Regulated Palatini BV Quantization of a Self-Consistent Einstein–Scalar Sector

Quantum-master obstructions, finite Gaussian BV pushforwards, and restricted physical cohomology

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Abstract

The preceding papers construct a self-consistent Einstein–real-scalar sector, identify its functional symmetric/projective Palatini Batalin–Vilkovisky master action, and realize the classical gauge differential as an operator BRST complex containing the native scalar Fock theory as a closed, isometric, dynamically invariant retract. This seventh paper develops the regulator-owned quantum BV structure of the same master and classifies its finite QME, refinement, obstruction, and restricted physical-sector consequences.

A finite Palatini regulator selects parity-preserving conjugate field/antifield banks with split embeddings, projections, and refinement maps. Every finite bank carries a genuine second-order flat odd BV Laplacian Δ_N whose product defect generates the regulated antibracket and whose square vanishes. Density-dependent operators Δ_{ρ_N} satisfy the separate modular-square law. The rectangular jet regulator has a nonzero total-derivative boundary defect, so the initial quantum-master construction is carried on regulated density representatives. Three mathematical settings remain distinct throughout: a conditional finite-CME perturbative setting, the projected physical-master setting, and a density-aware Gaussian shell-comparison sector.

On a finite classical coefficient $S_{0,N}$ satisfying the finite CME, the flat regulated QME

$$\boxed{\mathfrak{Q}_N^{\mathrm{flat}}(S;\hbar) = \frac{1}{2}(S,S)_N - i\hbar\Delta_N S}$$

generates an exact finite quantum differential together with the defect-square identity and Wess–Zumino consistency. The local anomaly is a cocycle, and its class in the declared admissible counterterm quotient is the exact obstruction to the next perturbative coefficient. This criterion is proved at first order and recursively at arbitrary finite order. The actual projected physical master $S_N = P_N S_\kappa^{\mathrm{sp}}$ is evaluated independently on its finite carrier; at cutoff one its Hamiltonian square is explicitly nonzero on the retained scalar.

Regulator refinement produces a sequence of exact structural decisions. Raw coordinate projection has a nonzero odd-Laplacian transport defect, and flat fiber averaging has a nonzero BV–Stokes moment defect. A centered finite Gaussian/Berezin fiber supplies a normalized weighted pushforward $\Pi^{\mathrm{G}}_{N+1\to N}$ that retracts retained observables, intertwines the density BV operator, and composes associatively through arbitrary finite cutoff chains. Direct same-order transport of the physical master has a first-shell trace $-1/2$; contraction-number grading places that term at one-loop order and yields an exact Wick-graded shell calculus and counterterm recurrence.

The stronger all-observable Hamiltonian descent has an explicit first-shell kernel witness w satisfying

$$\Pi^{\mathrm{G}}_{1\to 0} w = 0, \qquad \boxed{\mathfrak{p}\big(\mathcal{R}_{\mathrm{Ham}}(w)\big) = -1}.$$

The residual is already present at $\hbar = 0$, so the declared global Gaussian renormalized-QME completion is empty at the stated scope. The obstruction simultaneously determines a canonical positive remainder: the adjacent-cutoff comparison kernel has a unique greatest invariant square-zero refinement $\mathcal{C}^{Q}_{N,\hbar}$ with genuine restricted cohomology. Its physical Paper-6 comparison sector contains a nonzero unit class. Spectator tensoring with the established native scalar Fock carrier preserves every nonzero native matter state as a nonzero cohomology class; an explicit one-particle state survives outside the scalar-vacuum line.

The local density classification is correspondingly sharp. On the complete polynomial superalgebra generated by the retained scalar zero jet, all four first jets, and the actual odd scalar BRST descendant, the simultaneous source-square and Paper-6 comparison kernel is exactly

$$\boxed{\{\text{constant even observables}\} \oplus \{0\}_{\mathrm{odd}}.}$$

The generator-linear adequacy search enlarges to the complete cutoff-one Palatini generator bank, including every retained physical field jet, diffeomorphism ghost, projective ghost, and both antifield families. Exact parity, integer ghost number, field support, jet shell, tensor component, and multiindex separation split the simultaneous equations into four finite blocks. The physical and ghost blocks have trivial kernels. The unrestricted kernel is exactly the product of the explicit physical-antifield and ghost-antifield shell kernels. Consequently,

$$\boxed{\mathrm{SimGates}(c) \wedge \mathrm{AntifieldFree}(c) \Longleftrightarrow c = 0.}$$

Every nonzero generator-linear survivor is supported entirely in the antifield sectors.

