Master-Action Ownership of a Self-Consistent Einstein–Scalar Quantum Sector

Symmetric-projective BV closure, Palatini descent, and quantum Ward/source exchange

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Abstract

The first four papers in this series select a singular real-scalar operator law, construct its physical quantum evolution and conserved matter, and close a self-consistent nonlinear Einstein–scalar quantum sector. The present fifth paper moves upstream and identifies the functional gauge/master structure that generates the same Einstein–scalar action and exchanges with the established quantum observables.

The construction begins with an ambient untruncated spacetime-jet local-functional master modulo total divergences. Its functional antibracket satisfies the classical master equation, its generated Hamiltonian is square-zero on arbitrary antifield-dependent real local functionals, and the same bracket obeys generic graded Jacobi. The physical inverse metric density is then realized intrinsically on a symmetric-pair jet carrier. The induced cotangent normalization gives weight one to diagonal antifields and weight one half to off-diagonal antifields, with both canonical pairings normalized to one. The ambient quotient is proved not to be a global Poisson map; the intrinsic symmetric carrier carries its own canonical bracket and master.

Variation of the same Palatini action with respect to the independent connection is computed and solved exactly:

$$E_\Gamma S_{\mathrm{Pal}} = 0 \quad \Longleftrightarrow \quad \Gamma^\rho{}_{\mu\nu} = \Gamma[g]^\rho{}_{\mu\nu} + \delta^\rho_\nu A_\mu.$$

The physical action is exactly invariant under this projective shift, the pointwise linearized connection-Euler kernel is exactly the image of the projective generator, and that generator is injective. The torsion trace transforms as $\tau(P_B\Gamma) = \tau(\Gamma) - 3B$; each stationary projective orbit therefore has a unique zero-trace representative, equal to the Levi–Civita connection.

The symmetric carrier is enlarged by an odd projective covector ghost with its antifield. Its covector Lie-transport law combines with the diffeomorphism sector to give a nilpotent prolonged BRST differential. The projective cotangent functional satisfies its own classical master equation, the symmetric Palatini action obeys the corresponding Noether identities, and the enlarged real master

$$S^{\mathrm{sp}}_\kappa = \pi_{\mathrm{sym}}(S_\kappa) + S_{\Gamma^*\eta} + S_{\eta^* s\eta}$$

satisfies

$$\boxed{(S^{\mathrm{sp}}_\kappa, S^{\mathrm{sp}}_\kappa)_{\mathrm{sp}} = 0, \qquad (s^{\mathrm{sp}}_\kappa)^2 F = 0}$$

for arbitrary antifield-dependent local functionals on the intrinsic carrier. Classical projection returns the same Einstein–real-scalar density and the same native Paper 4 matter source.

For the coherent source state Ψ_z, the Fock first moment equals the coherent real mean field and its geometric jet moment equals the mean-field gradient. Along a finite coordinate-gauge orbit, the derivative of this Fock observable equals the coefficient generated by the master law. The matter projection has Hilbert metric response equal to the previously constructed coherent vacuum-relative quantum stress; the Einstein residual obeys exact pullback covariance; and gauge pullback intertwines with the positive-time physical clock.

Paper 5 therefore closes the classical master-action provenance through intrinsic symmetric-density and projective BV structure and binds its physical projection to the Einstein–scalar quantum sector of Papers 1–4. The continuing boundary begins with full functional properness/Koszul acyclicity, metric-only symplectic BV reduction, a direct quantum gauge differential on the Fock carrier, physical-state cohomology, and quantum master/anomaly analysis.

1 Introduction

The preceding four papers follow one real scalar field through a single chain of increasing physical structure. Paper 1 derives the singular Schwarzschild radial operator and proves that finite full-potential action selects its no-log Friedrichs law.[1] Paper 2 constructs the continuum and physical Cauchy quantum theories, including the complete signed real-field carrier, self-adjoint all-mode Hamiltonian, Bogoliubov transport, and physical clock.[2] Paper 3 differentiates that field into local vacuum-relative matter and establishes all-mode conservation on the transported regular carrier.[3] Paper 4 rebuilds the quantum theory on an independently constructed nonlinear geometry and closes the self-consistent physical cycle

$$\boxed{g_{\mathrm{NL}} \longrightarrow \mathcal{Q}(g_{\mathrm{NL}}) \longrightarrow \Delta T_{\mu\nu}^{\mathrm{quantum}} \longrightarrow g_{\mathrm{NL}}.} \tag{1}$$

The coherent source realization makes that closure especially explicit: the complete quantum field pairing yields the quadratic stress of its real mean field, and the resulting vacuum-relative stress satisfies the Einstein equation of the same nonlinear branch.[4]

Paper 5 asks the next question in the same trajectory:

$$\boxed{\begin{array}{c}\text{What functional gauge law generates the Einstein–scalar action}\\ \text{that generated this self-consistent quantum sector, and how does}\\ \text{that law exchange with the quantum observables and source equation?}\end{array}} \tag{2}$$

The answer begins with the real Palatini master action and then orders the field carrier on which its physical gauge structure is represented. The first construction is an ambient free local-functional BV carrier: ordered inverse-density jet symbols, an independent affine connection, scalar, diffeomorphism ghosts, and their antifields generate an untruncated polynomial/exterior jet algebra modulo total divergences. This carrier establishes the generic functional calculus, classical master equation, unrestricted square-zero law, graded Jacobi identity, and the original physical projection.

The independent connection variation then exposes a projective stationary direction. Requiring the master architecture to explain that direction reveals that the physical inverse metric density must be represented intrinsically as a symmetric tensor rather than as sixteen unrelated ordered symbols. Paper 5 therefore constructs a second, intrinsic symmetric-density Palatini BV carrier and equips it with its correctly normalized cotangent pairing. The old ambient quotient is proved not to be a global Poisson map into this carrier; the intrinsic carrier has its own canonical bracket. An odd projective covector ghost and antifield then enlarge the diffeomorphism master into the final classical master of the paper.

Write $\pi_{\mathrm{sym}} : \mathcal{J}_{\mathrm{Pal}}^{\mathrm{amb}} \to \mathcal{J}_{\mathrm{Pal}}^{\mathrm{sym}}$ for the proved ambient-to-symmetric quotient/transport map and $\iota_{\mathrm{sym}} : \mathcal{J}_{\mathrm{Pal}}^{\mathrm{sym}} \to \mathcal{J}_{\mathrm{Pal}}^{\mathrm{amb}}$ for its proved section. Thus

$$\pi_{\mathrm{sym}} \circ \iota_{\mathrm{sym}} = \mathrm{id}_{\mathcal{J}_{\mathrm{Pal}}^{\mathrm{sym}}}.$$

We use the same symbols for the induced maps on jet densities and local-functional classes when the level is clear from context. The resulting construction chain is

$$\boxed{\begin{array}{c} S_\kappa \xrightarrow{\pi_{\mathrm{sym}}} S_\kappa^{\mathrm{sym}} \longrightarrow S_\kappa^{\mathrm{sp}} \xrightarrow{\pi_{\mathrm{cl}}} S_{\text{Einstein-scalar}} \longrightarrow S_{\mathrm{native}},\\ S_{\mathrm{native}} \longrightarrow H_{\mathrm{NL}} \longrightarrow \mathcal{Q}(g_{\mathrm{NL}}) \longrightarrow \Delta T_{\mu\nu} \longrightarrow \mathcal{E}_{\mu\nu} = 0. \end{array}} \tag{3}$$

Each arrow carries a distinct theorem. The ambient master supplies the free variational calculus. The symmetric master realizes the actual symmetry of $\mathfrak{h}^{\mu\nu} = \mathfrak{h}^{\nu\mu}$. The enlarged master adds the projective gauge direction and satisfies its own classical master equation. Its classical projection is the same Einstein–real-scalar action. Variation with respect to the independent connection selects one projective orbit through Levi–Civita; the pointwise Euler kernel is exactly the projective image; and zero torsion trace selects the unique Levi–Civita representative. Evaluation on the nonlinear branch then reduces to the same native scalar action and frequency used by the Paper 4 quantum Hamiltonian. The coherent state turns the resulting field symbol into an actual Fock-space first moment, while the master-projected matter response closes on the previously computed quantum stress.

The final classical BV statement is therefore stronger than the original ambient closure. On the intrinsic symmetric/projective carrier,

$$\boxed{(S_\kappa^{\mathrm{sp}}, S_\kappa^{\mathrm{sp}})_{\mathrm{sp}} = 0, \qquad s_\kappa^{\mathrm{sp}} F := (S_\kappa^{\mathrm{sp}}, F)_{\mathrm{sp}}, \qquad (s_\kappa^{\mathrm{sp}})^2 F = 0} \tag{4}$$

for every real local functional, including arbitrary antifield dependence. The generic graded Jacobi theorem applies to the intrinsic bracket, and the enlarged construction retains full antifield feedback. The original ambient CME and its counterexample to unrestricted projective invariance remain part of the provenance: the physical projective symmetry is obtained by moving to the correct symmetric field carrier, not by retroactively changing the old one.

Two Hamiltonians remain mathematically distinct. The master Hamiltonian is the odd variational differential generated by the functional antibracket. The native quantum Hamiltonian $H_{\mathrm{NL}}(x)$ is the self-adjoint energy generator on the Fock carrier governing physical Cauchy evolution. Paper 5 proves shared action provenance and observable compatibility between these structures without identifying their carriers.

The exchange with the quantum sector occurs through quantities already established by the Paper 4 Fock theory. The coherent field first moment and its spacetime gradient are actual Hilbert-space expectation values. The diffeomorphism component of the master differential is visible as the derivative of these observables along finite coordinate-gauge paths. The quantum stress enters through the Hilbert metric derivative of the same classical matter projection. Gauge covariance is then tested at the level of the full Einstein residual before the source equation is imposed.

The projective ghost acts on the first-order gravitational connection sector; the scalar physical action and the downstream Paper 4 matter source are unchanged by the enlarged classical projection. A direct odd gauge differential on the Fock carrier, physical-state cohomology, and a quantum master/anomaly theorem remain separate quantum constructions.

Conventions. The manuscript retains the normalized conventions of Papers 1–4. The Einstein coupling κ, native terminal coordinate, native frequency Ω_k, quantum field normalization, and vacuum-relative stress use the same inherited definitions. The ambient, symmetric, and symmetric-projective masters have distinct symbols when their carrier matters. The functional master and quantum energy Hamiltonian remain separate objects. Detector proper time, detector energy, and SI-unit restoration remain external calibration maps, as in Paper 4.

Sections 2–4 construct the ambient Palatini functional master and its generic BV closure. Section 5 identifies its physical projection. Section 6 derives and solves the independent connection Euler equation and its projective orbit. Section 7 constructs the intrinsic symmetric-density carrier, proves the physical projective symmetry and the ambient obstruction, adds the projective ghost/antifield sector, and closes the enlarged CME and square-zero law. Section 8 binds the final classical projection to the native action and quantum frequency of Paper 4. Sections 9–12 establish the quantum field-moment Ward law, Hilbert-stress response, source covariance, and physical-clock compatibility. Section 14 states the final master–quantum exchange theorem, and Section 15 fixes the continuing mathematical boundary.

