Quantum Gauge Structure of a Self-Consistent Einstein–Scalar Sector

Operator BRST representation, a retracted physical Hilbert sector, and projective gauge reduction

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Abstract

The first five papers in this series examine one real scalar field from singular operator selection through quantum dynamics, local quantum matter, a self-consistent Einstein–scalar sector, and the functional Batalin–Vilkovisky master law of the same action. The present sixth paper declares the quantum gauge realization of that sector and identifies the precise relationship between its previously established scalar Fock theory and the completed BRST state space.

Let S^{sp}_{κ} be the intrinsic symmetric/projective Palatini master of Paper 5 and let $\mathcal{F}_{\mathrm{NL}}$ be the native scalar Fock space of Paper 4. On the algebraic BV/Fock carrier the quantum gauge charge is

$$\boxed{Q_{\mathrm{ES}} = s^{\mathrm{sp}}_{\mathrm{gauge}} \otimes I_{\mathcal{F}_{\mathrm{NL}}}},$$

and for every homogeneous finite jet observable F it satisfies

$$\boxed{[Q_{\mathrm{ES}}, \widehat{F}]_{\pm} = \widehat{s^{\mathrm{sp}}_{\mathrm{gauge}} F}}, \qquad \boxed{Q^2_{\mathrm{ES}} = 0}.$$

Thus the classical gauge differential has an operator BRST representation on an enlarged carrier containing the native scalar theory as the coefficient-vacuum lane.

The intrinsic monomial/exterior-wedge basis determines an unweighted Hilbert sum of native Fock fibers. Closing the exact algebraic graph of the charge gives a graph Hilbert carrier with bounded square-zero differential $\overline{Q}_{\mathrm{ES}}$. The graph realization is invariant, up to unitary equivalence, under every unitary change of coefficient presentation carrying the algebraic graph core to the corresponding presented core; this includes arbitrary relabeling of the intrinsic basis and independent wedge-sign conventions. Ordinary closability on the underlying counting Hilbert space is governed by the closed vertical-obstruction subspace

$$\mathcal{V}_Q = \{y : (0, y) \in \overline{\Gamma(Q_0)}\}, \qquad Q_0 \text{ closable} \iff \mathcal{V}_Q = \{0\}.$$

The graph differential raises integer ghost number by one. Its degree-zero Hausdorff BRST cohomology is formed from closed ghost-zero cycles modulo the closed represented boundary space. The latter has the equivalent descriptions

$$\boxed{\overline{Q_{\mathrm{graph}}(\Gamma_{\mathrm{core}})} = \overline{\operatorname{im} Q_{\mathrm{graph}}} = B^{\mathrm{Hilb}}_{Q}.}$$

The orthogonal complement of this boundary space is the kernel of $\overline{Q}^{*}_{\mathrm{ES}}$, and the BRST Hodge operator

$$\Delta_{\mathrm{BRST}} = \overline{Q}^{*}_{\mathrm{ES}}\overline{Q}_{\mathrm{ES}} + \overline{Q}_{\mathrm{ES}}\overline{Q}^{*}_{\mathrm{ES}}$$

has kernel equal to the harmonic representatives of the Hausdorff quotient.

The native scalar theory occupies a distinguished and fully controlled part of the degree-zero cohomology. Its class map

$$\iota^{H}_{\mathrm{phys}} : \mathcal{F}_{\mathrm{NL}} \longrightarrow H^{0,\mathrm{Hilb}}_{Q}$$

is an isometric embedding with closed range. The BV-vacuum coordinate descends to a continuous contraction

$$r_H : H^{0,\mathrm{Hilb}}_{Q} \longrightarrow \mathcal{F}_{\mathrm{NL}}, \qquad \boxed{r_H \iota^{H}_{\mathrm{phys}} = I}.$$

Consequently

$$\boxed{H^{0,\mathrm{Hilb}}_{Q} = \mathcal{H}_{\mathrm{native}} \oplus \mathcal{H}_{\mathrm{additional}}, \qquad \mathcal{H}_{\mathrm{native}} = \operatorname{Ran} \iota^{H}_{\mathrm{phys}}, \qquad \mathcal{H}_{\mathrm{additional}} = \ker r_H,}$$

as a topological direct sum. The present theorem surface supplies an exact decision criterion for exhaustion: $\mathcal{H}_{\mathrm{additional}} = 0$ if and only if $\iota^{H}_{\mathrm{phys}}$ is surjective. Its zero/nonzero status remains

an open structural question. Native time evolution descends to degree-zero BRST cohomology, preserves both summands, and restricts on $\mathcal{H}_{\text{native}}$ to the original Paper 4 unitary evolution.

The independent Palatini projective symmetry is controlled by the exact linear sequence

$$0 \longrightarrow \mathcal{P}_{\text{proj}} \xrightarrow{R_{\text{proj}}} \delta\Gamma \xrightarrow{L_\Gamma} \mathcal{E}_\Gamma, \qquad \ker R_{\text{proj}} = 0, \quad \ker L_\Gamma = \operatorname{im} R_{\text{proj}}.$$

Its torsion-trace gauge has Faddeev–Popov operator $-3I$, an explicit nonminimal doublet, a contractible quartet, and a strong deformation retract onto the quartet-reduced complex. On the stationary connection locus the unique zero-trace representative is Levi–Civita.

Scalar-field, stress-tensor, and Einstein-residual observables satisfy combined operator Ward identities on their declared common domains. The density-to-local-functional quotient is a BRST chain map and contains an explicit nonzero total derivative in its kernel, which yields a theorem that no function can universally recover every density from its local-functional equivalence class. The metric diffeomorphism stabilizer is exactly the Killing algebra. Carrier-level BRST square, time-intertwining, and Ward representation obstruction maps vanish on their declared algebraic and graph-Hilbert domains.

The resulting theorem surface gives a completed operator/BRST gauge realization of the self-consistent classical-metric/quantum-matter sector on the declared algebraic and graph-Hilbert carriers. Within ghost-number-zero BRST cohomology, the previously established native quantum theory is identified exactly as a closed, isometric, dynamically invariant retract, with any complementary BRST sector isolated as a separate structural question. The next projected boundary is the regulator-owned perturbative BV/EFT quantum-master problem.

1 Introduction

The preceding five papers establish one Einstein–scalar sector in successive theorem-owned layers. Paper 1 derives the Hardy-critical half-line operator of the spherically symmetric real scalar in the Schwarzschild interior, classifies its self-adjoint terminal laws, and proves that the finite full-potential form selects the no-log Friedrichs realization.[1] Paper 2 carries that selection into continuum bosonic Fock dynamics and into the physical Cauchy quantization of the same real scalar, with a self-adjoint all-mode Hamiltonian and unitary physical clock.[2] Paper 3 differentiates the quantum field into local vacuum-relative matter, establishes directed all-mode stress and conservation on dense transported regular fibers, and couples finite spherical quantum sources to an Einstein tangent.[3]

Paper 4 reconstructs the scalar quantum theory on an independently defined nonlinear Einstein–real-scalar geometry. Its central closure is

$$\boxed{g_{\mathrm{NL}} \longrightarrow \mathcal{Q}(g_{\mathrm{NL}}) \ni \Psi_\star \longrightarrow \Delta T_{\mu\nu}[\Psi_\star] \longrightarrow g_{\mathrm{NL}},} \tag{1}$$

with a classical Lorentz metric and quantum scalar matter sourced by the same native action.[4]

Paper 5 moves upstream to the functional gauge law of that action. Its final object is the intrinsic symmetric/projective Palatini BV master

$$\boxed{S^{\mathrm{sp}}_\kappa,} \tag{2}$$

with

$$\boxed{(S^{\mathrm{sp}}_\kappa, S^{\mathrm{sp}}_\kappa)_{\mathrm{sp}} = 0, \qquad (s^{\mathrm{sp}}_\kappa)^2 F = 0.} \tag{3}$$

It also solves the independent Palatini connection equation,

$$\boxed{E_\Gamma S_{\mathrm{Pal}} = 0 \Longleftrightarrow \Gamma^\rho{}_{\mu\nu} = \Gamma[g]^\rho{}_{\mu\nu} + \delta^\rho_\nu A_\mu,} \tag{4}$$

and identifies the free projective connection direction, its cotangent completion, and the same Einstein–scalar physical projection.[5]

The present paper addresses the state-space and operator consequences of that gauge structure. The central question is

$$\boxed{\begin{array}{c}\text{How does the classical symmetric/projective gauge differential act}\\ \text{on a quantum state carrier containing the native scalar theory,}\\ \text{which ghost-zero BRST classes recover the established Einstein–scalar}\\ \text{quantum sector, and how do their dynamics and observables descend?}\end{array}} \tag{5}$$

Three operators remain semantically distinct throughout:

$$\boxed{\begin{array}{rcl} S^{\mathrm{sp}}_\kappa & : & \text{classical local-functional BV master,}\\ Q_{\mathrm{ES}} & : & \text{operator representation of the gauge differential,}\\ H_{\mathrm{NL}}(x) & : & \text{native physical Hamiltonian of the scalar theory.}\end{array}} \tag{6}$$

The relation between the quantum gauge charge and the native scalar theory is

$$\boxed{Q_{\mathrm{ES}} = s^{\mathrm{sp}}_{\mathrm{gauge}} \otimes I_{\mathcal{F}_{\mathrm{NL}}}.} \tag{7}$$

The gauge differential acts on the BV coefficient factor, while the scalar Fock factor carries the previously established quantum dynamics.