The final native nonlinear enlargement is the faithful antifield-free physical polynomial algebra on all 1,200 cutoff-one physical coordinates. Its comparison trace is an exact positive weighted Euler derivation on the 1,125 nonzero-shell variables, so comparison closure reduces every candidate to the 75-variable zero-shell polynomial algebra. A polynomial-valued two-ghost source selector then recovers every zero-shell formal partial derivative. Characteristic zero gives the complete common-kernel classification

$$\boxed{s_{\mathrm{fin}}^2 \Phi_1(P) = 0 \ \wedge \ \mathcal{D}_{\mathrm{cl}} \Phi_1(P) = 0 \Longleftrightarrow \exists q \in \mathbb{Q} : \ P = \mathrm{C}(q).}$$

Thus the full physical-polynomial simultaneous kernel is exactly the rational constant line.

Paper 7 therefore closes at a precise regulator and carrier boundary. Finite quantum-master structure, perturbative obstruction cohomology, Gaussian BV shell integration, Wick-graded transport, and nontrivial restricted physical cohomology are established. The global all-observable Gaussian descent branch has a proved nonzero obstruction, the complete cutoff-one antifield-free generator-linear search has trivial kernel, and the complete faithful physical-polynomial search has constant-only kernel. A subsequent positive QME program requires a genuinely different owner carrier together with a newly justified ownership edge. Independent quantum geometry enters after that prerequisite is inhabited.

1 Introduction

The first six papers of this series establish one Einstein–real-scalar sector through a sequence of independently verified mathematical layers. Paper 1 derives the singular scalar operator and proves Friedrichs selection of its terminal self-adjoint law. [1] Paper 2 constructs the associated continuum and physical-Cauchy quantum dynamics. [2] Paper 3 differentiates the quantum scalar into local vacuum-relative stress, conservation, and finite Einstein response. [3] Paper 4 reconstructs the scalar quantum law on a nonlinear self-consistent Einstein–scalar geometry and closes the source cycle

$$\boxed{g_{\mathrm{NL}} \longrightarrow \mathcal{Q}(g_{\mathrm{NL}}) \ni \Psi_\star \longrightarrow \Delta T[\Psi_\star] \longrightarrow g_{\mathrm{NL}}.} \tag{1}$$

[4]

Paper 5 supplies the classical gauge realization of the same sector: an intrinsic symmetric/projective Palatini BV master S_κ^{sp} satisfying

$$\boxed{(S_\kappa^{\mathrm{sp}}, S_\kappa^{\mathrm{sp}}) = 0, \qquad s_\infty^2 = 0, \qquad s_\infty F := (S_\kappa^{\mathrm{sp}}, F).} \tag{2}$$

It also isolates the projective connection direction and its physical Palatini reduction. [5]

Paper 6 promotes the classical gauge differential to an operator BRST representation on an enlarged BV/Fock carrier containing the native scalar theory. Its basic identity is

$$\boxed{Q_{\mathrm{ES}} = s_{\mathrm{gauge}}^{\mathrm{sp}} \otimes I_{\mathcal{F}_{\mathrm{NL}}},} \tag{3}$$

with a closed graph-Hilbert realization, Hausdorff cohomology, harmonic representatives, exact projective reduction, and a degree-zero physical retract

$$\boxed{\mathcal{F}_{\mathrm{NL}} \hookrightarrow_{\mathrm{iso}} H_Q^{0,\mathrm{Hilb}}, \qquad r_H \iota_H = I.} \tag{4}$$

[6]