2 The ambient real Palatini functional master

Let $\mathcal{J}^{\mathrm{amb}}_{\mathrm{Pal}}$ denote the untruncated polynomial/exterior spacetime-jet superalgebra generated by ordered inverse-density symbols, an independent affine connection, real scalar, four diffeomorphism ghosts, and their antifields. At this ambient stage the symbols indexed by (μ,ν) and (ν,μ) are independent generators; tensor symmetry is imposed only by the later physical realization. Let $\mathbf{D} = (D_0, D_1, D_2, D_3)$ be the commuting total-jet derivative bank. The ambient real local-functional carrier is

$$\boxed{\mathfrak{F}_{\mathrm{Pal,amb}} := \mathbb{R} \otimes_{\mathbb{Q}} \left(\mathcal{J}^{\mathrm{amb}}_{\mathrm{Pal}} / \operatorname{Div}(\mathbf{D})\right).} \tag{5}$$

Here $\operatorname{Div}(\mathbf{D})$ is the linear span of total divergences. Jet multiindices remain unbounded and each representative retains arbitrary finite polynomial/exterior degree.

The quotient in Eq. (5) establishes the ambient local variational carrier. Section 7 constructs the intrinsic symmetric-density carrier required by the physical gravitational field and proves how the master reaches it. Boundary charges, horizon contributions, asymptotic generators, and boundary-completed gravitational actions belong to a boundary-aware extension in which the corresponding divergence data remain explicit.

Write the represented first-order variables as

$$\mathfrak{h}^{\mu\nu}, \qquad \Gamma^{\rho}{}_{\mu\nu}, \qquad \phi, \qquad c^{\mu},$$

where c^μ is the odd diffeomorphism ghost. On the physical Lorentz-metric realization, $\mathfrak{h}^{\mu\nu} = \sqrt{|g|}g^{\mu\nu}$ and is symmetric. The ambient jet symbols themselves remain ordered and independent. Their antifields are denoted $\mathfrak{h}^*_{\mu\nu}$, $(\Gamma^*)_\rho{}^{\mu\nu}$, ϕ^*, and c^*_μ. The classical Palatini density represented by the first two sectors is

$$\boxed{\mathcal{L}_{\mathrm{cl}} = \frac{1}{2\kappa}\,\mathfrak{h}^{\mu\nu}R_{\mu\nu}(\Gamma) - \frac{1}{2}\,\mathfrak{h}^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi.} \tag{6}$$

The master-generated gauge assignments used by the cotangent sector are, in component notation,

$$s\phi = c^\lambda\partial_\lambda\phi, \qquad\qquad sc^\mu = c^\lambda\partial_\lambda c^\mu, \tag{7}$$

$$s\mathfrak{h}^{\mu\nu} = c^\lambda\partial_\lambda\mathfrak{h}^{\mu\nu} + (\partial_\lambda c^\lambda)\mathfrak{h}^{\mu\nu} - (\partial_\lambda c^\mu)\mathfrak{h}^{\lambda\nu} - (\partial_\lambda c^\nu)\mathfrak{h}^{\mu\lambda}, \tag{8}$$

$$s\Gamma^\rho{}_{\mu\nu} = c^\lambda\partial_\lambda\Gamma^\rho{}_{\mu\nu} - (\partial_\lambda c^\rho)\Gamma^\lambda{}_{\mu\nu} + (\partial_\mu c^\lambda)\Gamma^\rho{}_{\lambda\nu} + (\partial_\nu c^\lambda)\Gamma^\rho{}_{\mu\lambda} + \partial_\mu\partial_\nu c^\rho. \tag{9}$$

These are the scalar Lie law, the weight-one contravariant inverse-density law, the affine connection law, and the ghost transport law implemented by the untruncated jet differential. The cotangent density fixes the field/antifield signs explicitly:

$$\boxed{\mathcal{L}_{\mathrm{cot}} = \mathfrak{h}^*_{\mu\nu}\,s\mathfrak{h}^{\mu\nu} + (\Gamma^*)_\rho{}^{\mu\nu}\,s\Gamma^\rho{}_{\mu\nu} + \phi^* s\phi - c^*_\mu sc^\mu.} \tag{10}$$

Thus the full real master is represented by

$$\boxed{\mathcal{L}_\kappa^{\mathrm{master}} = \mathcal{L}_{\mathrm{cl}} + \mathcal{L}_{\mathrm{cot}}, \qquad S_\kappa = \frac{1}{2\kappa} S_{\mathrm{curv}} - \frac{1}{2} S_{\mathrm{kin}} + S_{\mathrm{cot}}.} \tag{11}$$

The ambient canonical BV pairings are

$$(\mathfrak{h}^{\mu\nu}, \mathfrak{h}^*_{\alpha\beta})_{\mathrm{amb}} = \delta^\mu_\alpha \delta^\nu_\beta, \quad (\Gamma^\rho{}_{\mu\nu}, (\Gamma^*)_\sigma{}^{\alpha\beta}) = \delta^\rho_\sigma \delta^\alpha_\mu \delta^\beta_\nu, \quad (\phi, \phi^*) = 1, \quad (c^\mu, c^*_\nu) = \delta^\mu_\nu, \tag{12}$$

with the ghost-pair sign already encoded by Eq. (10). The ordered inverse-density pairing is intentionally distinct from the intrinsic symmetric pairing constructed later. The Lean carrier formalizes $\mathbb{Z}_2$ parity and pair orientation directly: $\mathfrak{h}, \Gamma, \phi$ are even, c is odd, their physical antifields are odd, and the ghost antifield is even. For standard BV bookkeeping, one may attach the conventional expository ghost numbers

$$\mathrm{gh}(\mathfrak{h}, \Gamma, \phi) = 0, \quad \mathrm{gh}(c) = 1, \quad \mathrm{gh}(\mathfrak{h}^*, \Gamma^*, \phi^*) = -1, \quad \mathrm{gh}(c^*) = -2.$$

The integer ghost-number labels provide reader orientation; the checked carrier uses parity/orientation as its formal grading data.[1][2]

The functional antibracket is a bilinear map

$$(\,\cdot\,,\,\cdot\,) : \mathfrak{F}_{\mathrm{Pal,amb}} \times \mathfrak{F}_{\mathrm{Pal,amb}} \longrightarrow \mathfrak{F}_{\mathrm{Pal,amb}}, \tag{13}$$

generated from the canonical field/antifield pairing and the full variational Euler maps. Its action on the master defines the variational Hamiltonian

$$H_\kappa^{\mathrm{BV}} F := (S_\kappa, F). \tag{14}$$

The same three physical sectors generate

$$H_\kappa^{\mathrm{BV}} = \frac{1}{2\kappa} H_{\mathrm{curv}} - \frac{1}{2} H_{\mathrm{kin}} + H_{\mathrm{cot}}, \tag{15}$$

with each operator obtained from the corresponding functional by the same antibracket construction.

The ambient master satisfies the functional classical master equation

$$\boxed{(S_\kappa, S_\kappa) = 0.} \tag{16}$$

The curvature and scalar Noether identities are derived from the actual diffeomorphism action of the same density; the action–cotangent and cotangent self-pairings then assemble Eq. (16).[3]

The classical projection π_{cl} sets ghost and antifield coordinates to their classical values while preserving the physical curvature and scalar coefficients. Section 5 evaluates this ambient projection on the physical metric and scalar. Section 7 then reconstructs the same classical density on the intrinsic symmetric carrier and enlarges its gauge sector projectively before the native reduction.

[1] *Formal counterpart:* `QGC12.BV.palatiniGaugeCotangentDensity`.
[2] *Formal counterpart:* `QGC12.BV.palatiniGaugeBRST_actual_connection_transport`.
[3] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealMasterFunctional_actual_full_S0_S1_CME`.

3 Unrestricted generated differential and antifield feedback

Write

$$s_\kappa F := H^{\mathrm{BV}}_\kappa F = (S_\kappa, F), \qquad F \in \mathfrak{F}_{\mathrm{Pal,amb}}. \tag{17}$$

The symbol s_κ is the master-generated odd variational differential on local functionals. The self-adjoint quantum Hamiltonian $H_{\mathrm{NL}}(x)$ appears on the Fock carrier in Section 8.

The sectorwise result follows from a stronger decomposition than the full CME alone. The curvature and kinetic functionals are even and antifield-free, hence

$$(S_{\mathrm{curv}}, S_{\mathrm{curv}}) = (S_{\mathrm{kin}}, S_{\mathrm{kin}}) = (S_{\mathrm{curv}}, S_{\mathrm{kin}}) = (S_{\mathrm{kin}}, S_{\mathrm{curv}}) = 0. \tag{18}$$

Their independently proved diffeomorphism Noether identities identify the action–cotangent cross brackets with zero,

$$(S_{\mathrm{curv}}, S_{\mathrm{cot}}) = (S_{\mathrm{cot}}, S_{\mathrm{curv}}) = (S_{\mathrm{kin}}, S_{\mathrm{cot}}) = (S_{\mathrm{cot}}, S_{\mathrm{kin}}) = 0, \tag{19}$$

and the cotangent sector satisfies

$$(S_{\mathrm{cot}}, S_{\mathrm{cot}}) = 0. \tag{20}$$

Consequently every sector separately satisfies its corresponding self-master equation, and every distinct pair has vanishing cross bracket. If $H_a = (S_a, \cdot)$, the generic even-CME nilpotence theorem gives $H_a^2 = 0$. Applying the same theorem to $S_a + S_b$ gives

$$0 = H_{a+b}^2 = H_a^2 + H_a H_b + H_b H_a + H_b^2, \tag{21}$$

so

$$\boxed{H_a^2 = 0, \qquad H_a H_b + H_b H_a = 0 \quad (a \neq b).} \tag{22}$$

Real base change preserves these identities with the original coefficients. Therefore

$$\boxed{s_\kappa(s_\kappa F) = 0 \quad \text{for every } F \in \mathfrak{F}_{\mathrm{Pal,amb}}.} \tag{23}$$

The theorem applies to arbitrary antifield dependence on the ambient real local-functional carrier.[4]

The cotangent Hamiltonian decomposes into the source gauge differential and antifield feedback,

$$H_{\mathrm{cot}} = Q_{\mathrm{source}} + F_{\mathrm{cot}}, \qquad s_\kappa = Q_{\mathrm{source},\mathbb{R}} + F_\kappa. \tag{24}$$

Consequently

$$s_\kappa^2 = Q_{\mathrm{source},\mathbb{R}} F_\kappa + F_\kappa Q_{\mathrm{source},\mathbb{R}} + F_\kappa^2, \tag{25}$$

and the complete feedback curvature satisfies

$$\boxed{Q_{\mathrm{source},\mathbb{R}} F_\kappa + F_\kappa Q_{\mathrm{source},\mathbb{R}} + F_\kappa^2 = 0} \tag{26}$$

on every real local functional.[5]

Equations (16), (15), (23), and (26) are bound together by the unrestricted real-master capstone.[6]

[4] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealHamiltonianVariation_actual_whole_square_zero`.
[5] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealMasterAntifieldFeedback_actual_whole_curvature_zero`.
[6] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealMaster_actual_unrestricted_Hamiltonian_capstone`.

