \langle \psi, \mathcal{A}^{\mathrm{cel}}_{\mathrm{hel}} J\rangle_{\mathrm{hel}}.} \tag{68}$$

[10]

The momentum-space radiative Hilbert space $\mathcal{H}^{(1)}_{\mathrm{rad}}$ and the celestial helicity space are unitarily equivalent:

$$U_{\mathrm{mom}\to\mathrm{cel}} : \mathcal{H}^{(1)}_{\mathrm{rad}} \xrightarrow{\simeq} \mathcal{H}^{\mathrm{cel}}_{\mathrm{hel}}. \tag{69}$$

The radiative extraction map used here is exactly

$$\boxed{\mathcal{E}_{\mathrm{rad}} = U^{-1}_{\mathrm{mom}\to\mathrm{cel}} \circ \iota^{*}_{\mathrm{pol}}.} \tag{70}$$

The target is the one-particle radiative Hilbert space constructed in Paper 3 [3], with its established Wigner/Poincaré representation. No equivariance theorem for the extraction map $\mathcal{E}_{\mathrm{rad}}$ itself is used or claimed here.[11]

7.1 Explicit positive-helicity witness

Let p be the nonzero mass-shell packet used to define the positive-helicity test wave

$$\psi_{+}(+2) = p, \qquad \psi_{+}(-2) = 0. \tag{71}$$

Let $e^{+}_{i}(z)$ denote the positive-helicity polarization vector in the north affine chart. The helicity-two tensor is its outer square,

$$\varepsilon^{+}_{ij}(z) = e^{+}_{i}(z) e^{+}_{j}(z). \tag{72}$$

Metric slot $m = 0$ corresponds to the spatial pair $(1, 1)$. Since

$$e^{+}_{1}(z) = \frac{2z}{1 + |z|^2}, \tag{73}$$

the relevant helicity-two component is

$$\boxed{\varepsilon^{+}_{0}(z) = \varepsilon^{+}_{11}(z) = e^{+}_{1}(z) e^{+}_{1}(z) = \left(\frac{2z}{1 + |z|^2}\right)^2.} \tag{74}$$

[10] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.
[11] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

Let θ_0 be the celestial point whose north affine coordinate is $z_0 = 1$. The north stereographic chart identifies a neighborhood of θ_0 with a neighborhood of z_0. At this point,

$$\varepsilon_0^+(z_0) = 1. \tag{75}$$

Continuity of the corresponding embedded polarization component therefore gives a celestial metric ball $B(\theta_0, r_0)$ with $r_0 > 0$ such that

$$\boxed{\left|\varepsilon_0^+(\theta)\right| > \frac{1}{2} \qquad \forall \theta \in B(\theta_0, r_0).} \tag{76}$$

The set $B(\theta_0, r_0) \times [1, 2)$ has positive mass-shell measure. We choose the nonzero packet p with support in this set and define the resulting slot-zero metric-source profile by

$$q := (\iota_{\mathrm{pol}}\psi_+)_0. \tag{77}$$

The lower bound in Eq. (76) and $p \neq 0$ give

$$q \neq 0. \tag{78}$$

[12] This is precisely the profile used in Eq. (57). Combining that realization with Eq. (55) gives

$$\boxed{(J_0^{\mathrm{grav}})_m = \begin{cases} -\dfrac{233}{115200}\, q, & m = 0, \\ 0, & m \neq 0. \end{cases}} \tag{79}$$

Consequently

$$\langle \iota_{\mathrm{pol}}\psi_+, J_0^{\mathrm{grav}} \rangle_{\mathrm{met}} = \sum_{m=0}^{5} \left\langle (\iota_{\mathrm{pol}}\psi_+)_m, (J_0^{\mathrm{grav}})_m \right\rangle \tag{80}$$

$$= -\frac{233}{115200} \langle q, q \rangle \tag{81}$$

$$\neq 0. \tag{82}$$

The adjoint identity in Eq. (68) therefore gives

$$\iota_{\mathrm{pol}}^* J_0^{\mathrm{grav}} \neq 0, \tag{83}$$

and the unitary Eq. (69) yields

$$\boxed{J_0^{\mathrm{rad}} := \mathcal{E}_{\mathrm{rad}} J_0^{\mathrm{grav}} \neq 0.} \tag{84}$$