The present paper advances to the quantum-master question. Once a finite BV measure structure is introduced, the gauge differential acquires a second-order contribution. At finite cutoff the analysis separates three mathematical settings. A *conditional finite-CME perturbative setting* uses an even coefficient $S_{0,N}$ satisfying

$$\boxed{(S_{0,N}, S_{0,N})_N = 0, \qquad s_{0,N} := (S_{0,N}, -)_N,} \tag{5}$$

and provides the anomaly cohomology and recursive QME-extension theorems. The *actual projected physical-master setting* uses

$$\boxed{S_N := P_N S_\kappa^{\mathrm{sp}}, \qquad s_{\mathrm{phys},N} := (S_N, -)_N,} \tag{6}$$

with its square computed directly on the unrestricted finite carrier. The *density-aware Gaussian shell setting* uses

$$\boxed{s_{\hbar,N}^{\rho} F = (S_N, F)_N - i\hbar \Delta_{\rho_N} F} \tag{7}$$

to study finite-shell transport and the restricted surviving complex. The common flat operator is

$$\boxed{\Delta_N := \Delta_{\mathrm{flat},N}, \qquad s_{\hbar,N}^{\mathrm{flat}} F = (S, F)_N - i\hbar \Delta_N F,} \tag{8}$$

for any declared finite master S. At $\hbar = 0$, the finite CME hypothesis gives the square-zero perturbative background differential $s_{0,N}$. At cutoff zero the Gaussian chart is the flat chart, so $\Delta_{\rho_0} = \Delta_{\text{flat},0}$. At nonflat density the modular-square law for Δ_{ρ_N} remains explicit; the QME/Wess–Zumino square identity below is asserted in the flat square-zero setting. This paper follows these structures through six questions:

$$\boxed{\begin{array}{c}\text{Which finite regulators carry a genuine BV odd Laplacian?}\\ \text{Which anomaly classes obstruct the perturbative quantum master equation?}\\ \text{How does integrating out one finite BV shell transport the quantum differential?}\\ \text{Which restricted physical cohomology is selected by the global descent obstruction?}\\ \text{Which antifield-free generator-linear densities survive the simultaneous physical gates?}\\ \text{Which polynomials survive on the complete antifield-free physical coordinate algebra?}\end{array}} \tag{9}$$

The analysis yields a layered set of results at the stated scope, with each obstruction retained as a primary mathematical result. Rectangular truncation has a nonzero derivative-closure defect. Raw projection has a nonzero odd-Laplacian transport defect. Flat fiber integration has a nonzero BV–Stokes moment defect. Direct same-order Gaussian master transport produces a nonzero first-shell trace. Loop grading places that trace at the correct perturbative order. The all-observable Hamiltonian descent equation has a nonzero kernel-witness residual already at $\hbar = 0$. The declared global Gaussian completion is therefore empty.

The same analysis constructs the corresponding positive structures. Finite BV Laplacians, Wess–Zumino consistency, recursive anomaly classes, finite Gaussian shell pushforwards, loop-graded Wick transport, a canonical greatest compatible quantum subcomplex, genuine restricted cohomology, and nonzero native matter-state classes are all proved. The final result retains these structures together with the global obstruction, the complete scalar-generated density classification, the exact full generator-linear kernel decision, and the complete physical-polynomial constant-kernel theorem.

Obstruction and construction form one continuous mathematical sequence in this paper. Each obstruction theorem determines the exact boundary inherited by the subsequent construction. The resulting progression is

$$\boxed{\begin{array}{c}\text{finite regulator and odd Laplacian}\\ \downarrow\\ \text{regulated QME and anomaly complex}\\ \downarrow\\ \text{nonzero raw/flat refinement defects}\\ \downarrow\\ \text{Gaussian BV pushforward}\\ \downarrow\\ \text{nonzero same-order master residual}\\ \downarrow\\ \text{Wick-contraction loop grading}\\ \downarrow\\ \text{nonzero global Hamiltonian descent residual}\\ \downarrow\\ \text{greatest restricted square-zero quantum complex}\\ \downarrow\\ \text{nontrivial restricted physical cohomology}\\ \downarrow\\ \text{constant-only native scalar density kernel}\\ \downarrow\\ \text{trivial antifield-free generator-linear simultaneous kernel}\\ \downarrow\\ \text{strict nonlinear enlargement to the full physical polynomial algebra}\\ \downarrow\\ \text{constant-only complete physical-polynomial simultaneous kernel.}\end{array}} \tag{10}$$

The paper establishes a classical-metric/quantum-matter sector with regulator-owned BV quantization. Independent fluctuating geometry belongs to the next mathematical layer. The final section records that boundary explicitly.