The paper has three principal theorem surfaces. The first is the graded operator representation

$$\boxed{[Q_{\mathrm{ES}}, \widehat{F}]_{\pm} = \widehat{s^{\mathrm{sp}}_{\mathrm{gauge}}F}, \qquad Q_{\mathrm{ES}}^2 = 0.} \tag{8}$$

The second is the physical-sector retract

$$\boxed{\iota^H_{\mathrm{phys}} : \mathcal{F}_{\mathrm{NL}} \hookrightarrow_{\mathrm{iso}} H^{0,\mathrm{Hilb}}_Q, \qquad r_H \iota^H_{\mathrm{phys}} = I,} \tag{9}$$

which yields

$$\boxed{H^{0,\mathrm{Hilb}}_Q = \mathcal{H}_{\mathrm{native}} \oplus \mathcal{H}_{\mathrm{additional}}, \qquad \mathcal{H}_{\mathrm{native}} \simeq_{\mathrm{iso}} \mathcal{F}_{\mathrm{NL}}.} \tag{10}$$

The third is the analytic gauge reduction: an exact graph-Hilbert BRST complex, Hausdorff/Hodge cohomology, unitary native-time descent, projective exactness and quartet reduction, and operator Ward identities for the physical observables.

Equation (10) preserves a precise interpretation. The subspace $\mathcal{H}_{\mathrm{native}}$ is the Einstein–scalar quantum sector already established in Papers 2–4. The complementary space $\mathcal{H}_{\mathrm{additional}}$ is the kernel of the descended BV-vacuum retraction. Its vanishing is equivalent to surjectivity of the native Fock embedding. The present theorem surface isolates this criterion and leaves its zero/nonzero status open.

The geometric metric remains classical throughout. The immediate next boundary is the regulator-owned perturbative BV/EFT quantum-master problem. A later quantum-gravity threshold requires a nontrivial quantum carrier for geometric degrees of freedom, gravitational observables, dynamics, and semiclassical recovery.

2 Input from Papers 4 and 5

Fix a positive Einstein coupling κ and one of the admissible nonlinear Einstein–real-scalar branches established in Paper 4. Let g_{NL} denote its classical Lorentz metric, $\mathcal{F}_{\mathrm{NL}}$ the native real scalar Fock carrier, $H_{\mathrm{NL}}(x)$ the self-adjoint native physical Hamiltonian, and

$$U_{\mathrm{NL}}(x_2, x_1)$$

the positive-time two-time unitary. It obeys

$$U_{\mathrm{NL}}(x, x) = I, \qquad U_{\mathrm{NL}}(x_3, x_2)U_{\mathrm{NL}}(x_2, x_1) = U_{\mathrm{NL}}(x_3, x_1), \tag{11}$$

and for $\Psi \in D(H_0)$,

$$\frac{d}{dx}U_{\mathrm{NL}}(x, x_0)\Psi = -iH_{\mathrm{NL}}(x)U_{\mathrm{NL}}(x, x_0)\Psi, \tag{12}$$

with joint strong continuity and two-way energy-domain standing.[1]

The classical gauge input is the final symmetric/projective master of Paper 5. Write

$$\mathfrak{F}^{\mathrm{sp}}_{\mathrm{Pal}}$$

[1] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.NativePhysicalOwnedFamily`.

for its real local-functional carrier and

$$s_\kappa^{\mathrm{sp}} F = (S_\kappa^{\mathrm{sp}}, F)_{\mathrm{sp}}.$$

Its field content includes the intrinsic symmetric inverse metric density $h^{\mu\nu}$, independent affine connection $\Gamma^\rho{}_{\mu\nu}$, real scalar ϕ, diffeomorphism ghost c^μ, projective covector ghost η_μ, and the corresponding antifields.

The classical projective action is

$$P_A(\Gamma)^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\mu\nu} + \delta^\rho_\nu A_\mu. \tag{13}$$

It is free, the stationary connection locus is exactly the projective orbit through Levi–Civita, and the torsion trace obeys

$$\boxed{\tau(P_B\Gamma) = \tau(\Gamma) - 3B.} \tag{14}$$

Thus every projective orbit contains exactly one zero-trace representative and every stationary zero-trace representative is $\Gamma[g]$.[2]

The physical projection satisfies $\pi_{\mathrm{cl}} S_\kappa^{\mathrm{sp}} = S_{\mathrm{Pal}}^{\mathrm{sym}}$, and holonomic evaluation gives the Einstein–real-scalar action of Paper 4. Its matter component reduces to the same native temporal action and mode frequency; hence the scalar Fock carrier, $H_{\mathrm{NL}}(x)$, and $U_{\mathrm{NL}}(x_2, x_1)$ remain the physical quantum data throughout the gauge reduction.

3 The direct BV/Fock complex

Let $\mathfrak{F}_{\mathrm{Pal}}^{\mathrm{sp}}$ be the symmetric/projective local-functional BV space and let $\mathcal{F}_{\mathrm{NL}}$ be the native scalar Fock space. Define

$$\mathcal{C}_{\mathrm{dir}} := \mathfrak{F}_{\mathrm{Pal}}^{\mathrm{sp}} \otimes_{\mathbb{R}} \mathcal{F}_{\mathrm{NL}}. \tag{15}$$

The direct differential acts only on the BV factor,

$$D_{\mathrm{dir}} := s_\kappa^{\mathrm{sp}} \otimes I. \tag{16}$$

Since $(s_\kappa^{\mathrm{sp}})^2 = 0$ on arbitrary local functionals,

$$\boxed{D_{\mathrm{dir}}^2 = 0} \tag{17}$$

on every algebraic tensor state.[3]

Native physical transport acts on the Fock factor,

$$U_{\mathrm{dir}}(x_2, x_1) := I \otimes U_{\mathrm{NL}}(x_2, x_1), \tag{18}$$

and therefore satisfies the exact intertwining law

$$\boxed{D_{\mathrm{dir}} U_{\mathrm{dir}} = U_{\mathrm{dir}} D_{\mathrm{dir}}.} \tag{19}$$

[2] *Formal counterpart:* `QGC12.BV.palatiniConnectionProjectiveShift_actual_stationary_gauge_slice`.
[3] *Formal counterpart:* `QGC12.BV.QuantumGeometry.directFockBVDifferential_actual_square_zero`.

[4]

Define algebraic cycles and boundaries by

$$Z_{\mathrm{dir}} = \ker D_{\mathrm{dir}}, \qquad B_{\mathrm{dir}} = \operatorname{im} D_{\mathrm{dir}}, \tag{20}$$

and the direct cohomology

$$H_{\mathrm{dir}} = Z_{\mathrm{dir}}/B_{\mathrm{dir}}. \tag{21}$$

The native unitary preserves both subspaces and descends with exact identity and cocycle laws. The actual master tensored with each normalized native coherent state is an explicit direct-complex cycle.[5]

The differential D_{dir} is the complete local-functional master Hamiltonian after quotienting total divergences, whereas Q_{ES} below represents the gauge-generator density differential. Section 13 relates the density complex to the local-functional quotient by the canonical chain map ρ.

4 Stationary connection quotient and exact projective sequence

Let

$$\mathcal{P}^{\Gamma}_{\mathrm{stat}} := \{(g,\Gamma) : E_\Gamma S_{\mathrm{Pal}}(g,\Gamma) = 0\} \tag{22}$$

be the stationary metric–connection locus. The scalar field is a separate argument of the reduced Einstein–scalar action. The projective action changes only the connection,

$$P_A(\Gamma)^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} + \delta^{\rho}_{\nu} A_\mu. \tag{23}$$

For stationary pairs, equality of the metric is equivalent to membership in the same spacetime-dependent projective connection orbit. Hence

$$\boxed{\mathcal{P}^{\Gamma}_{\mathrm{stat}}/\mathcal{G}_{\mathrm{proj}} \simeq \mathcal{M}_{\mathrm{Lor}},} \tag{24}$$

with Levi–Civita connection as the inverse representative.[6]

The linearized projective connection structure admits a sharper exact statement. Let

$$\mathcal{P}_{\mathrm{proj}} \cong \Omega^1$$

denote the projective parameter space, let $\delta\Gamma$ denote connection variations at a fixed metric and spacetime point, and let $\mathcal{E}_\Gamma$ denote linearized connection-Euler variations. Define

$$R_{\mathrm{proj}} : \mathcal{P}_{\mathrm{proj}} \to \delta\Gamma, \qquad (R_{\mathrm{proj}}A)^{\rho}{}_{\mu\nu} = \delta^{\rho}_{\nu} A_\mu, \tag{25}$$

and the linearized Euler map

$$L_\Gamma : \delta\Gamma \to \mathcal{E}_\Gamma. \tag{26}$$

[4] *Formal counterpart:* `QGC12.BV.QuantumGeometry.directFockBVDifferential_actual_transport_intertwining`.