4 Graded functional Jacobi and BV derivation structure

The local-functional bracket also satisfies the generic graded Jacobi identity. Let S and T be homogeneous functionals with parities $p, q \in \{0, 1\}$, and let F be arbitrary. Then

$$\boxed{(S,(T,F)) - (-1)^{(p+1)(q+1)}(T,(S,F)) = ((S,T),F).} \tag{27}$$

The source parities are measured by the descended parity operator of the same functional carrier. The third functional is arbitrary.[7]

For the even master S_κ, Eq. (27) makes s_κ a graded derivation of the bracket. If F has parity p,

$$\boxed{s_\kappa(F,G) = (s_\kappa F, G) + (-1)^{p+1}(F, s_\kappa G).} \tag{28}$$

Together, Eqs. (16), (23), and (27) give the ambient diffeomorphism gauge sector its full functional algebraic structure on the proved real carrier.[8]

Operationally, s_κ specifies how local field/ghost observables move along the gauge directions encoded by S_κ. The remainder of the paper identifies the physical symmetric carrier selected by the gravitational realization, enlarges the gauge law projectively, and then follows the unchanged classical projection into the quantum observables constructed in Paper 4.

5 Classical projection to the physical Einstein–scalar action

Let g be a smooth Lorentz metric, ϕ a smooth real scalar, and $\mathrm{ev}_{g,\phi}$ the holonomic physical jet evaluation that sends the Palatini inverse-density, connection, scalar, and their jet tower to the corresponding metric-derived fields. At density level the classical master projection satisfies

$$\boxed{\mathrm{ev}_{g,\phi}(\pi_{\mathrm{cl}}\mathcal{L}_\kappa)\,(x) = \mathcal{L}_{\mathrm{ES}}[g,\phi](x).} \tag{29}$$

The physical density is the sum

$$\mathcal{L}_{\mathrm{ES}}[g,\phi] = \frac{\sqrt{|g|}}{2\kappa}R[g] - \frac{1}{2}\sqrt{|g|}\sum_{i,j}(g^{-1})^{ij}(\partial_i\phi)(\partial_j\phi). \tag{30}$$

This is the explicit density represented in the formal definitions of the Einstein–Hilbert integrand and real-scalar metric action density.

Integration gives the same physical action,

$$\boxed{\mathrm{ev}_{g,\phi}(\pi_{\mathrm{cl}}S_\kappa) = S_{\text{Einstein-scalar}}[g,\phi].} \tag{31}$$

[9]

[7] *Formal counterpart:* `Tower.BV.realFunctionalJetBracket_actual_graded_Jacobi`.

[8] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealMaster_actual_graded_functional_capstone`.

[9] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealMasterDensity_actual_holonomic_same_integrated_physical_action`.

The projection also preserves every proved formal total derivative. For each spacetime jet multiindex J,

$$\boxed{\mathrm{ev}_{g,\phi}\Big(D^{\mathrm{jet}}_J\,\pi_{\mathrm{cl}}\mathcal{L}_\kappa\Big) = \partial_J\mathcal{L}_{\mathrm{ES}}[g,\phi].} \tag{32}$$

Here D^{jet}_J is the iterated formal total-jet derivative and ∂_J is the corresponding iterated physical coordinate derivative.[10]

Equations (29)–(32) give the ambient master's exact handoff to the physical Einstein–scalar theory. The same physical density is reconstructed intrinsically on the symmetric carrier in Section 7; the enlarged projective master has the identical classical projection and therefore inherits the same downstream native matter density.

6 Independent Palatini connection Euler equation and projective descent

The first-order carrier stores an independent affine connection Γ beside the Lorentz metric. The new variational result derives the connection equation directly from the same classical density in Eq. (6). The scalar kinetic polynomial contains no independent connection coordinate, and its connection Euler derivative therefore vanishes identically. The curvature polynomial supplies the entire connection response.[11][12]

Write

$$h^{bc} := \sqrt{|g|}\,g^{bc}. \tag{33}$$

For the repository component $\Gamma^a{}_{bc}$, the unscaled Euler residual computed from the literal Palatini curvature density is

$$\boxed{\begin{aligned} P_a{}^{bc} = \partial_a h^{bc} - \delta^b_a\,\partial_\ell h^{\ell c} + h^{br}\Gamma^c{}_{ar} + h^{\ell c}\Gamma^b{}_{\ell a} \\ - \delta^b_a h^{\ell r}\Gamma^c{}_{\ell r} - h^{bc}\Gamma^u{}_{ua}. \end{aligned}} \tag{34}$$

The real action derivative is exactly

$$\boxed{E_{\Gamma^a{}_{bc}}\mathcal{L}_{\mathrm{cl}} = \frac{1}{2\kappa}P_a{}^{bc}.} \tag{35}$$

Thus the sign, index order, and normalization of the connection equation arise directly from the represented action. The same identity is transported through the local-functional quotient and bound to the classical projection of the real master.[13][14]

Let

$$T^\rho{}_{\mu\nu} := \Gamma^\rho{}_{\mu\nu} - \Gamma^\rho{}_{\nu\mu}, \qquad \tau_a := T^u{}_{ua}. \tag{36}$$

[10] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealMasterDensity_actual_classical_physical_all_order`.
[11] *Formal counterpart:* `QGC12.BV.palatiniScalarKineticPolynomial_actual_no_connection_coordinate`.
[12] *Formal counterpart:* `QGC12.BV.palatiniScalarCurvaturePolynomial_actual_connection_Euler`.
[13] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealActionFunctional_actual_connection_Euler`.
[14] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealMasterDensity_actual_independent_connection_Euler_value`.

Using the weight-one covariant derivative of h^{bc}, the same computed residual admits the torsion-separated form

$$\boxed{P_a{}^{bc} = \nabla^\Gamma_a h^{bc} + h^{\ell c} T^b{}_{\ell a} - h^{bc}\tau_a - \delta^b_a \left(\nabla^\Gamma_\ell h^{\ell c} - h^{\ell c}\tau_\ell\right).} \tag{37}$$

This is a derived normalization of the actual Euler polynomial.[15]

The unrestricted stationary equation is

$$E_\Gamma S_{\mathrm{Pal}} = 0. \tag{38}$$

Solving its finite-dimensional difference system against the Levi–Civita connection gives the exact stationary class

$$\boxed{E_\Gamma S_{\mathrm{Pal}} = 0 \quad \Longleftrightarrow \quad \exists A_\mu \ : \ \Gamma^\rho{}_{\mu\nu} = \Gamma[g]^\rho{}_{\mu\nu} + \delta^\rho_\nu A_\mu.} \tag{39}$$

The one-form A is uniquely recoverable from the connection by

$$\boxed{A_\mu = \sum_\rho \left(\Gamma^\rho{}_{\rho\mu} - \Gamma[g]^\rho{}_{\rho\mu}\right).} \tag{40}$$

[16][17]

The projective degree of freedom is genuine. For every smooth Lorentz metric, the formal development constructs a stationary connection satisfying Eq. (38) outside the previously declared reduced connection condition. Hence unrestricted action stationarity selects the projective class, with Levi–Civita selected after the torsion fixing described below.[18]

The same projective form is measured directly by torsion. On the stationary class,

$$\boxed{\tau_\mu = \sum_\rho T^\rho{}_{\rho\mu} = -3A_\mu.} \tag{41}$$

Therefore zero torsion trace fixes the unique Levi–Civita representative:

$$\boxed{E_\Gamma S_{\mathrm{Pal}} = 0 \quad \text{and} \quad \tau = 0 \quad \Longleftrightarrow \quad \Gamma = \Gamma[g].} \tag{42}$$

Full torsion-freeness gives the stronger bridge to the existing reduced condition,

$$\boxed{E_\Gamma S_{\mathrm{Pal}} = 0 \quad \text{and} \quad T[\Gamma] = 0 \quad \Longleftrightarrow \quad \mathrm{ConnEq}(g,\Gamma) \quad \Longleftrightarrow \quad \Gamma = \Gamma[g].} \tag{43}$$

where ConnEq is the previously defined torsion-free, metric-compatible reduction. The earlier uniqueness theorem remains valid; the new result identifies its exact relation to the unrestricted action equation.[19][20][21][22]

[15] *Formal counterpart:* `QGC12.BV.palatiniConnectionEulerResidual_actual_covariant_density_torsion_decomposition`.

[16] *Formal counterpart:* `QGC12.BV.palatiniConnectionEulerEquation_actual_projective_solution_space`.

[17] *Formal counterpart:* `QGC12.BV.palatiniProjectiveConnection_actual_unique_form`.

[18] *Formal counterpart:* `QGC12.BV.palatiniConnectionEulerEquation_actual_nonunique_control`.

[19] *Formal counterpart:* `QGC12.BV.palatiniProjectiveConnection_actual_torsion_trace`.

[20] *Formal counterpart:* `QGC12.BV.palatiniConnectionEulerEquation_actual_torsion_trace_metric_locus`.

[21] *Formal counterpart:* `QGC12.BV.palatiniConnectionEulerEquation_actual_torsion_free_metric_locus`.

[22] *Formal counterpart:* `QGC12.BV.palatiniReducedConnectionEquation_actual_unique_metric_locus`.