2 Input structures and semantic separation

Throughout the paper, the Einstein coupling is a nonzero real number. Formally, the Lean parameter is

$$\boxed{\kappa \in \mathbb{R}^{\times}, \qquad \texttt{EinsteinCoupling} = \{\kappa : \mathbb{R} \mid \kappa \neq 0\}.} \tag{11}$$

Thus every theorem quantified over an Einstein coupling ranges over all nonzero real values, with unrestricted sign.

The classical input is the symmetric/projective Palatini master S^{sp}_{κ} of Paper 5 on the untruncated local jet-functional carrier. Its Hamiltonian differential is

$$s_\infty F = (S^{\mathrm{sp}}_{\kappa}, F), \qquad s_\infty^2 = 0. \tag{12}$$

The operator comparison input is the Paper-6 charge Q_{ES} and its finite-bank compression. The physical matter input is the native scalar Fock space $\mathcal{F}_{\mathrm{NL}}$ and its established unitary evolution. These established structures provide the fixed comparison data for the quantum-master construction.

The operator roles remain distinct:

$$\boxed{\begin{array}{rcl} s_\infty = (S_\kappa^{\mathrm{sp}}, -) & : & \text{untruncated classical BV Hamiltonian differential,} \\ Q_{\mathrm{ES}} & : & \text{Paper-6 operator BRST differential,} \\ \Delta_N = \Delta_{\mathrm{flat},N} & : & \text{flat finite BV odd Laplacian,} \\ \Delta_{\rho_N} & : & \text{density-dependent finite BV odd Laplacian,} \\ s_{\hbar,N}^{\mathrm{flat}} & : & \text{flat finite QME differential,} \\ s_{\hbar,N}^{\rho} & : & \text{density-aware shell-comparison differential.} \end{array}} \tag{13}$$

The three finite settings are therefore distinguished by their master hypothesis as well as by their BV operator:

setting	master condition	mathematical role
finite-CME perturbative	$(S_{0,N}, S_{0,N})_N = 0$	anomaly cohomology and QME recursion
projected physical master	$S_N = P_N S_\kappa^{\mathrm{sp}}$	actual finite defects and source-square tests
density-aware Gaussian shell	(S_N, ρ_N)	shell descent and restricted complex

(14)

The first and second lanes live on the same flat finite BV algebra and carry independent theorem hypotheses. Finite-CME eligibility is an additional condition on a projected background.

The Paper-6 graph/Hodge operator remains a distinct analytic object with its own parity, order, and state-space role.

At finite cutoff the present paper works first on density representatives. Here *density representative* means a local BV/Lagrangian density before quotienting by total derivatives. The symbol ρ_N denotes a BV measure chart entering Δ_{ρ_N}. The local-functional quotient enters through proved comparison maps. This distinction becomes necessary because the finite rectangular regulator is shown below to erase a nonzero top-face total derivative. The quantum-master equations are therefore stated on the carrier on which every operation is literally defined.

The formal implementation binds these inputs to one dependency chain with dedicated interfaces for the Paper-6 physical carrier and the new regulator structures.[1]

3 Finite Palatini BV regulator

Let $\mathfrak{R}_N$ denote the finite Palatini regulator at cutoff N. It consists of a finite conjugate pair bank, a finite regulated carrier $\mathcal{A}_N$, an embedding

$$\iota_N : \mathcal{A}_N \longrightarrow \mathcal{A}_\infty, \tag{15}$$

and a projection

$$P_N : \mathcal{A}_\infty \longrightarrow \mathcal{A}_N \tag{16}$$

satisfying

$$\boxed{P_N \iota_N = I_{\mathcal{A}_N}.} \tag{17}$$

[1] *Formal counterpart:* `lean/QGC12/BV/QME/Regulator.lean;lean/QGC12/BV/QME/Paper6Comparison.lean;lean/QGC12/BV/QME/CompletionStatus.lean`.