[5] *Formal counterpart:* `QGC12.BV.QuantumGeometry.nativeMasterCoherentCycle`.

[6] *Formal counterpart:* `QGC12.BV.QuantumGeometry.metricOnlyPalatiniEquiv`.

The sequence

$$\boxed{0 \longrightarrow \mathcal{P}_{\mathrm{proj}} \xrightarrow{R_{\mathrm{proj}}} \delta\Gamma \xrightarrow{L_\Gamma} \mathcal{E}_\Gamma} \tag{27}$$

is exact at the first two nonzero positions:

$$\boxed{\ker R_{\mathrm{proj}} = 0, \qquad \ker L_\Gamma = \mathrm{im}\, R_{\mathrm{proj}}.} \tag{28}$$

[7]

Equation (28) is the functional exactness statement used throughout the remainder of the paper. It gives the complete spacetime-dependent zero-mode content of the linearized Palatini connection equation in the projective direction.

5 Operator BRST representation

Let $\mathcal{K}_{\mathrm{sp}}$ denote the intrinsic symmetric/projective field–ghost key space and let $\mathcal{K}_{\mathrm{op}}$ denote the represented operator key space. There is an injective typed map

$$\iota_{\mathrm{BV}} : \mathcal{K}_{\mathrm{sp}} \hookrightarrow \mathcal{K}_{\mathrm{op}}, \qquad \iota_{\mathrm{BV}}(\eta_\mu) \neq \iota_{\mathrm{BV}}(c^\nu), \tag{29}$$

which preserves field/antifield role, parity, canonical-pair orientation, and opposite statistics.[8]

Let $\mathcal{A}_{\mathrm{ES}}^{\mathrm{BV}}$ denote the resulting intrinsic symmetric/projective finite-jet algebra. The algebraic operator-state carrier is

$$\boxed{\mathcal{H}_{\mathrm{ES}}^{\mathrm{BV,alg}} = \mathcal{A}_{\mathrm{ES}}^{\mathrm{BV}} \otimes_{\mathbb{Q}} \mathcal{F}_{\mathrm{NL}}.} \tag{30}$$

The native scalar theory enters through the coefficient-vacuum map

$$\iota_{\mathrm{phys}} : \mathcal{F}_{\mathrm{NL}} \to \mathcal{H}_{\mathrm{ES}}^{\mathrm{BV,alg}}, \qquad \psi \longmapsto 1 \otimes \psi, \tag{31}$$

and the BV-vacuum coefficient defines an algebraic retraction

$$r_{\mathrm{phys}} \iota_{\mathrm{phys}} = I_{\mathcal{F}_{\mathrm{NL}}}. \tag{32}$$

[9]

The intrinsic symmetric cotangent normalization of Paper 5 remains exact on this carrier. In particular, the diagonal and off-diagonal inverse-density field–antifield pairings are normalized to one after the intrinsic one-half off-diagonal antifield weight. The projective covector ghost and the diffeomorphism ghost remain independent typed coordinates.[10]

[7] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.functionalProjectiveConnectionSequence_exact_at_parameter_and_connection`.
[8] *Formal counterpart:* `QGC12.BV.QuantumGauge.paper5BVCoordinateEmbedding_actual_injective`.
[9] *Formal counterpart:* `QGC12.BV.QuantumGauge.nativePhysicalBVFockRetraction_embedding`.
[10] *Formal counterpart:* `QGC12.BV.QuantumGauge.paper5_intrinsic_density_offDiagonal_pairing_actual_one`.

5.1 Gauge charge and graded representation

The quantum gauge charge is

$$\boxed{Q_{\mathrm{ES}} = s^{\mathrm{sp}}_{\mathrm{gauge}} \otimes I_{\mathcal{F}_{\mathrm{NL}}}.} \tag{33}$$

[11] The conventional factor $i\hbar$ is absorbed into the normalization of the represented adjoint action. For every homogeneous finite jet observable F of parity sign ϵ_F,

$$\boxed{Q_{\mathrm{ES}}\widehat{F} - \epsilon_F \widehat{F} Q_{\mathrm{ES}} = \widehat{s^{\mathrm{sp}}_{\mathrm{gauge}}F}.} \tag{34}$$

[12]

The represented generator equations include the symmetric inverse density, independent connection, scalar, diffeomorphism ghost, projective covector ghost, and the represented antifield lane. The connection law contains the projective contribution explicitly,

$$[Q_{\mathrm{ES}}, \widehat{\Gamma}^{\rho}{}_{\mu\nu}]_{\pm} = \widehat{\mathcal{L}_c \Gamma^{\rho}{}_{\mu\nu}} + \cdots + \delta^{\rho}_{\nu}\widehat{\eta}_{\mu}, \tag{35}$$

where the omitted displayed terms are the standard affine connection response already contained in the finite-jet formula.[13] For the scalar,

$$\boxed{[Q_{\mathrm{ES}}, \widehat{\phi}]_{\pm} = \widehat{c^{\mu}\partial_{\mu}\phi}.} \tag{36}$$

Equation (33) fixes the interpretation of the representation. The gauge differential acts on the BV coefficient algebra, and the scalar Fock factor carries the already-established native quantum theory. In particular,

$$\boxed{Q_{\mathrm{ES}}(1 \otimes \psi) = 0 \qquad (\psi \in \mathcal{F}_{\mathrm{NL}}).} \tag{37}$$

The result of this paper is therefore an operator BRST representation containing the native Fock theory, together with its completed cohomological reduction.

5.2 Square-zero and native-time intertwining

The explicit diffeomorphism/projective Maurer–Cartan law gives

$$\boxed{Q_{\mathrm{ES}}^2 = 0.} \tag{38}$$

[14] The native two-time unitary acts on the Fock factor and obeys

$$\boxed{Q_{\mathrm{ES}}(I \otimes U_{\mathrm{NL}}(x_2, x_1)) = (I \otimes U_{\mathrm{NL}}(x_2, x_1))Q_{\mathrm{ES}}.} \tag{39}$$

[15] These identities form the algebraic BRST complex used below.

[11] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarQuantumBRSTCharge_actual_gauge_tensor_identity`.

[12] *Formal counterpart:* `QGC12.BV.QuantumGauge.einsteinScalarBRSTAdjoint_actual_represents`.

[13] *Formal counterpart:* `QGC12.BV.QuantumGauge.einsteinScalar_connection_commutator`.

[14] *Formal counterpart:* `QGC12.BV.QuantumGauge.einsteinScalarQuantumBRSTCharge_actual_square_zero`.

[15] *Formal counterpart:* `QGC12.BV.QuantumGauge.einsteinScalarQuantumBRSTCharge_actual_transport_intertwining`.

6 Algebraic BRST physical states

Define

$$Z_Q^{\mathrm{alg}} = \ker Q_{\mathrm{ES}}, \qquad B_Q^{\mathrm{alg}} = \operatorname{im} Q_{\mathrm{ES}}, \tag{40}$$

and

$$H_Q^{\mathrm{alg}} = Z_Q^{\mathrm{alg}} / B_Q^{\mathrm{alg}}. \tag{41}$$

The quotient H_Q^{alg} is the algebraic BRST cohomology, and its cycle representatives embed into the graph-Hilbert cycle space introduced in Section 7.

Every native scalar state is closed under the physical embedding,

$$Q_{\mathrm{ES}}(1 \otimes \psi) = 0. \tag{42}$$

More strongly, the BV-vacuum retraction annihilates every BRST image,

$$r_{\mathrm{phys}}(Q_{\mathrm{ES}}\Xi) = 0. \tag{43}$$

Therefore, if $1 \otimes \psi$ were exact, applying r_{phys} would give $\psi = 0$. Hence

$$\boxed{\psi \neq 0 \quad \Longrightarrow \quad [1 \otimes \psi]_Q \neq 0.} \tag{44}$$

[16]

In particular, every normalized native coherent source has a nonzero BRST class.[17]

Hence the map $\psi \mapsto [1 \otimes \psi]_Q$ is injective on the native physical sector, so every nonzero Papers 2–4 scalar state defines a nonzero BRST class.

7 Graph-Hilbert realization of the BRST complex

Let $\mathfrak{M}$ denote the countable intrinsic monomial/exterior-wedge index set of the finite-jet BV algebra. The analytic coefficient carrier is the unweighted Hilbert sum

$$\boxed{\mathcal{H}_{\mathrm{count}} = \ell^2(\mathfrak{M}; \mathcal{F}_{\mathrm{NL}}),} \tag{45}$$

with the finite algebraic core embedded injectively and densely.[18]

Let Q_0 denote the algebraic charge represented as a densely defined operator on $\mathcal{H}_{\mathrm{count}}$, and let

$$\Gamma(Q_0) \subset \mathcal{H}_{\mathrm{count}} \oplus \mathcal{H}_{\mathrm{count}} \tag{46}$$

be its exact algebraic graph. Define the graph Hilbert carrier by

$$\boxed{\mathcal{H}_{\mathrm{graph}} := \overline{\Gamma(Q_0)}.} \tag{47}$$

[16] *Formal counterpart:* `QGC12.BV.QuantumGauge.nativePhysicalBRSTClass_actual_nonzero`.
[17] *Formal counterpart:* `QGC12.BV.QuantumGauge.nativeZeroCoherentBRSTClass_actual_nonzero`.
[18] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarBVCoreInclusion_actual_dense`.