On the Levi–Civita representative, the verified first-order metric variation still agrees with the second-order Einstein–Hilbert variation,

$$\boxed{\delta S^{\mathrm{red}}_{\mathrm{Pal}}[g,\Gamma[g];h] = \delta S^{(2)}_{\mathrm{EH}}[g;h].} \tag{44}$$

[23][24]

Finite gauge pullback preserves the full unrestricted class. If d is a smooth spacetime diffeomorphism and d^*A denotes the corresponding covector pullback, then

$$\boxed{d^*\left(\Gamma[g] + \delta A\right) = \Gamma[d^*g] + \delta(d^*A).} \tag{45}$$

Consequently

$$\boxed{E_\Gamma S_{\mathrm{Pal}}[g,\Gamma] = 0 \quad \Longrightarrow \quad E_\Gamma S_{\mathrm{Pal}}[d^*g, d^*\Gamma] = 0.} \tag{46}$$

The Levi–Civita representative remains an invariant sublocus,

$$\boxed{d^*(\Gamma[g]) = \Gamma[d^*g].} \tag{47}$$

[25][26][27]

Let d_λ be a smooth diffeomorphism path through the identity with velocity ξ. On the Levi–Civita representative, the derivative of the pulled connection remains the master-generated connection coefficient:

$$\boxed{\left.\frac{d}{d\lambda}\right|_0 \Gamma^\rho{}_{\mu\nu}[d^*_\lambda g] = \delta^{\mathrm{master}}_\xi \Gamma^\rho{}_{\mu\nu}[g].} \tag{48}$$

Its component content is the affine Lie law

$$\begin{aligned} \delta^{\mathrm{master}}_\xi \Gamma^\rho{}_{\mu\nu} = {} & \xi^\sigma \partial_\sigma \Gamma^\rho{}_{\mu\nu} + \Gamma^\rho{}_{\mu\sigma}\partial_\nu \xi^\sigma - \partial_\sigma \xi^\rho\, \Gamma^\sigma{}_{\mu\nu} \\ & + \partial_\mu\partial_\nu \xi^\rho + \partial_\mu \xi^\sigma\, \Gamma^\rho{}_{\sigma\nu}. \end{aligned} \tag{49}$$

The same coefficient is generated at quotient level by the full real master Hamiltonian,

$$\boxed{s_\kappa(1 \otimes [\Gamma^\rho{}_{\mu\nu}]) = 1 \otimes [\delta^{\mathrm{raw}}_{\mathrm{cot}} \Gamma^\rho{}_{\mu\nu}]\,.} \tag{50}$$

[28][29]

The affine term remains explicitly nontrivial on the quadratic shear

$$d_t(x) = x + t(x^1)^2 e_0, \qquad d_0 = \mathrm{id}, \tag{51}$$

whose Jacobian and derivative are

$$J^a{}_b(d_t;x) = \delta^a{}_b + 2t\,x^1 \delta^a{}_0 \delta^1{}_b, \tag{52}$$

[23] *Formal counterpart:* `QGC05.reducedPalatini_eq_secondOrder`.
[24] *Formal counterpart:* `QGC05.w4_secondOrder_is_leviCivita_reduction`.
[25] *Formal counterpart:* `QGC12.BV.palatiniProjectiveConnection_actual_finite_pullback`.
[26] *Formal counterpart:* `QGC12.BV.palatiniConnectionEulerEquation_actual_full_gauge_covariance`.
[27] *Formal counterpart:* `QGC12.BV.palatiniFirstOrderGaugePullback_actual_metric_reduction_intertwining`.
[28] *Formal counterpart:* `QGC12.BV.palatiniMetricConnection_actual_finite_gauge_derivative`.
[29] *Formal counterpart:* `QGC12.BV.palatiniGaugeRealHamiltonianVariation_actual_same_connection_quotient`.

$$\partial_c J^a{}_b(d_t;x) = 2t\,\delta^a{}_0\delta^1{}_b\delta^1{}_c, \tag{53}$$

and hence

$$\boxed{\left.\frac{d}{dt}\right|_{t=0} \partial_1 J^0{}_1(d_t;x) = 2 \neq 0.} \tag{54}$$

[30][31][32]

The finite projective transformation is

$$\boxed{P_A(\Gamma)^\rho{}_{\mu\nu} := \Gamma^\rho{}_{\mu\nu} + \delta^\rho_\nu A_\mu.} \tag{55}$$

It obeys the exact additive action laws

$$P_0 = \mathrm{id}, \qquad P_A \circ P_B = P_{A+B}, \qquad P_{-A} \circ P_A = \mathrm{id}, \tag{56}$$

and acts freely on the connection carrier. The stationary locus is exactly one orbit through Levi–Civita,

$$\boxed{\{\Gamma : E_\Gamma S_{\mathrm{Pal}} = 0\} = \mathrm{Orb}_{\mathrm{proj}}(\Gamma[g])\,.} \tag{57}$$

Moreover the fixed-metric pointwise linearized connection Euler operator L_Γ has

$$\boxed{\ker L_\Gamma = \mathrm{im}\, R_{\mathrm{proj}}, \qquad R_{\mathrm{proj}} \text{ injective},} \tag{58}$$

where $R_{\mathrm{proj}}(A)^\rho{}_{\mu\nu} = \delta^\rho_\nu A_\mu$. Thus the classical connection zero mode is exactly the projective direction at the proved pointwise level.[33][34][35]

The torsion trace transforms on arbitrary connections as

$$\boxed{\tau(P_B\Gamma) = \tau(\Gamma) - 3B.} \tag{59}$$

Hence every projective orbit contains a unique zero-trace representative, with parameter $B = \tau(\Gamma)/3$. On the stationary orbit this representative is precisely $\Gamma[g]$.[36][37][38]

The physical Einstein–real-scalar Palatini action is also exactly invariant under Eq. (55). The Ricci change is the antisymmetric two-form

$$R_{\mu\nu}(P_A\Gamma) - R_{\mu\nu}(\Gamma) = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{60}$$

whose contraction with the symmetric inverse metric density vanishes; the scalar kinetic density is connection-independent. Therefore

$$\boxed{S_{\mathrm{Pal}}[g, P_A\Gamma, \phi] = S_{\mathrm{Pal}}[g, \Gamma, \phi].} \tag{61}$$

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[30] *Formal counterpart:* `QGC12.BV.metricGaugeQuadraticShear_actual_jacobian`.
[31] *Formal counterpart:* `QGC12.BV.metricGaugeQuadraticShear_actual_Hessian`.
[32] *Formal counterpart:* `QGC12.BV.metricGaugeQuadraticShear_actual_nonzero_affine_derivative`.
[33] *Formal counterpart:* `QGC12.BV.palatiniConnectionProjectiveShift_actual_stationary_orbit`.
[34] *Formal counterpart:* `QGC12.BV.palatiniConnectionLinearEuler_actual_projective_kernel_image`.
[35] *Formal counterpart:* `QGC12.BV.palatiniProjectiveGenerator_actual_injective`.
[36] *Formal counterpart:* `QGC12.BV.palatiniConnectionProjectiveShift_actual_torsion_trace`.
[37] *Formal counterpart:* `QGC12.BV.palatiniConnectionProjectiveShift_actual_unique_zero_trace_parameter`.
[38] *Formal counterpart:* `QGC12.BV.palatiniConnectionProjectiveShift_actual_stationary_gauge_slice`.

This physical symmetry is proved directly from the represented Ricci polynomial and all-order smooth jet evaluation.[39][40]

The same transformation does not define an off-shell symmetry of the ambient free carrier in Section 2: its ordered inverse-density generators include antisymmetric directions absent from the physical tensor. The resulting density change is proved nonzero in the ambient local-functional quotient. This obstruction motivates the intrinsic symmetric-density BV carrier constructed next and remains an explicit regression theorem rather than being erased by that construction.[41][42]

7 Intrinsic symmetric-density and projective BV completion

The physical inverse metric density is symmetric, so its independent components are indexed by unordered pairs. Let $\mathcal{J}_{\mathrm{Pal}}^{\mathrm{sym}}$ denote the untruncated jet superalgebra obtained by replacing the ambient ordered inverse-density key with an intrinsic symmetric-pair key while retaining the independent connection, scalar, diffeomorphism ghost, and their cotangent partners. Here

$$\pi_{\mathrm{sym}} : \mathcal{J}_{\mathrm{Pal}}^{\mathrm{amb}} \longrightarrow \mathcal{J}_{\mathrm{Pal}}^{\mathrm{sym}}$$

is the proved quotient/transport map, while

$$\iota_{\mathrm{sym}} : \mathcal{J}_{\mathrm{Pal}}^{\mathrm{sym}} \longrightarrow \mathcal{J}_{\mathrm{Pal}}^{\mathrm{amb}}$$

is its proved section. The proved retraction identity is therefore

$$\boxed{\pi_{\mathrm{sym}} \circ \iota_{\mathrm{sym}} = \mathrm{id}_{\mathcal{J}_{\mathrm{Pal}}^{\mathrm{sym}}}.} \tag{62}$$

The field map intertwines total derivatives at every jet order, so the symmetry $h^{\mu\nu} = h^{\nu\mu}$ is intrinsic throughout the complete jet tower rather than imposed only at zero order.[43]

The cotangent normalization is determined by the size of the ordered-index fiber. Diagonal inverse-density antifields retain weight one; off-diagonal antifields acquire weight one half. The fiber normalization is

$$\boxed{\sum_{q:\,\pi_{\mathrm{sym}}(q)=p} w(q) = 1,} \tag{63}$$

and consequently both diagonal and off-diagonal canonical pairings equal one:

$$\boxed{(h^{\mu\mu}, h^*_{\mu\mu})_{\mathrm{sym}} = 1, \qquad (h^{\mu\nu}, h^*_{\mu\nu})_{\mathrm{sym}} = 1 \quad (\mu < \nu).} \tag{64}$$

The ambient quotient itself is proved not to be a global Poisson map: for an off-diagonal field/antifield pair, quotienting the ambient bracket differs by the normalization factor from taking the intrinsic

[39] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSmoothConnection_actual_real_density_invariance`.
[40] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSmoothConnection_actual_real_integrated_action_invariance`.
[41] *Formal counterpart:* `QGC12.BV.palatiniProjectiveScalarDensityChange_actual_not_divergence`.
[42] *Formal counterpart:* `QGC12.BV.palatiniProjectiveScalarRealActionFunctional_actual_changes`.
[43] *Formal counterpart:* `QGC12.BV.palatiniSymmetricCarrierSection_actual_retraction`.

symmetric bracket after quotient. The symmetric carrier therefore carries its own canonical antibracket rather than inheriting the ambient one by decree.[44][45][46][47]

Write $\mathfrak{F}^{\mathrm{sym}}_{\mathrm{Pal}}$ for the real local-functional quotient of this intrinsic carrier. Transport of the Palatini curvature, kinetic, and diffeomorphism cotangent sectors gives a symmetric master $S^{\mathrm{sym}}_{\kappa}$ satisfying

$$\boxed{(S^{\mathrm{sym}}_{\kappa}, S^{\mathrm{sym}}_{\kappa})_{\mathrm{sym}} = 0, \qquad (s^{\mathrm{sym}}_{\kappa})^2 F = 0 \quad (F \in \mathfrak{F}^{\mathrm{sym}}_{\mathrm{Pal}}).} \tag{65}$$

The second identity holds for arbitrary antifield dependence.[48][49]

The projective gauge sector is represented by an odd covector ghost η_μ and its antifield. Its connection action is

$$s_{\mathrm{proj}}\Gamma^{\rho}{}_{\mu\nu} = \delta^{\rho}_{\nu}\eta_{\mu}, \tag{66}$$

while η itself transforms by the covector Lie-transport law induced by the diffeomorphism ghost. The combined prolonged differential, denoted s_{sp}, satisfies

$$\boxed{s^2_{\mathrm{sp}} X = 0} \tag{67}$$

for every jet generator and hence for every untruncated jet polynomial.[50]