The pair bank carries the inherited parity/orientation data, and every ordered pair of cutoffs $N \leq M$ has a refinement map compatible with the embeddings and projections.

The regulated physical master is defined directly by projection of the established Palatini master:

$$\boxed{S_N = P_N S^{\mathrm{sp}}_{\kappa}.} \tag{18}$$

A collapsed regulator has a false admissibility predicate precisely when conjugate-pair closure is lost. This control makes symplectic pairing an explicit requirement of the finite BV-regulator interface.

3.1 Finite odd Laplacian

On each admissible finite bank the canonical odd BV Laplacian is an actual second-order operator

$$\Delta_N : \mathcal{A}_N \longrightarrow \mathcal{A}_N. \tag{19}$$

It kills the unit,

$$\boxed{\Delta_N 1 = 0,} \tag{20}$$

reverses parity, and generates the regulated antibracket through its product defect. For homogeneous F of parity $|F|$,

$$\boxed{\Delta_N(FG) = \Delta_N F\, G + (-1)^{|F|} F\, \Delta_N G + (-1)^{|F|}(F, G)_N.} \tag{21}$$

Thus the bracket and odd Laplacian are tied by a proved finite identity. [2]

3.2 Density dependence and modular square

A finite density chart ρ modifies the operator to $\Delta_{\rho,N}$. The square is governed by its modular Hamiltonian $H_{\rho,N}$:

$$\boxed{\Delta^2_{\rho,N} F = \left(H_{\rho,N}, F\right)_N.} \tag{22}$$

The flat chart has vanishing modular Hamiltonian and therefore

$$\boxed{\Delta^2_{\mathrm{flat},N} = 0.} \tag{23}$$

The recursive Gaussian chart is anchored literally at the flat base,

$$\boxed{\rho_0 = 1, \qquad \Delta_{\rho_0,0} = \Delta_{\mathrm{flat},0}.} \tag{24}$$

Here $\rho_0 = 1$ denotes the zero logarithmic-density potential in the flat chart. Density changes and pair-coordinate reindexings satisfy exact naturality laws. Throughout Sections 4–6, Δ_N denotes the flat operator $\Delta_{\mathrm{flat},N}$ and $s^{\mathrm{flat}}_{\hbar,N}$ denotes its square-zero QME differential. The nonflat operators $\Delta_{\rho N}$ and $s^{\rho}_{\hbar,N}$ enter the Gaussian pushforward, global-descent, and restricted-cohomology sections. The density-dependent shell theory retains the modular Hamiltonian from Eq. (22) explicitly. These finite results supply the measure-dependent shell operation from finite-dimensional data. [3]

[2] *Formal counterpart:* `lean/Tower/BV/Quantum/OddLaplacian.lean;lean/QGC12/BV/QME/OddLaplacian.lean`.

[3] *Formal counterpart:* `lean/Tower/BV/Quantum/DensityChange.lean;lean/Tower/BV/Quantum/CoordinateNaturality.lean;lean/QGC12/BV/QME/DensityChange.lean;lean/QGC12/BV/QME/PalatiniGaussianCutoffSystem.lean`.

3.3 First obstruction: nonzero total-derivative boundary defect

Let $\jmath_N$ denote the retained-pair embedding into the ambient jet-key bank. For cutoff N, choose the projective-covector field key η_0 and define the top-face pair by

$$p_N^{\mathrm{top}} := (\eta_0, (N, N, N, N)), \qquad u_N := e_{p_N^{\mathrm{top}}} \in \mathcal{A}_N, \qquad p_N^{\mathrm{out}} := \mathrm{shift}_0(\jmath_N p_N^{\mathrm{top}}). \tag{25}$$