[13]

[12] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.GaussianActionRadiativeWitness`.
[13] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

8 Radiative source and its equality kernel

Linearity of Eq. (70) and Eq. (59) give

$$\boxed{J_a^{\mathrm{rad}} = s(a)J_0^{\mathrm{rad}} = \operatorname{Re}\omega_a(e_Y, e_Y)J_0^{\mathrm{rad}} = \frac{2}{1+a_Y}J_0^{\mathrm{rad}}.} \tag{85}$$

Because $s(a) > 0$ and $J_0^{\mathrm{rad}} \neq 0$,

$$\boxed{J_a^{\mathrm{rad}} \neq 0} \qquad \forall a \in (0,\infty)^3. \tag{86}$$

The equality kernel is

$$\boxed{J_a^{\mathrm{rad}} = J_b^{\mathrm{rad}} \quad \Longleftrightarrow \quad a_Y = b_Y.} \tag{87}$$

[14]

Thus the radiative extraction map preserves exactly the one-dimensional Gaussian information carried by the hypercharge amplitude.

9 Normalization and the repeated-source Bose pair

Normalize the nonzero radiative source by

$$r_a = \frac{J_a^{\mathrm{rad}}}{\|J_a^{\mathrm{rad}}\|}. \tag{88}$$

Then

$$\|r_a\| = 1, \qquad r_a \neq 0. \tag{89}$$

Equation (85) has a strictly positive real scalar, so

$$\boxed{r_a = \frac{J_0^{\mathrm{rad}}}{\|J_0^{\mathrm{rad}}\|}} \qquad \forall a \in (0,\infty)^3. \tag{90}$$

Hence

$$r_a = r_b \qquad \forall a, b. \tag{91}$$

Define the normalized repeated-source Bose pair

$$\boxed{\Phi_a^{(2)} = r_a \widehat{\otimes}_s r_a.} \tag{92}$$

It satisfies

$$\|\Phi_a^{(2)}\| = 1, \qquad \Phi_a^{(2)} \neq 0, \tag{93}$$

$$\mathsf{X}\Phi_a^{(2)} = \Phi_a^{(2)}, \tag{94}$$

and

$$U^{(2)}(g)\Phi_a^{(2)} = (U^{(1)}(g)r_a)\widehat{\otimes}_s(U^{(1)}(g)r_a). \tag{95}$$

[14] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

The associated Fock representative satisfies

$$\Psi^{(n)}_{a,\mathrm{rad}} = 0 \qquad n \neq 2, \tag{96}$$

$$\Psi^{(2)}_{a,\mathrm{rad}} \neq 0, \tag{97}$$

Equation (90) propagates to

$$\boxed{\Phi^{(2)}_a = \Phi^{(2)}_b} \qquad \forall a, b \in (0,\infty)^3. \tag{98}$$

Thus the normalized radiative pair is the same for every positive Gaussian scale in this hypercharge channel, although its upstream provenance still depends on a. The existing bounded affine cross-channel construction supplies an additional exact response corollary; Appendix A records it separately from the equality-kernel argument.

10 Information compression and equality kernels

The equality kernels are therefore as follows. The faithful covariance satisfies

$$\Sigma_a = \Sigma_b \iff a = b. \tag{99}$$

The quadratic-sector stress satisfies

$$T^{(2)}_{a,S,\sigma,g} = 0 \qquad \forall a, S, \sigma, g. \tag{100}$$

The hypercharge coefficient and the two unnormalized response sources satisfy

$$s(a) = s(b) \iff a_Y = b_Y, \tag{101}$$

$$J^{\mathrm{grav}}_a = J^{\mathrm{grav}}_b \iff a_Y = b_Y, \tag{102}$$

$$J^{\mathrm{rad}}_a = J^{\mathrm{rad}}_b \iff a_Y = b_Y. \tag{103}$$

Normalization and repeated Bose completion satisfy

$$r_a = r_b, \qquad \Phi^{(2)}_a = \Phi^{(2)}_b \qquad \forall a, b. \tag{104}$$

[15]

The equality criteria and retained Gaussian information are summarized in the following table.