The continuous graph shift

$$(x, y) \longmapsto (y, 0) \tag{48}$$

preserves $\mathcal{H}_{\text{graph}}$ and defines

$$\boxed{\overline{Q}_{\text{ES}} : \mathcal{H}_{\text{graph}} \to \mathcal{H}_{\text{graph}}, \qquad \overline{Q}_{\text{ES}}^2 = 0.} \tag{49}$$

[19] The algebraic graph core is dense in $\mathcal{H}_{\text{graph}}$.

7.1 Unitary presentation invariance

The graph realization carries a precise relative invariance. Let

$$V : \mathcal{H}_{\text{count}} \simeq_{\text{unitary}} \mathcal{H}'_{\text{count}}$$

be a unitary coefficient-space presentation change carrying the algebraic graph core to its presented image. The diagonal unitary

$$V \oplus V$$

maps the closed graph carrier onto the presented closed graph carrier and induces

$$\boxed{V_{\text{graph}} : \mathcal{H}_{\text{graph}}(Q_0) \simeq_{\text{unitary}} \mathcal{H}_{\text{graph}}(Q'_0),} \tag{50}$$

with

$$\boxed{V_{\text{graph}} \overline{Q}_{\text{ES}} = \overline{Q}'_{\text{ES}} V_{\text{graph}}.} \tag{51}$$

[20] Arbitrary relabeling of the intrinsic monomial/wedge basis and independent basis sign changes, including the signs generated by reordered odd wedges, are special cases of Eq. (50).[21]

The declared analytic input is therefore the unweighted coefficient norm together with its unitary presentation class. Inequivalent weighted Hilbert completions define a separate analytic choice.

7.2 Vertical obstruction and ordinary closability

Define the vertical-obstruction subspace

$$\boxed{\mathcal{V}_Q := \{y \in \mathcal{H}_{\text{count}} : (0, y) \in \overline{\Gamma(Q_0)}\}.} \tag{52}$$

It is a closed linear subspace and satisfies

$$\boxed{Q_0 \text{ is closable} \Longleftrightarrow \mathcal{V}_Q = \{0\}.} \tag{53}$$

[22] The vertical subspace is exactly the kernel of the source projection from the closed graph relation. Native horizontal graph states have trivial intersection with this vertical lane.[23]

[19] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarClosedBRST_actual_square_zero`.

[20] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.graphBRST_unitaryPresentationInvariant_actual_intertwines`.

[21] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.graphBRST_basisRelabelingInvariant`.

[22] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarCountingBRSTOperator_closable_iff_verticalSubspace_eq_bot`.

[23] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.nativeCountingGraphState_actual_disjoint_vertical`.

Every source column of the BRST matrix in the intrinsic monomial/wedge basis has finite support. The transpose finiteness required for a dense formal adjoint is isolated by the finite-row condition

$$\forall\beta, \qquad \#\{\alpha : Q_{\beta\alpha} \neq 0\} < \infty. \tag{54}$$

A densely defined formal adjoint implies ordinary counting-space closability. Thus Eq. (54) supplies the precise remaining combinatorial route from the graph realization to a conventional closed operator on $\mathcal{H}_{\mathrm{count}}$.[24]

8 Ghost number, Hausdorff boundaries, and Hodge realization

The intrinsic pair orientation determines integer ghost number:

$$\mathrm{gh}(h, \Gamma, \phi) = 0, \qquad \mathrm{gh}(c, \eta) = 1, \tag{55}$$

$$\mathrm{gh}(h^*, \Gamma^*, \phi^*) = -1, \qquad \mathrm{gh}(c^*, \eta^*) = -2. \tag{56}$$

Jet prolongation preserves degree and monomial degree is additive. The closed graph differential raises ghost number by one:

$$\boxed{\overline{Q}_{\mathrm{ES}} : \mathcal{H}^{n}_{\mathrm{graph}} \to \mathcal{H}^{n+1}_{\mathrm{graph}}.} \tag{57}$$

[25]

Let

$$Z_Q := \ker \overline{Q}_{\mathrm{ES}} \tag{58}$$

and define the closed boundary subspace by

$$\boxed{B^{\mathrm{Hilb}}_Q := \overline{\mathrm{im}\, \overline{Q}_{\mathrm{ES}}}.} \tag{59}$$

The dense algebraic graph core generates exactly the same closed boundary subspace:

$$\boxed{\overline{\overline{Q}_{\mathrm{ES}}(\Gamma_{\mathrm{core}})} = \overline{\mathrm{im}\, \overline{Q}_{\mathrm{ES}}} = B^{\mathrm{Hilb}}_Q.} \tag{60}$$

[26] The Hausdorff BRST cohomology is

$$H^{\mathrm{Hilb}}_Q := Z_Q / B^{\mathrm{Hilb}}_Q. \tag{61}$$

Its physical-degree component is the literal ghost-number-zero quotient

$$\boxed{H^{0,\mathrm{Hilb}}_Q.} \tag{62}$$

The closed boundary space has orthogonal complement

$$(B^{\mathrm{Hilb}}_Q)^{\perp} = \ker \overline{Q}^{*}_{\mathrm{ES}}. \tag{63}$$

[24] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarCounting_denseFormalAdjoint_implies_closable`.

[25] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarClosedBRST_actual_maps_graphGhostDegree`.

[26] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.graphBoundaryClosure_eq_coreBoundaryClosure`.

[27] Hence the harmonic representatives are

$$\mathcal{H}_{\mathrm{BRST}} := \ker \overline{Q}_{\mathrm{ES}} \cap \ker \overline{Q}_{\mathrm{ES}}^{*}. \tag{64}$$

The quotient and harmonic realizations are canonically unitarily equivalent.

Define

$$\boxed{\Delta_{\mathrm{BRST}} := \overline{Q}_{\mathrm{ES}}^{*}\overline{Q}_{\mathrm{ES}} + \overline{Q}_{\mathrm{ES}}\overline{Q}_{\mathrm{ES}}^{*}.} \tag{65}$$

The energy identity gives

$$\mathrm{Re}\langle \psi, \Delta_{\mathrm{BRST}}\psi\rangle = \|\overline{Q}_{\mathrm{ES}}\psi\|^2 + \|\overline{Q}_{\mathrm{ES}}^{*}\psi\|^2, \tag{66}$$

and therefore

$$\boxed{\ker \Delta_{\mathrm{BRST}} = \ker \overline{Q}_{\mathrm{ES}} \cap \ker \overline{Q}_{\mathrm{ES}}^{*} = \mathcal{H}_{\mathrm{BRST}}.} \tag{67}$$

[28] The operator Δ_{BRST} is the Hodge operator of the completed BRST complex. The regulator-owned BV quantum Laplacian enters at the subsequent perturbative quantum-master boundary.

9 The native Einstein–scalar sector as an isometric retract

The coefficient-vacuum embedding descends to literal ghost-number-zero cohomology,

$$\iota^{H}_{\mathrm{phys}} : \mathcal{F}_{\mathrm{NL}} \longrightarrow H^{0,\mathrm{Hilb}}_{Q}. \tag{68}$$

The BV-vacuum coordinate extends continuously across the graph completion, annihilates B^{Hilb}_{Q}, and descends to

$$r_H : H^{0,\mathrm{Hilb}}_{Q} \longrightarrow \mathcal{F}_{\mathrm{NL}}. \tag{69}$$

The two maps satisfy

$$\boxed{r_H \iota^{H}_{\mathrm{phys}} = I_{\mathcal{F}_{\mathrm{NL}}}.} \tag{70}$$

[29] The quotient norm is exact on the native image:

$$\boxed{\|\iota^{H}_{\mathrm{phys}}\psi\|_{H^{0,\mathrm{Hilb}}_{Q}} = \|\psi\|_{\mathcal{F}_{\mathrm{NL}}}.} \tag{71}$$

[30] Thus $\iota^{H}_{\mathrm{phys}}$ is a linear isometry with closed range.

Define

$$\mathcal{H}_{\mathrm{native}} := \mathrm{Ran}\, \iota^{H}_{\mathrm{phys}}, \qquad \mathcal{H}_{\mathrm{additional}} := \ker r_H. \tag{72}$$

The idempotent

$$P_{\mathrm{native}} := \iota^{H}_{\mathrm{phys}} r_H \tag{73}$$

[27] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.graphBoundaryClosure_orthogonal_eq_adjointKernel`.

[28] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.graphBRST_harmonic_eq_hodgeKernel`.

[29] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.nativeGhostZeroCohomologyRetraction_native`.