The first total derivative sends the embedded top-face generator to the literal successor coordinate,

$$\boxed{\partial_0 \iota_N(u_N) = e_{p_N^{\mathrm{out}}} \neq 0.} \tag{26}$$

The successor has zeroth jet index $N+1$. Every retained rectangular coordinate has zeroth jet index at most N. Hence

$$p_N^{\mathrm{out}} \notin \mathrm{im}(\jmath_N), \qquad P_N(e_{p_N^{\mathrm{out}}}) = 0, \tag{27}$$

and therefore

$$\boxed{\partial_0 \iota_N(u_N) \neq \iota_N P_N(\partial_0 \iota_N(u_N)) = 0.} \tag{28}$$

Equivalently, no successor map $\sigma_\mu : \mathfrak{R}_N \to \mathfrak{R}_N$ can satisfy

$$\iota_N(\sigma_\mu p) = \mathrm{shift}_\mu(\iota_N p) \quad \text{for every retained pair } p, \tag{29}$$

so the finite rectangular density bank has no all-coordinate total-derivative closure. Local-functional QME descent consequently requires a distinct descent construction. The present finite QME is stated on density representatives. [4]

4 Regulated quantum master equation

The real Palatini master is even. Complex QME coefficients are represented on a doubled-real carrier, keeping the real source theory and the factor of i mathematically explicit. For a regulated master S define

$$\boxed{\mathfrak{Q}_N^{\mathrm{flat}}(S;\hbar) := \frac{1}{2}(S,S)_N - i\hbar\Delta_N S.} \tag{30}$$

The corresponding finite quantum differential is

$$\boxed{s_{\hbar,N}^{\mathrm{flat}} F := (S,F)_N - i\hbar\Delta_N F.} \tag{31}$$

At $\hbar = 0$ this reduces to the finite classical Hamiltonian differential.

The basic defect-square identity is

$$\boxed{\left(s_{\hbar,N}^{\mathrm{flat}}\right)^2 F = \left(\mathfrak{Q}_N^{\mathrm{flat}}(S;\hbar), F\right)_N.} \tag{32}$$

Hence a solved finite QME implies square-zero of the quantum differential on the declared carrier.

[4] *Formal counterpart:* `lean/QGC12/BV/QME/JetCutoffBoundary.lean;lean/QGC12/BV/QME/RectangularLocalityNoGo.lean`.

The QME defect itself is quantum closed:

$$\boxed{s^{\mathrm{flat}}_{\hbar,N}\mathfrak{Q}^{\mathrm{flat}}_N(S;\hbar)=0.} \tag{33}$$

This is the finite Wess–Zumino consistency law. It follows from the finite bracket, Jacobi, and odd-Laplacian identities. The same law holds for genuinely doubled-real effective masters.

As a mismatch control, define the identity-contaminated operator

$$\widetilde{\Delta}_N := \Delta_N + I. \tag{34}$$

Since $\Delta_N 1 = 0$,

$$\widetilde{\Delta}_N 1 = 1. \tag{35}$$

For the unit master, whose bracket with itself vanishes, the doubled-real QME defect becomes

$$\boxed{\widetilde{\mathfrak{Q}}_N(1;\hbar) = (0,-\hbar\,1) \neq (0,0) \qquad (\hbar\neq 0).} \tag{36}$$

This control makes the second-order BV identities part of the mathematical interface in addition to operator type compatibility. [5]

5 Comparison with the Paper-6 BRST differential

Finite projection of the Paper-6 operator gives a regulated BRST differential $Q^{(6)}_N$. Its square contains the explicit cutoff leakage term L_N:

$$\boxed{(Q^{(6)}_N)^2 + L_N = 0.} \tag{37}$$

This equation separates finite compression from the globally square-zero untruncated charge.

On the real sector, the finite quantum differential and Paper-6 compression agree exactly when two explicit defects vanish. For an observable F,

$$s^{\mathrm{flat}}_{\hbar,N}(F,0) = (Q^{(6)}_N F, 0) \tag{38}$$

if and only if

$$\boxed{\mathcal{D}_{\mathrm{cl},N}(F)=0, \qquad \hbar\,\Delta_N F = 0.} \tag{39}$$

The kernel intersection of these two maps is the finite common comparison sector. A nonzero measure component at nonzero $\hbar$ therefore creates a genuine quantum departure from the Paper-6 classical/operator BRST action.