Object	Equality criterion	Gaussian information retained
Σ_a	$\Sigma_a = \Sigma_b \iff a = b$	all three positive moduli
$T^{(2)}_{a,S,\sigma,g}$	identically zero	none
$s(a)$	$s(a) = s(b) \Leftrightarrow a_Y = b_Y$	a_Y
J^{grav}_a	equality iff $a_Y = b_Y$	a_Y
J^{rad}_a	equality iff $a_Y = b_Y$	a_Y
r_a	$r_a = r_b$ for all a, b	none
$\Phi^{(2)}_a$	$\Phi^{(2)}_a = \Phi^{(2)}_b$ for all a, b	none

Each map identifies part of the original three-parameter Gaussian family, and its equality kernel determines exactly how much information is lost. The provenance record retains the upstream data even after normalization enlarges the equality kernel.

[15] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

11 Provenance and the downstream radiative pair

For each positive Gaussian scale a, finite algebraic datum S, discrete sector scale σ and smooth metric g, we summarize the construction schematically by

$$\mathfrak{P}_a = (a, K_a, \Sigma_a, \omega_a, Q_2, \mathfrak{Q}_a, \rho_a^{(2)}, T^{(2)}_{a,S,\sigma,g}; e_Y, s(a), J_a^{\mathrm{grav}}, \mathcal{E}_{\mathrm{rad}}, J_a^{\mathrm{rad}}, r_a, \Phi_a^{(2)}). \tag{105}$$

The semicolon separates the variational and radiative branches.

The downstream handoff is the generic radiative pair

$$\boxed{\Phi_a^{(2)} \in \mathcal{H}^{(2)}_{\mathrm{phys}}.} \tag{106}$$

This completed Bose-symmetric Hilbert space has physical-helicity one-particle factors and diagonal Poincaré action.[16]

The celestial-coordinate space used in Eqs. (64)–(69) is an already qualified one-particle realization,

$$\mathcal{H}^{(1)}_{\mathrm{rad}} \simeq \mathcal{H}^{\mathrm{cel}}_{\mathrm{hel}}. \tag{107}$$

The next paper addresses a distinct two-particle problem. It starts from an arbitrary

$$\Phi^{(2)} \in \mathcal{H}^{(2)}_{\mathrm{phys}} \tag{108}$$

and constructs its source-aware multiplicity-free celestial principal-series decomposition independently of the Gaussian construction used here.

12 Discussion

The first four papers form a sequence from completion to generative structure, representation and finally parameter-sensitive response. Paper 1 establishes the completed state space required by the closed current. Paper 2 identifies the finite pair law that generates the nontruncatable Gaussian tower. Paper 3 establishes its Bose/Fock, analytic and Poincaré structure and closes with a natural prospective picture in which faithful Gaussian covariance could feed an expectation-valued stress and then a radiative response. Paper 4 tests that expectation. The resulting structure bifurcates: after the action and radiative maps are applied, the two derived branches retain different amounts of the original Gaussian information.

The two branches answer that question differently. The quadratic Taylor density is independent of the ambient metric, so its variational stress vanishes for the entire Gaussian family. The radiative branch probes only the hypercharge diagonal of the covariance, and therefore retains exactly a_Y before normalization removes that final amplitude. These are distinct constructions with distinct equality kernels. The prospective single stress-to-radiation route suggested at the end of Paper 3 therefore resolves here into a zero-stress variational branch and a separately selected nonzero radiative branch.

[16] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.GaussianDerivedRadiativePair.toPhysicalRadiativeBosePair`.

This gives the organizing principle

$$\boxed{\text{faithful covariance} \longrightarrow \text{derived quantities with exact equality kernels.}} \tag{109}$$

The equality kernels quantify the information lost by each map: none at the covariance stage, two Gaussian directions in the unnormalized hypercharge response, and all scale dependence after normalization. The zero-stress branch collapses the full Gaussian family independently.

The current framework does not yet define a map from the smooth variational stress to the completed metric source. Establishing such a map would permit a direct comparison between the two branches. The present result therefore keeps their provenances separate while giving an exact parameter-level account of both.