[30] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.nativeGhostZeroHilbertBRSTClass_actual_isometric`.

has

$$\operatorname{Ran} P_{\text{native}} = \mathcal{H}_{\text{native}}, \qquad \ker P_{\text{native}} = \mathcal{H}_{\text{additional}}. \tag{74}$$

Both subspaces are closed, and they form an exact complemented pair:

$$\boxed{H_Q^{0,\text{Hilb}} = \mathcal{H}_{\text{native}} \oplus \mathcal{H}_{\text{additional}}.} \tag{75}$$

[31] The projection is contractive in the quotient norm. The theorem surface establishes the topological direct sum in Eq. (75).

The subspace $\mathcal{H}_{\text{native}}$ is the sector already given direct physical interpretation by Papers 2–4. The complementary space $\mathcal{H}_{\text{additional}}$ records any additional ghost-zero BRST classes carried by the enlarged gauge complex. Its zero criterion is exact:

$$\boxed{\mathcal{H}_{\text{additional}} = \{0\} \iff r_H \text{ is injective} \iff \iota_{\text{phys}}^H \text{ is surjective.}} \tag{76}$$

[32] Equation (76) records the remaining structural decision. Current authority establishes the complemented sector and its exact exhaustion criterion; a future witness or exhaustion theorem will decide its zero/nonzero status and any further physical interpretation.

10 Unitary physical time and block invariance

The native two-time unitary lifts to the graph carrier and commutes with $\overline{Q}_{\text{ES}}$:

$$\boxed{\overline{Q}_{\text{ES}} U_{\text{graph}}(x_2, x_1) = U_{\text{graph}}(x_2, x_1) \overline{Q}_{\text{ES}}.} \tag{77}$$

It preserves cycles and the closed boundary space and therefore descends to a unitary

$$\boxed{U_Q(x_2, x_1) : H_Q^{0,\text{Hilb}} \simeq_{\text{unitary}} H_Q^{0,\text{Hilb}}.} \tag{78}$$

[33]

The embedding intertwines the descended clock with the native scalar clock,

$$\boxed{U_Q(x_2, x_1)\iota_{\text{phys}}^H = \iota_{\text{phys}}^H U_{\text{NL}}(x_2, x_1),} \tag{79}$$

and the retraction intertwines in the reverse direction,

$$\boxed{r_H U_Q(x_2, x_1) = U_{\text{NL}}(x_2, x_1) r_H.} \tag{80}$$

[34] Consequently

$$U_Q \mathcal{H}_{\text{native}} = \mathcal{H}_{\text{native}}, \qquad U_Q \mathcal{H}_{\text{additional}} = \mathcal{H}_{\text{additional}}. \tag{81}$$

Relative to Eq. (75), the physical clock therefore preserves the native and additional blocks. Its restriction to $\mathcal{H}_{\text{native}}$ is exactly the Paper 4 native evolution transported through the isometric embedding.

[31] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.native_additional_ghostZero_BRSTCohomology_actual_isCompl`.

[32] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.additionalGhostZeroBRSTCohomologySector_eq_bot_iff_retraction_injective`.

[33] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarHilbertBRSTCohomologyTransportUnitary_actual_norm`.

[34] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.nativeGhostZeroCohomologyRetraction_actual_intertwines_evolution`.

11 Projective gauge fixing

The projective symmetry is free and has the particularly simple torsion-trace action Eq. (14). Introduce a new projective antighost $\bar{\eta}_\mu$ and Lautrup field b_μ, with ghost numbers

$$\mathrm{gh}(\bar{\eta}) = -1, \qquad \mathrm{gh}(b) = 0. \tag{82}$$

The antighost is a new nonminimal field with its own typed role, and the projective ghost antifield retains its original BV-cotangent role.[35]

The natural projective gauge fermion density is

$$\Psi_{\mathrm{proj}} = \bar{\eta}^\mu \tau_\mu. \tag{83}$$

Because $\tau(P_B\Gamma) - \tau(\Gamma) = -3B$, the Faddeev–Popov operator is

$$\boxed{M_{\mathrm{FP}} = -3I, \qquad M_{\mathrm{FP}}^{-1} = -\frac{1}{3}I.} \tag{84}$$

[36][37]

The derived gauge-fixed density has the form

$$b \cdot \tau + \bar{\eta} \cdot M_{\mathrm{FP}}\eta. \tag{85}$$

The nonminimal differential is

$$s\bar{\eta} = b, \qquad sb = 0, \tag{86}$$

and has an explicit contracting homotopy. Together with the projective minimal pair this gives a projective quartet with

$$\boxed{dh + hd = I.} \tag{87}$$

[38]

11.1 Strong deformation retract and cohomology equivalence

Let $\mathcal{C}_{\mathrm{red}}$ denote the completed graph carrier after projecting to the quartet-reduced factor and

$$\mathcal{C}_{\mathrm{gf}} = \mathcal{C}_{\mathrm{red}} \times \mathcal{C}_{\mathrm{quartet}}. \tag{88}$$

There are explicit chain maps

$$i : \mathcal{C}_{\mathrm{red}} \to \mathcal{C}_{\mathrm{gf}}, \qquad p : \mathcal{C}_{\mathrm{gf}} \to \mathcal{C}_{\mathrm{red}},$$

with

$$\boxed{pi = I, \qquad dh + hd = I - ip.} \tag{89}$$

[35] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.projectiveAntighost_actual_distinct_from_ghostAntifield`.

[36] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.projectiveFaddeevPopov_actual_inverse_left`.

[37] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.projectiveFaddeevPopov_actual_inverse_right`.

[38] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.projectiveQuartet_actual_contractible`.

[39]

The induced cohomology maps are mutually inverse:

$$\boxed{H^\bullet(\mathcal{C}_{\rm gf}) \simeq H^\bullet(\mathcal{C}_{\rm red}).} \tag{90}$$

[40]

Thus the projective quartet is cohomologically contractible and the reduced carrier contains the full physical cohomology. On the stationary locus the unique $\tau = 0$ representative is the Levi–Civita connection:

$$\boxed{E_\Gamma S_{\rm Pal} = 0, \quad \tau = 0 \quad \Longrightarrow \quad \Gamma = \Gamma[g].} \tag{91}$$

[41]

This result fixes the projective gauge direction cohomologically. BV-canonical elimination of the connection pair (Γ, Γ^*) forms the separately defined reduced-symplectic program recorded in Section 19.

12 Combined operator Ward laws

For a BV coefficient C and native core operator $\widehat{O}$, the BRST differential acts on the coefficient factor while $\widehat{O}$ acts on the physical Fock factor. Thus

$$Q(C \otimes \widehat{O}) - (C \otimes \widehat{O})Q = (sC) \otimes \widehat{O}$$

for even C on the common core, with the graded sign inserted for odd C.

For a BV/spacetime coefficient C and native core operator $\widehat{O}$, define

$$\widehat{\mathcal{O}}_{C,O} := C \otimes \widehat{O}. \tag{92}$$

For even C, the exact common-core Ward identity is

$$\boxed{Q_{\rm ES}\widehat{\mathcal{O}}_{C,O} - \widehat{\mathcal{O}}_{C,O}Q_{\rm ES} = \widehat{\mathcal{O}}_{sC,O}.} \tag{93}$$

[42]

12.1 Scalar field

For each native real scalar mode,

$$\boxed{Q_{\rm ES}\widehat{\Phi} - \widehat{\Phi}Q_{\rm ES} = \widehat{s\Phi}} \tag{94}$$

[39] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.projectiveGaugeFixed_actual_deformation_retract`.

[40] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.projectiveGaugeFixedCohomologyEquiv`.

[41] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.projectiveGaugeFixing_actual_LeviCivita_endpoint`.

[42] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.representedCombinedCoreOperator_actual_even_Ward`.

on the actual finite physical core, with the spacetime coefficient carrying the diffeomorphism transformation.[43]

The derivative-correct jet identity follows from commutation of the BRST differential with total spacetime derivatives. The Paper 5 expectation-value Ward theorem is therefore recovered as a downstream consequence of the operator realization.

12.2 Stress and Einstein residual

Taking $\widehat{O}$ to be the finite-bank stress operator gives

$$\boxed{Q_{\mathrm{ES}}\widehat{\Delta T}^{(B)}_{\mu\nu} - \widehat{\Delta T}^{(B)}_{\mu\nu} Q_{\mathrm{ES}} = s\widehat{\Delta T^{(B)}_{\mu\nu}}.} \tag{95}$$

The projective sector is inert on the matter stress; diffeomorphism covariance is carried by the tensorial spacetime coefficient.[44]

Define the classical-Einstein/quantum-stress residual

$$\widehat{\mathcal{E}}_{\mu\nu} := G_{\mu\nu}[g]I - \kappa\widehat{\Delta T}_{\mu\nu}. \tag{96}$$

The Einstein tensor remains classical. The residual obeys the same combined Ward law on the finite core.[45]

12.3 All-mode termination

For finite-occupation core states, the directed stress expectations have the all-mode limit of Paper 4, while each regulating bank satisfies the operator Ward identity. For any represented observable-vector net converging in graph Hilbert topology, continuity of $\overline{Q}_{\mathrm{ES}}$ passes the BRST transform to the limit.