The projective variation of the symmetric Ricci density is derived directly:

$$\boxed{s_{\mathrm{proj}} R_{\mu\nu} = D_\mu \eta_\nu - D_\nu \eta_\mu.} \tag{68}$$

Its contraction with the symmetric inverse density vanishes in the local-functional quotient, while the scalar kinetic density is projectively inert. The curvature and kinetic sectors therefore satisfy the required projective Noether identities, and the enlarged evolutionary cotangent functional satisfies

$$\boxed{(S^{\mathrm{sp}}_{\mathrm{cot}}, S^{\mathrm{sp}}_{\mathrm{cot}})_{\mathrm{sp}} = 0.} \tag{69}$$

[51][52][53][54]

The final real master of Paper 5 is

$$\boxed{S^{\mathrm{sp}}_{\kappa} = \pi_{\mathrm{sym}}(S_\kappa) + S_{\Gamma^*\eta} + S_{\eta^* s\eta}.} \tag{70}$$

This identity is a proved ownership statement: the new master is the ambient-to-symmetric image of the existing real master plus precisely the earned projective connection-antifield and covector-ghost cotangent terms. Its functional antibracket satisfies

$$\boxed{(S^{\mathrm{sp}}_{\kappa}, S^{\mathrm{sp}}_{\kappa})_{\mathrm{sp}} = 0,} \tag{71}$$

[44] *Formal counterpart:* `QGC12.BV.palatiniSymmetricCotangentWeight_actual_fiber_normalization`.
[45] *Formal counterpart:* `QGC12.BV.palatiniSymmetricPair_actual_diagonal`.
[46] *Formal counterpart:* `QGC12.BV.palatiniSymmetricPair_actual_off_diagonal`.
[47] *Formal counterpart:* `QGC12.BV.palatiniSymmetricFunctionalQuotient_actual_not_global_Poisson`.
[48] *Formal counterpart:* `QGC12.BV.palatiniSymmetricRealMasterFunctional_actual_full_S0_S1_CME`.
[49] *Formal counterpart:* `QGC12.BV.palatiniSymmetricRealHamiltonianVariation_actual_whole_square_zero`.
[50] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricBRST_actual_nilpotence`.
[51] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricRicciDensity_actual_projective_variation`.
[52] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricBRST_actual_curvature_Noether`.
[53] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricBRST_actual_kinetic_Noether`.
[54] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricCotangentFunctional_actual_CME`.

and the associated Hamiltonian variation obeys

$$\boxed{s^{\mathrm{sp}}_{\kappa}(s^{\mathrm{sp}}_{\kappa}F) = 0 \quad \text{for every } F \in \mathfrak{F}^{\mathrm{sp}}_{\mathrm{Pal}}.} \tag{72}$$

Generic functional Jacobi and full antifield feedback therefore hold on the intrinsic enlarged carrier.[55][56][57]

Classical projection removes the odd/antifield sector and returns the symmetric Palatini action:

$$\boxed{\pi_{\mathrm{cl}} S^{\mathrm{sp}}_{\kappa} = S^{\mathrm{sym}}_{\mathrm{Pal}}.} \tag{73}$$

Holonomic physical evaluation then gives the same Einstein–real-scalar density as the original master, and subtraction of the Einstein–Hilbert term gives the same native matter density used by Paper 4.[58][59]

The paper-facing classical gauge capstone now joins the enlarged CME to physical projective action invariance, the exact stationary orbit, Eq. (58), injectivity of the generator, the unique zero-trace slice, and the Levi–Civita representative. Full functional properness or Koszul acyclicity is a stronger statement and remains outside the proved scope; the earned exactness statement is the pointwise fixed-metric connection kernel/image theorem above. Likewise, Levi–Civita substitution is not identified with symplectic elimination of the connection antifield.[60]

8 Reduction to the native action and quantum frequency

Evaluate the classical matter projection of the intrinsic symmetric-projective master on the nonlinear geometry of Paper 4. For a native radial point $x > 0$ and radial field data $(q, \dot{q}, q_{\mathrm{sp}})$, the reduced master density equals the metric-derived native temporal density:

$$\boxed{\mathcal{L}^{\mathrm{red}}_{\mathrm{master,matter}}(x, q, \dot{q}, q_{\mathrm{sp}}) = \mathcal{L}_{\mathrm{native}}(x, q, \dot{q}, q_{\mathrm{sp}}).} \tag{74}$$

[61]

The native temporal density has the explicit form

$$\mathcal{L}_{\mathrm{native}} = \frac{1}{2}\left[\left(\dot{q} + b(x)q\right)^2 - q_{\mathrm{sp}}^2\right], \tag{75}$$

[55] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricRealMasterDensity_actual_same_ambient_master_plus_earned_terms`.

[56] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricRealMasterFunctional_actual_full_S0_S1_CME`.

[57] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricRealHamiltonianVariation_actual_whole_square_zero`.

[58] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricRealMasterDensity_actual_classical_projection`.

[59] *Formal counterpart:* `QGC12.BV.palatiniProjectiveSymmetricRealMasterDensity_actual_same_native_matter_density`.

[60] *Formal counterpart:* `QGC12.BV.palatiniSymmetricProjectivePaper5ClassicalGaugeCapstone`.

[61] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeMasterReducedMatterDensity_actual_same_native_action`.

where $b(x)$ is the native area connection. At this stage the construction has returned from the abstract local-functional class to the holonomically evaluated radial representative density. Accordingly,

$$\begin{aligned} [\mathcal{L}_{\text{native}}] &= [\mathcal{L}_{\text{area}}] && \text{in the local total-divergence quotient,} \\ \mathcal{L}_{\text{native}} &= \mathcal{L}_{\text{area}} + \frac{d}{dx}B[q] && \text{as representative densities.} \end{aligned} \tag{76}$$

The second equality retains the boundary derivative because the radial boundary-completion calculation is performed on the representative action density. Its boundary completion is

$$\mathcal{L}_{\text{native}} = \mathcal{L}_{\text{area}} + \frac{d}{dx}B[q], \qquad \mathcal{L}_{\text{area}} = \frac{1}{2}\left(\dot{q}^{\,2} - q_{\text{sp}}^2 + V_{\text{NL}}(x)q^2\right). \tag{77}$$

[62]

The conjugate momentum is $\pi = \dot{q}$, and the Legendre transform gives

$$\boxed{\mathcal{H}_{\text{area}} = \frac{1}{2}\left(\pi^2 + q_{\text{sp}}^2 - V_{\text{NL}}(x)q^2\right).} \tag{78}$$

For signed Fourier mode k, $q_{\text{sp}} = 2\pi k\, q$ in the inherited compact temporal convention, so

$$\boxed{\mathcal{H}_k(x) = \frac{1}{2}\left(\pi_k^2 + \Omega_k^2(x)q_k^2\right),} \qquad \Omega_k^2(x) = (2\pi k)^2 - V_{\text{NL}}(x). \tag{79}$$

The exact formal implementation uses the signed-mode spectral label carried by Paper 4.[63]

Thus the final classical master and quantum energy generator meet through an explicit reduction chain:

$$\boxed{S_\kappa^{\text{sp}} \xrightarrow{\pi_{\text{cl}}} S_{\text{ES}} \xrightarrow{\rho_{\text{rad}}} S_{\text{native}} \xrightarrow{\text{Legendre}} \mathcal{H}_{\text{area}} \xrightarrow{k} \Omega_k^2(x) \longrightarrow H_{\text{NL}}(x).} \tag{80}$$

The final self-adjoint $H_{\text{NL}}(x)$ and strong Schrödinger evolution retain the independent operator-theoretic results of Paper 4.

9 Coherent quantum moments and the master Ward law

Let Ψ_z be the genuine coherent source state of Paper 4. For a complex mode value v, its exact field first moment is

$$\boxed{\langle \Psi_z, \Phi(v)\Psi_z\rangle = 2\,\text{Re}(vz).} \tag{81}$$

[64]

Define the geometric quantum jet moment

$$J_i^z(x) := \text{Re}\langle \Psi_z, \nabla_i \Phi(x)\Psi_z\rangle. \tag{82}$$

[62] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeMasterReducedMatterDensity_actual_same_boundary_completed_action`.

[63] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeMasterCompletedHamiltonian_actual_same_quantum_frequency`.

[64] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeZeroCoherentField_actual_first_moment`.

The Fock expectation equals the gradient of the real coherent mean field:

$$\boxed{J_i^z(x) = \partial_i \phi_z(x).} \tag{83}$$

[65]

For a smooth gauge vector ξ, the master-generated scalar coefficient is

$$\boxed{\delta_\xi^{\text{master}} \phi_z(x) = \sum_i \xi^i(x) J_i^z(x).} \tag{84}$$

The generated gradient coefficient is derivative-correct:

$$\boxed{\delta_\xi^{\text{master}} J_a^z = \sum_i \left[(\partial_a \xi^i) J_i^z + \xi^i \partial_a J_i^z \right].} \tag{85}$$

[66]

Let d_λ be a smooth diffeomorphism path through the identity with tangent ξ. The finite-orbit intertwining theorem gives

$$\boxed{\left. \frac{d}{d\lambda} \right|_0 \langle \Psi_z, \Phi(d_\lambda x) \Psi_z \rangle = \delta_\xi^{\text{master}} \phi_z(x).} \tag{86}$$

[67]

Equations (81)–(86) establish a master–quantum *observable intertwining*: the classical master differential is visible on actual Fock expectation values. The quantum carrier itself continues to use the Paper 4 self-adjoint Hamiltonian and unitary evolution; an intrinsic odd quantum gauge operator is part of the subsequent program. For the constant native radial gauge vector $\xi_{\mathrm{r}} = e_1$, the generated scalar coefficient is

$$\boxed{\delta_{\xi_{\mathrm{r}}}^{\text{master}} \phi_z(x) = \frac{\operatorname{Re} F_z}{\mathcal{T}_z(x^1)},} \qquad F_z := \text{nativeCoherentSourceFlux}(z), \tag{87}$$

where $\mathcal{T}_z$ is the branch transport factor. On the canonical coherent source $z = i$,

$$\boxed{\operatorname{Re} F_i \neq 0, \qquad \mathcal{T}_i(x^1) < 0, \qquad \delta_{\xi_{\mathrm{r}}}^{\text{master}} \phi_i(x) \neq 0.} \tag{88}$$

This supplies an explicit nonvacuous gauge-response control on the distinguished self-consistent branch.[68][69]

[65] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentSource_actual_quantum_jet_first_moment`.

[66] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentSource_actual_generated_master_quantum_gradient_Ward_law`.

[67] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentQuantumFieldMoment_actual_master_finite_gauge_intertwining`.

[68] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentSource_actual_generated_radial_gauge_coefficient`.

[69] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCanonicalCoherentSource_actual_generated_master_gauge_nonzero`.