13 Conclusion

The Gaussian covariance is faithful to all three positive parameters. Its quadratic Taylor density has zero variational stress. The profile-based hypercharge branch retains only a_Y until normalization removes that dependence.

The normalized repeated-source Bose pair is a nonzero unit vector with diagonal Poincaré covariance and exact degree-two Fock support. It supplies the generic radiative two-particle input for the principal-series analysis developed in Paper 5.

A Bounded cross-channel corollary

The qualified source also enters the existing bounded off-diagonal interaction between the Gaussian hard channel and the radiative Fock channel. Let h_a denote the normalized Gaussian hard state and let q_a denote the corresponding nonzero radiative Fock source. In the direct-sum Hilbert space,

$$\langle q_a, h_a \rangle = 0, \tag{110}$$

and the bounded operator X_a satisfies

$$X_a h_a = q_a. \tag{111}$$

[17]

For $g \in \mathbb{R}$, define

$$\Psi_a(g) = h_a + g X_a h_a. \tag{112}$$

The channel amplitude is

$$\mathcal{A}_a(g) = \langle q_a, \Psi_a(g) \rangle, \tag{113}$$

so

$$\boxed{\mathcal{A}_a(g) = g \, \langle q_a, q_a \rangle.} \tag{114}$$

The phase-insensitive response

$$\mathcal{R}_a(g) = |\mathcal{A}_a(g)|^2 \tag{115}$$

satisfies

$$\boxed{\mathcal{R}_a(g) = g^2 \left| \langle q_a, q_a \rangle \right|^2.} \tag{116}$$

For $g \neq 0$,

$$\mathcal{R}_a(g) > 0. \tag{117}$$

These identities characterize the defined bounded affine cross-channel construction.[18]

B Formal verification map

The main text uses invariant mathematical language. Repository carrier names and implementation labels appear here solely to identify exact formal counterparts. The Paper 4 manuscript authority is pinned to repository commit

`4a621c0226d93bef2fe7899b6cc90e06a365cb31`.

[17] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

[18] *Formal counterpart:* `Tier30.Research.GaussianPhysicalization.GaussianBoundedCrossChannelResponseClosure`.

Mathematical result	Formal authority
Paper 3 predecessor and pair/covariance surface	`GaussianPairDynamicalPredecessor.current`
Faithful positive Gaussian covariance	`positiveGaussianCovarianceMatrix_injective`
Positive Hermitian two-point functional	`gaussianTwoPoint;gaussianTwoPoint_conj_symm;gaussianTwoPoint_positive`
Exact expectation of a native quadratic observable	`gaussianNativeQuadraticExpectation_exact`
Connection-quadratic Taylor symbol	`gaussianQuadraticTaylorSymbol;gaussianQuadraticTaylorSymbol_apply`
Taylor covariance functional	`gaussianQuadraticTaylorCovarianceFunctional_eq_covariance`
Quadratic-sector density realization	`gaussianQuadraticSectorDensityRealization;gaussianQuadraticSectorDensityRealization_apply`
Pointwise/pathwise quadratic-sector stress closure	`gaussianQuadraticSectorStressClosure`
Selected hypercharge covariance moment	`gaussianSelectedHyperchargeMoment_exact;gaussianSelectedHyperchargeMoment_positive`
Selected moment equality kernel	`gaussianSelectedHyperchargeMoment_eq_iff`
Action residual coefficient vector and nonzero gravitational coefficient	`gaussianPaper4_jointSectorResponse_exact;gaussianPaper4BaseActionResidual_exact;gaussianPaper4BaseActionResidual_zero_ne_zero`
Profile-realized source nonvanishing	`gaussianIndependentW4ActionResidual;actionResidualW4MetricSource_zeroActionSurface_apply;sourceProfile_ne_zero;gaussianIndependentW4ActionResidual_ne_zero`
Physical polarization embedding	`paper4PhysicalPolarizationEmbedding;paper4PhysicalPolarizationEmbedding_injective`
Adjoint helicity pairing	`paper4PhysicalHelicityAdjoint_pairing`
Momentum/celestial factorization	`actionResidualRadiativeSource_factorization;paper4BaseRadiativeSource_factorization`
Radiative bridge qualification	`paper4PhysicalRadiativeBridgeClosure`
Exact positive-helicity witness	`paper4PhysicalHelicityWitness_pairing_exact`
Nonzero witness and base radiative source	`paper4PhysicalHelicityWitness_pairing_ne_zero;paper4BaseRadiativeSource_ne_zero`
Selected radiative branch	`paper4SelectedRadiativeBranch_exact`
Typed information-compression theorem	`paper4TypedInformationCompression`
Repeated-source Bose-pair qualification	`gaussianRepeatedSourceBosePairQualification`
Bounded cross-channel response	`gaussianBoundedCrossChannelResponseClosure`
Frozen P4-GPR-03 publication authority	`paper4GaussianRadiativePublicationAuthority`
P4-REV-01 clarification authority	`paper4RefereeClarificationAuthority`
Generic downstream physical-pair handoff	`GaussianDerivedRadiativePair.toPhysicalRadiativeBosePair_exact`