For any supplied net of represented observable vectors converging in graph Hilbert topology,

$$O_B \to O \quad \Longrightarrow \quad \overline{Q}_{\mathrm{ES}} O_B \to \overline{Q}_{\mathrm{ES}} O. \tag{97}$$

[46]

For the actual stress, the directed expectation values satisfy

$$\boxed{\langle \psi, \widehat{\Delta T}^{(B)}_{\mu\nu}\psi\rangle \longrightarrow \Delta T^{(\infty)}_{\mu\nu}[\psi],} \tag{98}$$

and Eq. (95) holds at every regulating bank. The Einstein residual inherits the corresponding affine expectation limit.[47][48]

[43] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.realModeField_actual_combined_BRST_Ward`.

[44] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.realScalarFieldBankMetricStressCore_actual_combined_Ward`.

[45] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.nativeEinsteinResidual_actual_combined_Ward`.

[46] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.AllModeGraphWardCertificate.actual_transformed_converges`.

[47] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.realScalarAllModeStress_actual_expectation_and_finiteBank_Ward`.

[48] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.realScalarAllModeResidual_actual_expectation_limit`.

Accordingly, the all-mode Ward statement is exact on the maximal domain currently carried by these observables: graph-state convergence for represented vectors and expectation-valued convergence for stress and residuals.

13 Density and local-functional BRST comparison

The operator BRST complex acts on finite-jet density representatives. The local-functional complex is obtained by quotienting total jet divergences. Let

$$\rho : \mathcal{A}_{\mathrm{ES}}^{\mathrm{BV}} \longrightarrow \mathfrak{F}_{\mathrm{local}} \tag{99}$$

be the quotient map. Commutation of the projective-symmetric gauge differential with total derivatives gives

$$\boxed{s_{\mathrm{functional}}\rho(F) = \rho(s_{\mathrm{density}}F).} \tag{100}$$

[49] Thus ρ induces a canonical forward map on gauge-sector cohomology.

There is an explicit density d_{div} with

$$\boxed{d_{\mathrm{div}} \neq 0, \qquad \rho(d_{\mathrm{div}}) = 0.} \tag{101}$$

[50] Hence ρ is not injective. It follows that no function

$$\sigma : \mathfrak{F}_{\mathrm{local}} \longrightarrow \mathcal{A}_{\mathrm{ES}}^{\mathrm{BV}} \tag{102}$$

can recover every density through

$$\boxed{\sigma \circ \rho = I_{\mathcal{A}_{\mathrm{ES}}^{\mathrm{BV}}}.} \tag{103}$$

[51] Equation (103) is the no-universal-density-recovery theorem. A quotient section addresses the opposite composition $\rho \circ \sigma = I_{\mathfrak{F}_{\mathrm{local}}}$ and is a separate algebraic choice.

The forward chain map therefore carries precisely the canonical information shared by the density and local-functional complexes, while the divergence quotient records the information discarded on passage to local-functional equivalence classes.

14 Diffeomorphism stabilizers and the effective gauge space

Let g be a smooth Lorentz metric on the spacetime carrier used throughout the series. The infinitesimal metric gauge generator is

$$R_g(\xi) = \mathcal{L}_\xi g. \tag{104}$$

The actual metric stabilizer is therefore

$$\mathrm{Stab}(g) = \ker R_g. \tag{105}$$

[49] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarDensityToFunctional_actual_chain_map`.

[50] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalarNonzeroDivergence_actual_ne_zero`.

[51] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.einsteinScalar_no_universal_density_recovery`.

On the formal smooth carrier, this is exactly the Killing algebra:

$$\boxed{\mathrm{Stab}(g) = \mathfrak{Kill}(g).} \tag{106}$$

[52]

Thus the diffeomorphism sector is organized by metric stabilizers. The effective infinitesimal gauge space at g is naturally

$$\boxed{\mathfrak{X}(M)/\mathfrak{Kill}(g).} \tag{107}$$

This is the precise sense in which the diffeomorphism sector is stabilizer-stratified.

The resulting gauge inventory is therefore:

Gauge direction	Structure	Physical treatment
Projective connection symmetry	Free with unique $\tau = 0$ slice	Gauge fixed; quartet contractible
Diffeomorphism symmetry	Stabilizer-stratified by $\mathfrak{Kill}(g)$	BRST quotient modulo stabilizer

15 Carrier-level BRST obstruction maps

The relevant obstruction maps are defined directly from the represented equations. On the algebraic carrier set

$$\mathfrak{D}_Q := Q_{\mathrm{ES}}^2, \tag{108}$$

$$\mathfrak{D}_U(x_2, x_1) := Q_{\mathrm{ES}} U_{\mathrm{dir}}(x_2, x_1) - U_{\mathrm{dir}}(x_2, x_1) Q_{\mathrm{ES}}, \tag{109}$$

and for a homogeneous represented observable F,

$$\mathfrak{D}_W(F) := [Q_{\mathrm{ES}}, \widehat{F}]_{\pm} - \widehat{sF}. \tag{110}$$

The established representation theorems give

$$\boxed{\mathfrak{D}_Q = 0, \qquad \mathfrak{D}_U = 0, \qquad \mathfrak{D}_W = 0} \tag{111}$$

on their declared algebraic domains.[53]

On the graph Hilbert carrier the corresponding square and transport obstruction maps also vanish:

$$\boxed{\overline{Q}_{\mathrm{ES}}^2 = 0, \qquad \overline{Q}_{\mathrm{ES}} U_{\mathrm{graph}} - U_{\mathrm{graph}} \overline{Q}_{\mathrm{ES}} = 0.} \tag{112}$$

The all-mode Ward statement retains the maximal domains established in Section 12: graph-state convergence for represented observable vectors and expectation-valued directed limits for stress and Einstein residuals.

These equations are carrier-level BRST representation statements. The perturbative quantum-master anomaly problem begins once a regulator-owned BV Laplacian or equivalent renormalized anomaly functional is supplied together with its subtraction and continuum data.

[52] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.qgc12MetricStabilizer_actual_eq_killingAlgebra`.
[53] *Formal counterpart:*
`QGC12.BV.QuantumGauge.Analytic.einsteinScalarCarrierBRSTObstructions_actual_zero`.

16 Paper-level theorem

The preceding results assemble into the following statement.

Theorem (quantum gauge realization of the self-consistent Einstein–scalar sector). Fix a positive Einstein coupling and an admissible self-consistent nonlinear Einstein–real-scalar branch from Paper 4. Then the symmetric/projective gauge structure of Paper 5 admits an operator and analytic BRST realization with the following properties.

(i) The algebraic charge is

$$Q_{\mathrm{ES}} = s^{\mathrm{sp}}_{\mathrm{gauge}} \otimes I_{\mathcal{F}_{\mathrm{NL}}}$$

and represents the gauge differential by the graded commutator law

$$[Q_{\mathrm{ES}}, \widehat{F}]_{\pm} = \widehat{sF}.$$

(ii) The exact algebraic graph completes to a graph Hilbert carrier with bounded square-zero differential $\overline{Q}_{\mathrm{ES}}$. Its unitary presentation class is invariant under every unitary coefficient-space change carrying the algebraic graph core to the corresponding presented core.

(iii) Integer ghost number is preserved by jet prolongation and raised by one under $\overline{Q}_{\mathrm{ES}}$. Degree-zero Hausdorff BRST cohomology is formed using the closed boundary space

$$B^{\mathrm{Hilb}}_Q = \overline{\overline{Q}_{\mathrm{ES}}(\Gamma_{\mathrm{core}})} = \overline{\mathrm{im}\, \overline{Q}_{\mathrm{ES}}}.$$

(iv) The harmonic realization satisfies

$$(B^{\mathrm{Hilb}}_Q)^{\perp} = \ker \overline{Q}^{*}_{\mathrm{ES}}, \qquad \ker \Delta_{\mathrm{BRST}} = \ker \overline{Q}_{\mathrm{ES}} \cap \ker \overline{Q}^{*}_{\mathrm{ES}}.$$

(v) The native scalar Fock theory embeds isometrically into literal ghost-number-zero cohomology and admits a continuous left inverse:

$$\iota^{H}_{\mathrm{phys}} : \mathcal{F}_{\mathrm{NL}} \hookrightarrow_{\mathrm{iso}} H^{0,\mathrm{Hilb}}_{Q}, \qquad r_H \iota^{H}_{\mathrm{phys}} = I.$$

Its range is closed.

(vi) The degree-zero cohomology decomposes as

$$H^{0,\mathrm{Hilb}}_{Q} = \mathcal{H}_{\mathrm{native}} \oplus \mathcal{H}_{\mathrm{additional}}, \qquad \mathcal{H}_{\mathrm{native}} = \mathrm{Ran}\, \iota^{H}_{\mathrm{phys}}, \quad \mathcal{H}_{\mathrm{additional}} = \ker r_H.$$

The equality $\mathcal{H}_{\mathrm{additional}} = 0$ is equivalent to surjectivity of the native embedding.