10 Master-projected matter and quantum Hilbert stress

Define the matter component of the classical master projection by removing its Einstein–Hilbert summand:

$$\mathcal{L}_{\text{master,matter}} := \text{ev}_{g,\phi}(\pi_{\text{cl}}\mathcal{L}_{\kappa}^{\text{sp}}) - \mathcal{L}_{\text{EH}}. \tag{89}$$

The resulting density is exactly

$$\boxed{\mathcal{L}_{\text{master,matter}}(x) = -\frac{1}{2}\sqrt{|g(x)|}\sum_{i,j}(g^{-1})^{ij}(x)\partial_i\phi(x)\partial_j\phi(x).} \tag{90}$$

[70]

The coherent-state reduction explains why the independently constructed quantum stress has this Hilbert form. Vacuum subtraction of the full coherent two-field pairing gives

$$\boxed{\Delta_z W(u,v) := \langle\Psi_z, \Phi(u)\Phi(v)\Psi_z\rangle - \langle\Psi_0, \Phi(u)\Phi(v)\Psi_0\rangle = 4\,\Re(uz)\Re(vz).} \tag{91}$$

The coherent mean map is

$$\phi_z(u) = 2\Re(uz), \qquad \Delta_z W(u,v) = \phi_z(u)\phi_z(v). \tag{92}$$

Applying this to the spacetime gradient values gives the rank-one relative jet pair

$$P^z_{ij}(x) = J^z_i(x)J^z_j(x) = \partial_i\phi_z(x)\partial_j\phi_z(x). \tag{93}$$

The metric stress map is linear in this pair,

$$T_{ij}[P] = P_{ij} - \frac{1}{2}g_{ij}g^{ab}P_{ab}, \tag{94}$$

so the complete normal-plus-anomalous coherent expectation after reference-vacuum subtraction satisfies

$$\boxed{\Delta T^z_{ij} = T_{ij}[P^z] = T_{ij}[\phi_z].} \tag{95}$$

This factorization is the bridge: the Fock calculation and the classical-looking scalar Hilbert tensor meet because the coherent relative two-point function is exactly the outer product of the coherent mean-field jets.[71][72]

Let g_λ be the admissible inverse-metric exponential variation used in the formal development, with tangent h^{ij} at $\lambda = 0$. On the coherent source branch,

$$\boxed{\left.\frac{d}{d\lambda}\right|_0 \mathcal{L}_{\text{master,matter}}[g_\lambda, \phi_z] = -\frac{1}{2}\sqrt{|g|}\sum_{i,j}\Delta T^z_{ij}h^{ij}.} \tag{96}$$

[73]

[70] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeMasterProjectedMatterDensity_actual_same_real_density`.

[71] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeZeroCoherentField_actual_relative_pairing`.

[72] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeZeroCoherentGeometricStress_actual_real_mean_field`.

[73] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentSource_actual_master_projected_quantum_Hilbert_response`.

The same tensor ΔT^z_{ij} was independently constructed in Paper 4 from the coherent Fock field, including its normal and anomalous sectors. Hence the two routes close on one stress tensor:

$$\boxed{\begin{array}{ccccc} S^{\rm sp}_\kappa & \xrightarrow{\pi_{\rm cl}} & S^{\rm sym}_{\rm Pal} & \xrightarrow{\pi_{\rm matter}} & S_{\rm matter} \\ & & & & \downarrow \delta_g \\ \Psi_z & \xrightarrow{\text{quadratic Fock expectation}} & \Delta T^z_{ij} & = & \text{Hilbert response}(S_{\rm matter}). \end{array}} \tag{97}$$

This identity gives the source tensor simultaneous quantum-observable and master-action provenance.

The stress remains vacuum-relative at the present mathematical boundary. Paper 4 identifies the Hadamard/microlocal route to an absolute locally covariant renormalized stress under a common prescription.[8, 9, 10]

11 Gauge covariance of the quantum Einstein residual

Define

$$\mathcal{E}_{ij}[g,T] := G_{ij}[g] - \kappa T_{ij}. \tag{98}$$

Paper 4 establishes

$$\boxed{\mathcal{E}_{ij}[g_z, \Delta T^z] = 0} \tag{99}$$

for every admissible coherent source branch.

The quantum stress entering Paper 5 is rebuilt after gauge pullback from the pulled Fock-valued jets. If $\mathcal{G}_d$ denotes this gauge realization, then

$$\boxed{\mathcal{G}_d \Delta T^z = d^* \Delta T^z.} \tag{100}$$

[74] The Einstein tensor is recomputed from the pulled metric and satisfies

$$G[d^* g] = d^* G[g]. \tag{101}$$

Therefore, before imposing the source equation,

$$\boxed{\mathcal{E}[d^* g, d^* T] = d^* \mathcal{E}[g,T].} \tag{102}$$

[75]

The geometric covariance in Eq. (102) is standard tensor structure. The paper-specific result is the identification

$$\boxed{\mathcal{G}_d(\Delta T^z_{\text{Paper 4}}) = d^* \Delta T^z_{\text{Paper 4}},} \tag{103}$$

where the left side is recomputed from the previously constructed coherent state, reference vacuum, and pulled field jets on their constructed Fock carrier.

Specializing the off-shell identity to the self-consistent branch yields

$$\boxed{\mathcal{E}[g_z, \Delta T^z] = 0 \quad \Longrightarrow \quad \mathcal{E}[d^* g_z, \mathcal{G}_d \Delta T^z] = 0.} \tag{104}$$

[74] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentGaugeStress_actual_original_quantum_observable_covariance`.

[75] *Formal counterpart:* `QGC12.Matter.quantumSourceResidual_actual_gauge_pullback`.

[76] For any smooth gauge path d_λ, the stronger componentwise orbit statement is

$$\boxed{\frac{d}{d\lambda}\mathcal{E}_{ij}[d_\lambda^* g_z,\, \mathcal{G}_{d_\lambda}\Delta T^z]\,(x) = 0.} \tag{105}$$

Thus the zero residual is constant along every established gauge orbit at the level of each spacetime component.[77]

12 Physical-clock compatibility

Let $J^{\mathrm{phys}}_{B,i}(x;\psi)$ denote the finite-bank physical Cauchy field jet at native time x, and let $\mathcal{G}_d$ denote weak coordinate-gauge jet pullback. Write U_x for the physical unitary from the native anchor X_a to x. The clock-intertwining theorem is

$$\boxed{\mathcal{G}_d J^{\mathrm{phys}}_{B,i}(X_a; U_x\psi) = U_x\left(\mathcal{G}_d J^{\mathrm{phys}}_{B,i}(x;\psi)\right).} \tag{106}$$

[78]

Equation (106) states the compatibility of the geometric gauge map with the physical Fock evolution. The corresponding stress statement is equally explicit. If the mapped point satisfies $x(d(p)) = x$, then

$$\boxed{\left(\mathcal{G}_d\Delta T^z\right)_{ij}(p) = \sum_{a,b} J^a{}_i(d;p)\, J^b{}_j(d;p)\, T^{z,\mathrm{phys}}_{ab}(x, d(p))\,.} \tag{107}$$

The right-hand side is the previously constructed evolving physical quantum source at native time x, evaluated at the mapped point and pulled back by the actual Jacobian.[79]

The time domain is the strictly positive native interval inherited from Paper 4. Equations (106) and (107) determine the positive-time gauge/clock exchange on both field jets and the resulting relative stress.

13 Observable content and comparison surface

The master/quantum exchange adds a small set of directly evaluable quantities to the Paper 4 comparison surface. Define

$$\Phi_z(x) := \langle \Psi_z, \Phi(x)\Psi_z\rangle, \qquad J^z_i(x) := \mathrm{Re}\langle \Psi_z, \nabla_i\Phi(x)\Psi_z\rangle, \tag{108}$$

$$\delta^{\mathrm{master}}_\xi \phi_z := \sum_i \xi^i J^z_i, \qquad \mathcal{E}_{ij} := G_{ij} - \kappa\Delta T^z_{ij}. \tag{109}$$

[76] *Formal counterpart:*
`QGC12.Quantum.NonlinearFlux.nativeCoherentSourceResidual_actual_gauge_preserves_self_consistency`.
[77] *Formal counterpart:*
`QGC12.Quantum.NonlinearFlux.nativeCoherentSourceResidual_actual_gauge_orbit_derivative_zero`.
[78] *Formal counterpart:*
`QGC12.Quantum.NonlinearFlux.nativePhysicalCauchyJetGaugePullback_actual_clock_intertwining`.
[79] *Formal counterpart:*
`QGC12.Quantum.NonlinearFlux.nativeCoherentGaugeStress_actual_same_evolving_quantum_source`.

The established identities are

$$\boxed{\Phi_z = \phi_z, \qquad J_i^z = \partial_i \phi_z, \qquad \mathcal{E}_{ij}[g_z, \Delta T^z] = 0.} \tag{110}$$

The matter response adds

$$\boxed{\delta_g S_{\mathrm{master,matter}} = -\frac{1}{2} \int \sqrt{|g|}\, \Delta T_{ij}^z \, \delta g^{ij},} \tag{111}$$

whenever the corresponding physical integral is admissible; the local density identity of Eq. (96) is the theorem used throughout this paper.

The resulting consistency tests are deterministic. The connection/gauge tests are

$$\boxed{E_\Gamma S_{\mathrm{Pal}} = 0 \overset{?}{\Longleftrightarrow} \Gamma = \Gamma[g] + \delta A, \qquad \ker L_\Gamma \overset{?}{=} \mathrm{im}\, R_{\mathrm{proj}}, \qquad \tau(P_B \Gamma) \overset{?}{=} \tau(\Gamma) - 3B.} \tag{112}$$

The BV-side regression additionally checks the intrinsic symmetric pairing, the enlarged CME $(S_\kappa^{\mathrm{sp}}, S_\kappa^{\mathrm{sp}})_{\mathrm{sp}} = 0$, and arbitrary-antifield square-zero of s_κ^{sp}. The observable/source tests are

$$\left.\frac{d}{d\lambda}\right|_0 \Phi_z(d_\lambda x) \overset{?}{=} \delta_\xi^{\mathrm{master}} \phi_z(x), \tag{113}$$

$$\delta_g \mathcal{L}_{\mathrm{master,matter}} \overset{?}{=} -\frac{1}{2} \sqrt{|g|}\, \Delta T_{ij}^z \, \delta g^{ij}, \tag{114}$$

$$\mathcal{E}[d^* g, \mathcal{G}_d \Delta T^z] \overset{?}{=} d^* \mathcal{E}[g, \Delta T^z], \tag{115}$$

$$\Omega^2_{k,\mathrm{master}}(x) \overset{?}{=} \Omega^2_{k,\mathrm{NL}}(x). \tag{116}$$

Each equality is proved on the declared carrier and supplies a direct regression target for later numerical or symbolic evaluation.