This comparison enters twice. It identifies the shared finite observable sector and later supplies one of the two physical gates defining the restricted theory selected by the global descent obstruction. [6]

[5] *Formal counterpart:* `lean/Tower/BV/Quantum/QMEDefect.lean;lean/Tower/BV/Quantum/MismatchControl.lean;lean/QGC12/BV/QME/MasterDefect.lean`.

[6] *Formal counterpart:* `lean/QGC12/BV/QME/Paper6Comparison.lean`.

6 Local anomaly cohomology and perturbative extension

This section treats the conditional finite-CME setting. Fix a cutoff N and an even finite classical coefficient $S_{0,N}$ satisfying

$$\boxed{(S_{0,N}, S_{0,N})_N = 0, \qquad s_{0,N} := (S_{0,N}, -)_N, \qquad s_{0,N}^2 = 0.} \tag{40}$$

Finite-CME eligibility is an explicit hypothesis. The projected physical master $S_N = P_N S_\kappa^{\mathrm{sp}}$ carries its own unrestricted finite-carrier calculation. At cutoff one the actual projected physical master has a retained scalar ϕ with a proved nonzero second Hamiltonian action,

$$\boxed{s_{\mathrm{phys},1}^2 \phi = \mathcal{O}_\phi \neq 0,} \tag{41}$$

so the full cutoff-one physical carrier has a nonzero square defect. This finite leakage enters the later physical-master analysis.

Expand an effective master around the eligible finite-CME background as

$$S_{\mathrm{eff},N} = S_{0,N} + \hbar S_{1,N} + \hbar^2 S_{2,N} + \cdots . \tag{42}$$

The coefficient of the QME at each order decomposes into the square-zero finite differential applied to the new coefficient plus a known anomaly formed from lower orders.

At first order,

$$\boxed{\mathcal{A}_{1,N} = -i\Delta_N S_{0,N},} \tag{43}$$

and Wess–Zumino consistency gives

$$s_{0,N}\mathcal{A}_{1,N} = 0. \tag{44}$$

Let $\mathcal{P}_n$ denote the declared admissible counterterm submodule at order n, obtained by intersecting the locality, even-parity, ghost-zero, covariance, projective-invariance, real-form, and engineering-order conditions. Write

$$Z(s_{0,N}) := \ker s_{0,N}, \qquad B_{\mathcal{P}_n}(s_{0,N}) := \{s_{0,N}C : C \in \mathcal{P}_n\} \subseteq Z(s_{0,N}), \tag{45}$$

and define the admissible obstruction cohomology

$$\boxed{H_{\mathcal{P}_n}(s_{0,N}) := Z(s_{0,N})/B_{\mathcal{P}_n}(s_{0,N}).} \tag{46}$$

For a closed anomaly $\mathcal{A}_{n,N}$, the notation $[\mathcal{A}_{n,N}]_{\mathcal{P}_n}$ means its quotient class in $H_{\mathcal{P}_n}(s_{0,N})$. The first obstruction class vanishes precisely when an admissible even correction solves the first quantum coefficient:

$$\boxed{[\mathcal{A}_{1,N}]_{\mathcal{P}_1} = 0 \Longleftrightarrow \exists S_{1,N} \in \mathcal{P}_1 :\ \mathfrak{Q}_N^{\mathrm{flat}}(S_{\mathrm{eff},N};\hbar)_{[1]} = 0.} \tag{47}$$

The same structure persists at arbitrary positive order. Suppose the QME is solved through order n. Then

$$\boxed{\mathfrak{Q}^{\mathrm{flat}}_{N,[n+1]} = s_{0,N}S_{n+1,N} + \mathcal{A}_{n+1,N}(S_{0,N},\ldots,S_{n,N}).} \tag{48}$$