C Formal development and authority boundary

The results in this paper are supported by a Lean 4 formal development maintained in a private research repository. The final manuscript surface is collected by

`Tier30.Research.GaussianPhysicalization.Paper4RefereeClarificationAuthority`.

This premise-free capstone contains the frozen P4-GPR-03 authority and adds the publication-facing

Taylor scope, stress closure, selected covariance theorem, helicity adjoint bridge, nonzero witness, compression theorem, repeated-source pair qualification and bounded cross-channel response.

The formal distinction between Eqs. (34) and (39) is part of the publication authority. Exact Gaussian expectation is attached to native quadratic observables. For a general finite action observable, the half-Hessian connection-quadratic Taylor symbol is evaluated by $\mathfrak{Q}_a$.

The variational theorem is scoped to the quadratic-sector density realized in the metric-variation carrier. Its pointwise and admissible-path Fréchet derivative laws are theorem-owned, and its ambient metric derivative is exactly zero. The current manuscript authority closes at this local variational surface. Integrated-action and covariant-conservation interfaces require additional hypotheses beyond this density realization.

The radiative theorem exposes the complete physical-helicity bridge used in the body. The two-helicity celestial carrier embeds injectively into the six-component completed metric-source carrier; its Hilbert adjoint defines the celestial response; the one-particle momentum/celestial unitary returns the momentum-space radiative source. The target carries the existing Wigner/Poincaré representation and celestial intertwining square, and the concrete source carries the existing BRST physical representative. The present theorem surface assigns these properties to the target carrier and its source realization. It does not assert an intertwining identity for $\mathcal{E}_{\mathrm{rad}}$ itself.

The smooth variational-stress carrier and the finite metric-source carrier remain separated by an open typed interface. The theorem map records this status as ‘stressConstructedSourceMapOpen‘. The present paper therefore carries the quadratic-sector variational branch and the selected radiative branch through their independently established maps.

The normalized repeated-source Bose pair is qualified by norm, nonvanishing, exchange symmetry, exact degree-two Fock support, diagonal Poincaré covariance and BRST representative. The BRST terminology here follows the standard constrained gauge-system and cohomological framework [8]. Its interpretation in the manuscript is restricted to those carrier properties and to its role as the generic radiative two-particle input of Paper 5. Paper 4 uses the existing one-particle momentum/celestial realization; Paper 5 establishes the two-particle multiplicity-free principal-series decomposition.

The Paper 5 formal closure remains dependency-separated from every ‘GaussianPhysicalization‘ module. The updated Python verifier requires the P4-REV-01 clarification authority, checks its publication declarations, preserves the Paper 4/Paper 5 firewall and rejects downstream celestial CG/OPE dependencies from the present closure.

Repository surface names carry formal provenance. The manuscript body uses invariant terms tied directly to the equations above: Gaussian coordinate carrier, quadratic Taylor covariance functional, selected matter density, variational stress, gravitational metric source, physical-helicity adjoint, radiative one-particle carrier and repeated-source Bose pair.

The full research repository is not publicly accessible. Paper-scoped source review, theorem-level verification and access to relevant formal modules may be arranged in the context of professional collaboration, technical review or institutional partnership.

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