The detector-facing comparison observables inherited from Paper 4 remain

$$\delta g, \qquad \delta V, \qquad \delta N_k, \qquad \delta T_{ij}, \qquad \delta R. \tag{117}$$

Paper 5 supplies the transformation and action-provenance laws constraining these quantities. The eventual quantum-gauge stage can therefore be tested against observables already fixed by the preceding papers.

14 Master–quantum exchange theorem

The preceding results assemble into one paper-level theorem.

Theorem (symmetric-projective master–quantum exchange closure). Fix a positive Einstein coupling κ, an admissible coherent amplitude z with nonzero real mean-field flux, and the corresponding self-consistent branch g_z. Let $\mathfrak{F}_{\mathrm{Pal}}^{\mathrm{sp}}$ be the intrinsic symmetric/projective real local-functional carrier. Then the final classical master obeys

$$\boxed{(S_\kappa^{\mathrm{sp}}, S_\kappa^{\mathrm{sp}})_{\mathrm{sp}} = 0, \qquad s_\kappa^{\mathrm{sp}} F = (S_\kappa^{\mathrm{sp}}, F)_{\mathrm{sp}}, \qquad (s_\kappa^{\mathrm{sp}})^2 F = 0 \quad (F \in \mathfrak{F}_{\mathrm{Pal}}^{\mathrm{sp}}).} \tag{118}$$

Its provenance is

$$\boxed{S_\kappa^{\mathrm{sp}} = \pi_{\mathrm{sym}}(S_\kappa) + S_{\Gamma^*\eta} + S_{\eta^* s\eta},} \tag{119}$$

and the intrinsic bracket inherits the generic graded Jacobi theorem on homogeneous functionals.

The classical projection satisfies

$$\boxed{\pi_{\mathrm{cl}} S_\kappa^{\mathrm{sp}} = S_{\mathrm{Pal}}^{\mathrm{sym}}, \qquad \mathrm{ev}_{g_z,\phi_z}(\pi_{\mathrm{cl}} \mathcal{L}_\kappa^{\mathrm{sp}}) = \mathcal{L}_{\mathrm{ES}}[g_z, \phi_z].} \tag{120}$$

The matter component of this projection is the same native matter density used downstream in Paper 4.

The independent connection equation, projective action, and zero mode satisfy

$$\boxed{E_\Gamma S_{\mathrm{Pal}} = 0 \quad \Longleftrightarrow \quad \Gamma^\rho{}_{\mu\nu} = \Gamma[g]^\rho{}_{\mu\nu} + \delta^\rho_\nu A_\mu,} \tag{121}$$

$$\boxed{S_{\mathrm{Pal}}[g, P_A\Gamma, \phi] = S_{\mathrm{Pal}}[g, \Gamma, \phi], \qquad \ker L_\Gamma = \operatorname{im} R_{\mathrm{proj}}, \qquad R_{\mathrm{proj}} \text{ injective.}} \tag{122}$$

The torsion trace obeys

$$\boxed{\tau(P_B\Gamma) = \tau(\Gamma) - 3B,} \tag{123}$$

so every stationary orbit has exactly one zero-trace representative and

$$\boxed{E_\Gamma S_{\mathrm{Pal}} = 0, \quad \tau = 0 \quad \Longleftrightarrow \quad \Gamma = \Gamma[g].} \tag{124}$$

Finite diffeomorphism pullback transports the projective action covariantly and preserves the stationary orbit.

Radial reduction and Legendre transformation of the same classical matter projection give

$$\boxed{\rho_{\mathrm{rad}}(\pi_{\mathrm{matter}} \mathcal{L}_\kappa^{\mathrm{sp}}) = \mathcal{L}_{\mathrm{native}}, \qquad \mathcal{H}_k(x) = \frac{1}{2}\left(\pi_k^2 + \Omega_k^2(x) q_k^2\right),} \tag{125}$$

with the same Ω_k^2 used by the Paper 4 quantum Hamiltonian.

The quantum observable exchange remains

$$\boxed{\langle \Psi_z, \Phi(v)\Psi_z\rangle = 2\,\mathrm{Re}(vz), \qquad J_i^z = \partial_i \phi_z, \qquad \left.\frac{d}{d\lambda}\right|_0 \Phi_z(d_\lambda x) = \delta_\xi^{\mathrm{master}} \phi_z(x).} \tag{126}$$

The matter and gravitational responses satisfy

$$\boxed{\delta_g \mathcal{L}_{\mathrm{master,matter}} = -\frac{1}{2}\sqrt{|g|}\,\Delta T_{ij}^z\,\delta g^{ij}, \qquad \mathcal{E}[d^* g, \mathcal{G}_d \Delta T^z] = d^* \mathcal{E}[g, \Delta T^z].} \tag{127}$$

Hence

$$\boxed{\mathcal{E}[g_z, \Delta T^z] = 0 \quad \Longrightarrow \quad \mathcal{E}[d^* g_z, \mathcal{G}_d \Delta T^z] = 0.} \tag{128}$$

Finally, the physical Cauchy jets satisfy the positive-time clock intertwining

$$\boxed{\mathcal{G}_d J_B^{\mathrm{phys}}(X_a; U_x\psi) = U_x(\mathcal{G}_d J_B^{\mathrm{phys}}(x;\psi)).} \tag{129}$$

Equations (118)–(129) close the classical master-action/projective-connection/observable/source exchange layer. The continuing boundary starts beyond this classical closure: full functional properness, metric-only symplectic BV reduction, and an intrinsic quantum gauge/cohomological realization on the Fock carrier.

15 Scope and continuing mathematical boundary

Paper 5 closes the classical master law on an intrinsic symmetric/projective Palatini carrier and binds that law to the Paper 4 quantum sector through the unchanged physical action, native reduction, observables, stress, clock, and Einstein residual. The remaining questions begin beyond the enlarged CME and its proved pointwise projective exactness.

Full functional properness and cohomological exactness. The fixed-metric pointwise connection result proves

$$\ker L_\Gamma = \operatorname{im} R_{\mathrm{proj}}$$

and injectivity of the projective generator. The enlarged master also has arbitrary-antifield square-zero and full antifield feedback. A stronger functional statement would establish properness or Koszul–Tate acyclicity for the complete field/ghost/antifield complex, including the mixed metric/connection sector. That theorem is not identified with the pointwise kernel/image result and remains a subsequent homological stage.

Gauge fixing and metric-only BV reduction. The torsion trace gives a unique classical slice on every projective orbit,

$$\tau(P_B\Gamma) = 0 \quad \Longleftrightarrow \quad B = \frac{1}{3}\tau(\Gamma),$$

and on stationary connections that representative is Levi–Civita. A gauge-fixing fermion or equivalent BV Lagrangian-submanifold implementation is a stronger construction. Likewise, replacing the stationary connection by $\Gamma[g]$ does not by itself eliminate the connection/connection-antifield pair symplectically. A metric-only master therefore requires a separate proof that the antibracket and generated differential descend under the corresponding BV reduction.

Quantum gauge representation and physical state space. The enlarged classical master differential acts on local functionals, while the Paper 4 Hamiltonian acts on the Fock carrier. A subsequent construction can ask for a genuine quantum gauge operator or differential with a proved domain on that carrier. A natural target is a nilpotent operator Q_{q} with dynamical compatibility

$$Q_{\mathrm{q}}(x)^2 = 0, \qquad Q_{\mathrm{q}}(x_2)U_{\mathrm{NL}}(x_2, x_1) = U_{\mathrm{NL}}(x_2, x_1)Q_{\mathrm{q}}(x_1), \tag{130}$$

followed by a physical-state construction from its kernel/image or cohomology once the required closed-domain standing is proved. The field-moment Ward theorem and stress covariance established here provide exact observables that such a representation must reproduce.

Quantum master equation and anomaly analysis. Classical BV closure now includes the intrinsic diffeomorphism-plus-projective master. Quantization asks whether this gauge architecture survives the quantum and renormalization steps. At schematic BV level the target is

$$\frac{1}{2}(S, S) - i\hbar\,\Delta S = 0, \tag{131}$$

or the corresponding renormalized identity on the eventual quantum gauge carrier. Establishing the quantum BV Laplacian or its replacement, its domains, and the resulting anomaly criterion forms a distinct mathematical stage.[5, 6]

Boundary and asymptotic extension. The local-functional carrier identifies total divergences inside the variational quotient. A boundary-aware extension can retain horizon terms, asymptotic charges, and boundary action contributions as explicit data. The interior symmetric/projective master supplies the variational law to which those structures must attach.

Concrete next observable. The natural quantitative test of an intrinsic quantum gauge realization is a Ward or cohomological calculation returning an observable already established by Papers 2–4. Equation (130) gives one concrete target: a nilpotent quantum gauge operator whose physical subspace is preserved by U_{NL}, with a Ward identity reproducing the field-moment or stress transformation established here.

These directions continue the same series trajectory. The classical projective ambiguity itself is no longer part of the open boundary: its physical action symmetry, intrinsic ghost/antifield realization, enlarged CME, pointwise zero-mode identification, and unique Levi–Civita representative are now established. The next boundary is homological, symplectic-reductive, and quantum.

16 Conclusion

Paper 5 moves the series upstream from a closed Einstein–scalar quantum sector to the functional gauge law that generates the same physical action. The development now distinguishes three classical layers. The ambient free Palatini master supplies the untruncated jet/local-functional BV calculus and satisfies its own functional CME, unrestricted square-zero law, generic graded Jacobi identity, and antifield-feedback closure. The intrinsic symmetric-density carrier then represents the physical inverse metric density with its correct cotangent normalization and its own canonical antibracket. Finally, the projective covector ghost and antifield enlarge that symmetric master into the diffeomorphism-plus-projective master S_κ^{sp}.

The final master satisfies

$$\boxed{(S_\kappa^{\mathrm{sp}}, S_\kappa^{\mathrm{sp}})_{\mathrm{sp}} = 0, \qquad (s_\kappa^{\mathrm{sp}})^2 F = 0 \quad (F \in \mathfrak{F}_{\mathrm{Pal}}^{\mathrm{sp}}),} \tag{132}$$

including arbitrary antifield dependence. Its density is formally bound to the image of the ambient master plus the projective connection-antifield and projective-ghost cotangent terms, so the later construction retains explicit provenance rather than replacing the original master retrospectively.

The independent connection Euler derivative of the same physical Palatini action is computed and solved. Its stationary locus is exactly the projective orbit

$$\Gamma = \Gamma[g] + \delta A.$$

The physical action is invariant under that transformation, the pointwise linearized Euler kernel is exactly the image of the injective projective generator, and the torsion trace transforms as $\tau(P_B\Gamma) = \tau(\Gamma) - 3B$. Every stationary orbit therefore contains exactly one zero-trace representative, namely the Levi–Civita connection. The old ambient projective counterexample remains valid and shows why the intrinsic symmetric carrier is mathematically necessary.