(vii) The descended physical clock is unitary, preserves both summands, and satisfies

$$U_Q \iota^{H}_{\mathrm{phys}} = \iota^{H}_{\mathrm{phys}} U_{\mathrm{NL}}, \qquad r_H U_Q = U_{\mathrm{NL}} r_H.$$

(viii) The projective connection sequence

$$0 \to \mathcal{P}_{\mathrm{proj}} \xrightarrow{R_{\mathrm{proj}}} \delta\Gamma \xrightarrow{L_{\Gamma}} \mathcal{E}_{\Gamma}$$

is exact at the projective-parameter and connection-variation positions. The torsion-trace gauge has Faddeev–Popov operator $-3I$, the projective quartet is contractible, and the gauge-fixed and quartet-reduced cohomologies are equivalent.

(ix) Scalar-field, stress-tensor, and Einstein-residual observables satisfy the combined BRST Ward identity on their declared common domains, with the all-mode passage retained at the maximal topology supplied by the observable theory.

(x) The density-to-local-functional quotient is a BRST chain map. An explicit nonzero total derivative lies in its kernel, and no universal density-recovery map σ can satisfy $\sigma\rho = I$.

(xi) The infinitesimal diffeomorphism stabilizer is the Killing algebra, and the carrier-level BRST square, time-intertwining, and Ward obstruction maps vanish on their declared domains.

[54]

17 Interpretation of the state spaces

The completed BRST theory contains two mathematically distinct levels of state interpretation. The full quotient

$$H_Q^{0,\mathrm{Hilb}} \tag{113}$$

is the literal ghost-number-zero Hausdorff BRST Hilbert cohomology. The closed subspace

$$\boxed{\mathcal{H}_{\mathrm{native}} = \iota^H_{\mathrm{phys}}(\mathcal{F}_{\mathrm{NL}})} \tag{114}$$

is the Einstein–scalar quantum sector already given physical semantics by Papers 2–4. Its norm, clock, coherent states, and physical observables are inherited exactly through the isometric embedding and the retraction.

The complementary subspace

$$\mathcal{H}_{\mathrm{additional}} = \ker r_H \tag{115}$$

contains every ghost-zero BRST class whose BV-vacuum coordinate vanishes. The current theorem surface gives its closedness, complementarity, and time invariance. Its vanishing is equivalent to exhaustion of ghost-zero BRST cohomology by the native Fock theory. This criterion isolates the remaining state-space question without assigning additional physical interpretation before an exhaustion theorem or a nonnative witness is obtained.

The graph Hilbert realization has an equally precise analytic interpretation. It is the Hilbert closure of the exact algebraic graph under the declared unweighted coefficient norm. Its unitary presentation invariance covers relabelings and sign conventions of the intrinsic monomial/wedge basis. The vertical-obstruction space $\mathcal{V}_Q$ records the exact relation between this graph realization and ordinary closure of the counting-space charge. When $\mathcal{V}_Q = 0$, the graph relation is the conventional graph of the closed operator $\overline{Q}_0$.

18 Relation to BRST/BV quantization

BRST/BV quantization organizes gauge symmetry through a square-zero differential and an extended field/antifield action satisfying the classical master equation.[6, 7, 8] The present paper establishes

[54] *Formal counterpart:* `QGC12.BV.QuantumGauge.Analytic.paper6ReviewClosure_actual_capstone`.

this structure for the exact Einstein–scalar sector carried through Papers 1–5 at the level required for operator BRST states and their Hilbert completion.

The physical scalar theory enters through the split isometric map

$$\mathcal{F}_{\mathrm{NL}} \hookrightarrow_{\mathrm{iso}} H_Q^{0,\mathrm{Hilb}},$$

the projective connection direction is controlled by the exact sequence Eq. (27), and the graph differential admits quotient, harmonic, and Hodge realizations. These results determine the complete Paper 6 state and gauge surface.

The subsequent perturbative stage introduces the regulator-owned BV quantum-master data. There the relevant anomaly object is attached to the regulated BV Laplacian or its equivalent renormalized obstruction functional, together with counterterms and controlled refinement or continuum limits.

19 Scope and next mathematical boundary

Paper 6 ends with the completed operator/graph-Hilbert BRST theory of a self-consistent classical-metric/quantum-matter Einstein–scalar sector. The following results belong to this endpoint:

$$\boxed{\begin{array}{c}\text{operator representation of the classical gauge differential,}\\ \text{closed graph BRST complex and ghost-zero Hausdorff cohomology,}\\ \text{Hodge realization of the closed quotient,}\\ \text{closed isometric native Fock retract and complementary sector,}\\ \text{unitary time-block invariance,}\\ \text{projective exactness, gauge fixing, and quartet reduction,}\\ \text{combined Ward laws and density/local-functional comparison.}\end{array}} \tag{116}$$

Two adjacent problems have independent exact formulations. The counting-space operator satisfies

$$Q_0 \text{ closable} \iff \mathcal{V}_Q = \{0\}, \tag{117}$$

and the remaining direct proof route is the finite-row/dense-formal-adjoint problem for the untruncated jet generator. The first-order Palatini carrier also retains the BV canonical pair (Γ, Γ^*) after projective gauge reduction; a BV-canonical elimination of this pair defines a separate reduced-symplectic problem.

The projected next paper boundary is regulator-owned perturbative BV/EFT. Its starting data are explicit regulated integration, a regulator-compatible BV quantum operator or equivalent anomaly functional, local counterterm spaces, Wess–Zumino consistency, and controlled regulator refinement or continuum limits. The outcome of that program is determined by the formal obstruction and limit theorems it earns.

A later quantum-gravity boundary requires a nontrivial quantum carrier for geometric degrees of freedom together with gravitational observables, dynamics or transitions, and semiclassical recovery.

20 Conclusion

Paper 6 completes the quantum gauge realization of the self-consistent Einstein–scalar sector developed across the preceding five papers. The symmetric/projective gauge differential of Paper 5 acts as

$$Q_{\mathrm{ES}} = s^{\mathrm{sp}}_{\mathrm{gauge}} \otimes I_{\mathcal{F}_{\mathrm{NL}}}$$

on a BV/native-Fock carrier and satisfies the exact graded representation law

$$[Q_{\mathrm{ES}}, \widehat{F}]_{\pm} = \widehat{sF}, \qquad Q_{\mathrm{ES}}^2 = 0.$$

Its graph completion carries a bounded square-zero differential with integer ghost degree, unitary presentation invariance, and the exact vertical-obstruction criterion for ordinary counting-space closure.

The Hausdorff boundary space is generated equivalently by the dense algebraic graph core or by the full graph differential range. Its orthogonal complement is the adjoint kernel, and the BRST Hodge operator realizes the same harmonic state space:

$$B_Q^{\mathrm{Hilb}} = \overline{\overline{Q}_{\mathrm{ES}}(\Gamma_{\mathrm{core}})} = \overline{\operatorname{im}\overline{Q}_{\mathrm{ES}}}, \qquad \ker \Delta_{\mathrm{BRST}} = \ker \overline{Q}_{\mathrm{ES}} \cap \ker \overline{Q}^{*}_{\mathrm{ES}}.$$

The central physical-state theorem is the split isometric embedding

$$\boxed{\iota^H_{\mathrm{phys}} : \mathcal{F}_{\mathrm{NL}} \hookrightarrow_{\mathrm{iso}} H_Q^{0,\mathrm{Hilb}}, \qquad r_H \iota^H_{\mathrm{phys}} = I.} \tag{118}$$

It yields

$$\boxed{H_Q^{0,\mathrm{Hilb}} = \mathcal{H}_{\mathrm{native}} \oplus \mathcal{H}_{\mathrm{additional}}.} \tag{119}$$

The native summand is the previously established Einstein–scalar Fock theory with its original norm and time evolution. The additional summand is the exact closed complement selected by the vanishing BV-vacuum coordinate. Its zero/nonzero status is equivalent to the exhaustion question for ghost-zero BRST cohomology.

The projective connection symmetry is governed by an injective generator whose image equals the kernel of the linearized connection Euler map. Its torsion-trace gauge has Faddeev–Popov operator $-3I$, its quartet retracts cohomologically, and the unique stationary zero-trace representative is Levi–Civita. Diffeomorphism stabilizers are exactly Killing algebras. Scalar, stress, and Einstein-residual observables obey the corresponding operator Ward laws on their maximal declared domains.

The six-paper chain therefore reaches

$$\boxed{\begin{array}{c}\text{terminal-law selection} \longrightarrow \text{quantum dynamics} \longrightarrow \text{local quantum matter} \\ \longrightarrow \text{self-consistent Einstein–scalar sector} \longrightarrow \text{classical master/gauge ownership} \\ \longrightarrow \text{operator BRST realization and gauge-reduced Hilbert state theory.}\end{array}} \tag{120}$$

The next projected boundary is the regulator-owned perturbative quantum-master problem.

A Reader theorem map

The principal paper-facing propositions and their formal counterparts are collected below. Internal formal names appear here for verification and provenance while the main body retains field-standard semantics.