Classical projection of S_κ^{sp} returns the same Einstein–real-scalar density used by the nonlinear geometry of Paper 4. Its matter component reduces to the same native temporal action, Hamiltonian density, and mode frequency used by the physical quantum theory. The coherent Fock first moment equals the real mean field, the diffeomorphism master law is visible on finite-orbit quantum observables, the Hilbert response equals the existing coherent vacuum-relative quantum stress, the Einstein residual transforms covariantly, and gauge pullback remains compatible with the positive-time physical clock.

The resulting series-level chain is

$$\boxed{\begin{gathered} S_\kappa \xrightarrow{\pi_{\mathrm{sym}}} S_\kappa^{\mathrm{sym}} \longrightarrow S_\kappa^{\mathrm{sp}} \longrightarrow S_{\text{Einstein-scalar}} \longrightarrow S_{\mathrm{native}}, \\ S_{\mathrm{native}} \longrightarrow H_{\mathrm{NL}} \longrightarrow \mathcal{Q}(g_{\mathrm{NL}}) \longrightarrow \Delta T_{\mu\nu} \longrightarrow \mathcal{E}_{\mu\nu} = 0. \end{gathered}} \tag{133}$$

The connection-side descent inside this chain is itself structured:

$$\boxed{E_\Gamma S_{\mathrm{Pal}} = 0 \iff \Gamma \in \mathrm{Orb}_{\mathrm{proj}}(\Gamma[g]) \xrightarrow{\tau=0} \Gamma[g].} \tag{134}$$

The continuing gradient begins beyond this classical closure. Full functional properness/Koszul acyclicity, a metric-only symplectic BV reduction, and connection-antifield elimination remain separate mathematical problems. On the quantum side, the next primitive is an intrinsic gauge differential on the Fock carrier together with its physical-state cohomology, dynamical compatibility, and quantum master/anomaly analysis. The projective direction itself is no longer an unresolved classical ambiguity: it is now part of the proved BV architecture of the Einstein–real-scalar sector.

A Reader theorem map

The table below gives the principal propositions first in ordinary notation and then by Lean theorem name. It separates the ambient provenance, intrinsic symmetric construction, projective BV extension, and physical/quantum handoff.

Role	Mathematical proposition	Lean theorem
Ambient master	$(S_\kappa, S_\kappa)_{\mathrm{amb}} = 0$ on the ordered free Palatini local-functional carrier.	`palatiniGaugeRealMasterFunctional_actual_full_S0_S1_CME`
Ambient square-zero	$s_\kappa^2 F = 0$ for arbitrary antifield-dependent ambient real local functionals.	`palatiniGaugeRealMaster_actual_unrestricted_Hamiltonian_capstone`
Symmetric carrier	The intrinsic symmetric carrier is a retraction of its section and carries normalized diagonal/off-diagonal cotangent pairing.	`palatiniSymmetricCarrierSection_actual_retraction`; `palatiniSymmetricCotangentWeight_actual_fiber_normalization`
Poisson boundary	The ambient quotient is not a global Poisson map to the intrinsic symmetric bracket.	`palatiniSymmetricFunctionalQuotient_actual_not_global_Poisson`
Symmetric master	$(S_\kappa^{\mathrm{sym}}, S_\kappa^{\mathrm{sym}})_{\mathrm{sym}} = 0$ and $(s_\kappa^{\mathrm{sym}})^2 = 0$.	`palatiniSymmetricRealMasterFunctional_actual_full_S0_S1_CME`; `palatiniSymmetricRealHamiltonianVariation_actual_whole_square_zero`
Projective BRST	The diffeomorphism-plus-projective prolonged differential squares to zero; the projective Ricci response is $D_\mu\eta_\nu - D_\nu\eta_\mu$.	`palatiniProjectiveSymmetricBRST_actual_nilpotence`; `palatiniProjectiveSymmetricRicciDensity_actual_projective_variation`
Enlarged master	$(S_\kappa^{\mathrm{sp}}, S_\kappa^{\mathrm{sp}})_{\mathrm{sp}} = 0$ and $(s_\kappa^{\mathrm{sp}})^2 F = 0$ for arbitrary antifield dependence.	`palatiniProjectiveSymmetricRealMasterFunctional_actual_full_S0_S1_CME`; `palatiniProjectiveSymmetricRealHamiltonianVariation_actual_whole_square_zero`

Role	Mathematical proposition	Lean theorem
Master ownership	S^{sp}_{κ} is the symmetric image of the ambient master plus the projective connection-antifield and projective-ghost cotangent terms.	`palatiniProjectiveSymmetricRealMasterDensity_actual_same_ambient_master_plus_earned_terms`
Connection Euler	$E_{\Gamma} S_{\mathrm{Pal}} = 0 \iff \Gamma^{\rho}{}_{\mu\nu} = \Gamma[g]^{\rho}{}_{\mu\nu} + \delta^{\rho}_{\nu} A_{\mu}$.	`palatiniConnectionEulerEquation_actual_projective_solution_space`
Projective symmetry	The physical action is invariant under P_A, and the stationary set is exactly one projective orbit through Levi–Civita.	`palatiniProjectiveSmoothConnection_actual_real_integrated_action_invariance`; `palatiniConnectionProjectiveShift_actual_stationary_orbit`
Zero-mode identity	The fixed-metric pointwise linearized connection-Euler kernel is exactly the image of the injective projective generator.	`palatiniConnectionLinearEuler_actual_projective_kernel_image`; `palatiniProjectiveGenerator_actual_injective`
Levi–Civita slice	$\tau(P_B\Gamma) = \tau(\Gamma) - 3B$; each orbit has a unique zero-trace representative, equal to $\Gamma[g]$ on the stationary locus.	`palatiniConnectionProjectiveShift_actual_unique_zero_trace_parameter`; `palatiniConnectionProjectiveShift_actual_stationary_gauge_slice`
Classical projection	The enlarged master projects to the symmetric Palatini density and the same native Paper 4 matter density.	`palatiniProjectiveSymmetricRealMasterDensity_actual_classical_projection`; `palatiniProjectiveSymmetricRealMasterDensity_actual_same_native_matter_density`
Quantum Ward law	$\langle \Psi_z, \Phi(v)\Psi_z \rangle = 2\Re(vz)$ and $\frac{d}{d\lambda}\Phi_z(d_\lambda x) = \delta^{\mathrm{master}}_{\xi} \phi_z(x)$.	`nativeCoherentQuantumFieldMoment_actual_master_finite_gauge_intertwining`
Stress bridge	$\Delta_z W(u,v) = \phi_z(u)\phi_z(v)$, hence the Fock stress equals the master-projected Hilbert response.	`nativeZeroCoherentField_actual_relative_pairing`; `nativeCoherentSource_actual_master_projected_quantum_Hilbert_response`
Einstein residual	$\mathcal{E}[d^*g, \mathcal{G}_d \Delta T] = d^*\mathcal{E}[g, \Delta T]$, preserving $\mathcal{E} = 0$ on the established gauge orbit.	`nativeCoherentSourceResidual_actual_quantum_gauge_covariance`; `nativeCoherentSourceResidual_actual_gauge_preserves_self_consistency`
Clock intertwining	Gauge pullback of the physical Cauchy jet commutes with the positive-time physical evolution in the proved sense.	`nativePhysicalCauchyJetGaugePullback_actual_clock_intertwining`

B Verification and scope

The Paper 5 verification revision is frozen at repository revision

`cfd7f003eb6622a90951e0773ce5cb0b29633df0`,

covering the ambient master/action bridge, independent connection-Euler closure, physical projective symmetry and obstruction analysis, and the intrinsic symmetric-projective BV completion through M2647. The principal focused audit roots are

`QGC12PalatiniFunctionalAudit`, `QGC12MasterQuantumGaugeAudit`, `QGC12WholeMasterAudit`, `QGC12FunctionalJacobiAudit`, `QGC12MetricLocusAudit`, `QGC12PalatiniConnectionEulerAudit`, `QGC12PalatiniProjectiveGaugeAudit`, `QGC12SymmetricProjectiveBVAudit`.

The M2455–M2483 connection-Euler tranche computes the actual independent-connection Euler derivative from the represented Palatini density, proves the scalar kinetic contribution zero, normalizes the residual into covariant-density/torsion form, solves the stationary equation as the projective Levi–Civita class, proves the torsion-trace and torsion-free Levi–Civita selections, constructs a stationary non-reduced control, and establishes finite gauge covariance of the complete projective class.

The M2568–M2604 projective tranche proves exact physical action invariance from the literal Ricci polynomial and symmetric physical inverse density, the additive/free projective action laws, the exact stationary orbit, the torsion-trace transformation and unique zero-trace slice, finite diffeomorphism compatibility, and the pointwise kernel/image theorem with injective projective generator. It also retains a formal negative control: projective invariance fails on the old ambient ordered-density carrier, even modulo total divergences. This counterexample is preserved as part of the final provenance rather than removed by the later carrier refinement.

The M2633–M2647 symmetric-projective tranche constructs the intrinsic symmetric-pair carrier, derives the diagonal/off-diagonal cotangent normalization, proves that the ambient quotient is not a global Poisson map, establishes the symmetric Palatini CME and whole square-zero law, adds the covector projective ghost and antifield, proves prolonged BRST nilpotence and projective Noether closure, establishes the enlarged real CME and arbitrary-antifield square-zero, proves exact ownership by the ambient-master image plus the new earned cotangent terms, and binds the enlarged classical projection to the same Einstein–real-scalar and native Paper 4 matter density.

The dedicated symmetric-projective audit checks the new theorem surfaces, dependency ownership, source hygiene, and axiom reports; its static controls reject `sorry`, `admit`, new axioms, hidden `native_decide`, and prototype authority. The authority boundary recorded in the repository marks arbitrary-antifield functional square-zero, generic functional Jacobi reuse, full antifield feedback, pointwise projective properness, and the Levi–Civita gauge representative as earned. Full functional properness/Koszul acyclicity, metric-only BV reduction, connection-antifield symplectic elimination, direct quantum gauge implementation, and quantum-state cohomology remain deferred.

The checked Paper 5 scope therefore includes: the ambient free Palatini CME and its generic functional calculus; the intrinsic symmetric-density cotangent realization; the symmetric CME; the projective covector ghost/antifield extension; the enlarged diffeomorphism-plus-projective CME; square-zero on arbitrary antifield-dependent local functionals; exact physical projective action invariance; the stationary projective orbit; the pointwise Euler-kernel/projective-image identity; the unique zero-torsion-trace Levi–Civita representative; the unchanged physical Einstein–scalar projection and native matter reduction; the Paper 4 coherent quantum sector; field-moment Ward intertwining; master-projected Hilbert stress; coordinate-gauge covariance of the quantum source residual; and positive native physical-time clock compatibility.

The continuing mathematical boundary is correspondingly narrower: full functional BV properness and homological acyclicity, a metric-only symplectic BV reduction, explicit connection-antifield elimination, a boundary-aware extension of the local variational quotient, a genuine quantum gauge operator and physical state space, and quantum master/anomaly analysis. Each requires its own formal verification.

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