Role	Mathematical proposition	Formal counterpart
Direct BV/Fock square-zero	$D_{\mathrm{dir}}^2 = 0$.	`directFockBVDifferential_actual_square_zero`
Direct native intertwining	$D_{\mathrm{dir}} U = U D_{\mathrm{dir}}$.	`directFockBVDifferential_actual_transport_intertwining`
Stationary connection quotient	$\mathcal{P}^{\Gamma}_{\mathrm{stat}}/\mathcal{G}_{\mathrm{proj}} \simeq \mathcal{M}_{\mathrm{Lor}}$.	`metricOnlyPalatiniEquiv`
Projective exact sequence	$\ker R_{\mathrm{proj}} = 0$ and $\ker L_{\Gamma} = \operatorname{im} R_{\mathrm{proj}}$.	`functionalProjectiveConnectionSequence_exact_at_parameter_and_connection`
Gauge tensor identity	$Q_{\mathrm{ES}} = s^{\mathrm{sp}}_{\mathrm{gauge}} \otimes I$.	`einsteinScalarQuantumBRSTCharge_actual_gauge_tensor_identity`
Operator BRST representation	$[Q_{\mathrm{ES}}, \widehat{F}]_{\pm} = \widehat{sF}$.	`einsteinScalarBRSTAdjoint_actual_represents`
Operator square-zero	$Q_{\mathrm{ES}}^2 = 0$.	`einsteinScalarQuantumBRSTCharge_actual_square_zero`
Graph square-zero	$\overline{Q}_{\mathrm{ES}}^2 = 0$.	`einsteinScalarClosedBRST_actual_square_zero`
Unitary presentation invariance	Unitary coefficient changes induce graph unitaries intertwining the closed charge.	`graphBRST_unitaryPresentationInvariant_actual_intertwines`
Vertical obstruction	Q_0 closable iff $\mathcal{V}_Q = 0$.	`einsteinScalarCountingBRSTOperator_closable_iff_verticalSubspace_eq_bot`
Boundary closure bridge	$\overline{Q(\Gamma_{\mathrm{core}})} = \overline{\operatorname{im} Q}$.	`graphBoundaryClosure_eq_coreBoundaryClosure`
Boundary/adjoint bridge	$B_Q^{\perp} = \ker Q^*$.	`graphBoundaryClosure_orthogonal_eq_adjointKernel`
Hodge kernel	$\ker \Delta_{\mathrm{BRST}} = \ker Q \cap \ker Q^*$.	`graphBRST_harmonic_eq_hodgeKernel`
Ghost-zero retraction	$r_H \iota^H_{\mathrm{phys}} = I$.	`nativeGhostZeroCohomologyRetraction_native`
Native isometry	$\lVert \iota^H_{\mathrm{phys}} \psi \rVert = \lVert \psi \rVert$.	`nativeGhostZeroHilbertBRSTClass_actual_isometric`
Native range closed	$\operatorname{Ran} \iota^H_{\mathrm{phys}}$ is closed.	`nativeGhostZeroBRSTCohomologySector_actual_closed`
Native/additional split	$H_Q^{0,\mathrm{Hilb}} = \mathcal{H}_{\mathrm{native}} \oplus \mathcal{H}_{\mathrm{additional}}$.	`native_additional_ghostZero_BRSTCohomology_actual_isCompl`
Exhaustion criterion	Additional sector vanishes iff the retraction is injective / native map is surjective.	`additionalGhostZeroBRSTCohomologySector_eq_bot_iff_retraction_injective`
Reverse time intertwining	$r_H U_Q = U_{\mathrm{NL}} r_H$.	`nativeGhostZeroCohomologyRetraction_actual_intertwines_evolution`
Projective FP inverse	$M_{\mathrm{FP}} = -3I$ with inverse $-\frac{1}{3}I$.	`projectiveFaddeevPopov_actual_inverse_left`
Projective quartet contraction	$dh + hd = I$.	`projectiveQuartet_actual_contractible`
Gauge-fixed cohomology equivalence	$H^{\bullet}(\mathcal{C}_{\mathrm{gf}}) \simeq H^{\bullet}(\mathcal{C}_{\mathrm{red}})$.	`projectiveGaugeFixedCohomologyEquiv`
Combined scalar Ward law	Physical scalar observable obeys the combined BRST Ward identity.	`realModeField_actual_combined_BRST_Ward`
Combined stress Ward law	Finite-bank stress obeys the combined BRST Ward identity.	`realScalarFieldBankMetricStressCore_actual_combined_Ward`
All-mode stress endpoint	Directed expectation limit plus exact finite-bank Ward laws.	`realScalarAllModeStress_actual_expectation_and_finiteBank_Ward`
Density/function chain map	$s_{\mathrm{functional}} \rho = \rho s_{\mathrm{density}}$.	`einsteinScalarDensityToFunctional_actual_chain_map`
Density-recovery theorem	No function can satisfy $\sigma\rho = I$ on all densities.	`einsteinScalar_no_universal_density_recovery`

Role	Mathematical proposition	Formal counterpart
Metric stabilizer	$\mathrm{Stab}(g) = \mathfrak{Kill}(g)$.	`qgc12MetricStabilizer_actual_eq_killingAlgebra`
Carrier-level obstructions	Square, transport, and Ward obstruction maps vanish on their declared carriers.	`einsteinScalarCarrierBRSTObstructions_actual_zero`
Review-closure capstone	Retract, split, evolution, graph, Hodge, projective, and density comparison results assembled.	`paper6ReviewClosure_actual_capstone`

B Verification and authority boundary

The review-closed Paper 6 theorem surface is fixed at repository revision

`0b8e63e9b1190f82cf17c90f9723f15852e33005`.

The focused audit target is

`QGC12Paper6ReviewClosureAudit`.

The audit checks the review-closure capstone, the native retraction and isometry, the topological decomposition, the vertical-obstruction criterion, unitary graph-presentation invariance, the boundary/Hodge bridge, projective exactness, the density-recovery theorem, and the carrier-level BRST obstruction maps. The audit also maintains the firewall separating this Paper 6 authority from the regulator-owned quantum-master program developed afterward.

The review closure strengthens the manuscript in four places. First, native physical states form a closed isometric retract of literal ghost-number-zero BRST cohomology. Second, the additional ghost-zero sector is defined exactly as the kernel of the descended retraction, with an exact zero criterion and an open zero/nonzero status. Third, the graph-Hilbert carrier has a unitary presentation-invariance theorem and an explicit closed vertical-obstruction subspace controlling ordinary closability. Fourth, algebraic-core boundaries, full graph boundaries, adjoint kernels, and Hodge representatives are linked by explicit equalities.

C Statement and status map

Statement	Status	Role
Direct full-master BV/Fock complex	proved	local-functional state complex
Stationary metric–connection quotient	proved	projective reduction to Lorentz metrics
Projective linear exactness	proved	complete linearized projective zero-mode statement
Typed operator BRST representation	proved	operator realization of the classical gauge differential
Operator square-zero and native-time intertwining	proved	algebraic BRST complex
Unweighted coefficient realization	fixed analytic input	counting Hilbert carrier
Graph-Hilbert BRST differential	proved	bounded closed square-zero completion

Statement	Status	Role
Unitary presentation invariance	proved	relabeling/sign/presentation robustness within the declared norm class
Counting-space closability	exact criterion isolated	equivalent to $\mathcal{V}_Q = 0$; finite-row/formal-adjoint route isolated
Ghost-number grading	proved	degree-zero BRST sector
Closed boundary equality	proved	core/full analytic boundary identification
Hausdorff/Hodge realization	proved	quotient, harmonic, and Hodge descriptions
Native Fock map into ghost-zero cohomology	proved isometric with closed range	established Einstein–scalar physical sector
Native cohomology retraction	proved continuous	split embedding
Native/additional topological direct sum	proved	exact state-space decomposition
Additional sector vanishing	exact criterion; undecided	equivalent to exhaustion by native Fock states
Physical quotient evolution	proved unitary	gauge-reduced clock
Native/additional block invariance	proved	dynamical preservation of both summands
Projective Faddeev–Popov operator	proved invertible	nondegenerate torsion-trace gauge
Projective quartet	proved contractible	cohomological gauge removal
Gauge-fixed / reduced cohomology equivalence	proved	strong deformation retract endpoint
Finite scalar/stress/residual Ward laws	proved	physical observable gauge law
All-mode Ward passage	maximal declared domains	graph-state and expectation-valued limits
Density-to-functional BRST map	proved	canonical forward comparison
Nonzero total divergence	proved	quotient kernel witness
Universal density recovery	proved impossible	no left inverse $\sigma\rho = I$
Metric stabilizer equals Killing algebra	proved	stabilizer-stratified diffeomorphism sector
Carrier-level BRST obstruction maps	proved zero	square/time/Ward representation closure
Connection–antifield BV-canonical elimination	separate problem	reduced symplectic presentation
Regulator-owned BV quantum-master theory	projected next boundary	perturbative anomaly/counterterm/refinement problem
Quantized geometry / quantum gravity	later conditional boundary	geometric quantum carrier, observables, dynamics, semiclassical recovery

D Formal source note

The theorem package belongs to the larger Concordia Lean 4 development underlying the series. The internal formal vocabulary, repository paths, and theorem names are retained in this appendix and in the formal-reference footnotes for reproducibility. The mathematical body uses the corresponding standard operator, BRST/BV, Palatini, and Hilbert-space semantics.

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