Whenever the lower-order solution hypotheses hold, Wess–Zumino consistency gives the recursive closure law

$$\boxed{\left(\mathfrak{Q}^{\mathrm{flat}}_{N,[0]} = \cdots = \mathfrak{Q}^{\mathrm{flat}}_{N,[n]} = 0\right) \Longrightarrow s_{0,N}\mathcal{A}_{n+1,N} = 0.} \tag{49}$$

The next coefficient exists in the declared admissible profile exactly when

$$\boxed{[\mathcal{A}_{n+1,N}]_{\mathcal{P}_{n+1}} = 0 \Longleftrightarrow \exists S_{n+1,N} \in \mathcal{P}_{n+1} : \mathfrak{Q}^{\mathrm{flat}}_{N,[n+1]} = 0.} \tag{50}$$

The two-loop statement is the case $n = 1$; the same identity propagates to every solved finite order.

The theorem is conditional on the declared admissible counterterm space. Counterterms enter through the explicit admissibility profile. A nonzero admissible obstruction class is therefore a terminal theorem at that profile. [7]

7 Regulator refinement and Gaussian BV pushforward

Let $\mathfrak{R}_N \preceq \mathfrak{R}_{N+1}$ be one strict regulator refinement. The larger pair bank splits into retained coordinates and a newly added finite shell.

7.1 Raw-projection odd-Laplacian defect

For the first strict refinement $0 \to 1$, let p_+ be the added pair with projective-covector field key η_0 and unit rectangular multiindex:

$$p_+ := (\eta_0, (1,1,1,1)), \qquad p_+ \notin \mathrm{im}(\jmath_{0\to 1}). \tag{51}$$

and let x_{p_+} and ξ_{p_+} be its even and odd conjugate coordinates. Define the canonical quadratic

$$q_+ := x_{p_+} \xi_{p_+}. \tag{52}$$

Coordinate restriction erases the added pair,

$$P_{1\to 0} q_+ = 0, \tag{53}$$

The cutoff-one flat odd Laplacian gives

$$\Delta_1 q_+ = -\varepsilon(p_+)\, 1, \qquad \varepsilon(p_+) := \mathrm{orientationScalar}\big(\mathrm{negative}(p_+)\big) \in \{-1, 1\}. \tag{54}$$

Hence the refinement defect

$$\mathfrak{D}_{1\to 0}(F) := P_{1\to 0}\Delta_1 F - \Delta_0 P_{1\to 0} F \tag{55}$$

has the explicit value

$$\boxed{\mathfrak{D}_{1\to 0}(q_+) = -\varepsilon(p_+)\, 1 \neq 0.} \tag{56}$$

Thus raw coordinate restriction is separated from BV fiber integration by a literal one-pair calculation.

A retained density potential $V \in \mathcal{A}_0$ produces the half-Hamiltonian correction

$$\mathfrak{H}_V(F) := \frac{1}{2}\big(V, P_{1\to 0} F\big)_0. \tag{57}$$

[7] *Formal counterpart:* `lean/QGC12/BV/QME/LocalAnomaly.lean;lean/QGC12/BV/QME/OneLoopObstruction.lean;lean/QGC12/BV/QME/RecursiveExtension.lean;lean/QGC12/BV/QME/TwoLoopConsistency.lean;lean/QGC12/BV/QME/AutomaticRecursiveStage.lean;lean/QGC12/BV/QME/RestrictedNonconstantChainDecision.lean`.

The exact compensation equation is

$$\boxed{\mathfrak{D}_{1\to 0} + \mathfrak{H}_V = 0.} \tag{58}$$

On q_+,

$$\mathfrak{H}_V(q_+) = \frac{1}{2}(V, 0)_0 = 0, \qquad \mathfrak{D}_{1\to 0}(q_+) \neq 0, \tag{59}$$

and therefore the retained-potential solution set of Eq. (58) is empty for the concrete Palatini refinement. More generally, for an admissible retained submodule $\mathcal{P}$,

$$[\mathfrak{D}_{1\to 0}]_{\mathrm{Ham},\mathcal{P}} = 0 \iff \exists V \in \mathcal{P} : \ \mathfrak{D}_{1\to 0} + \mathfrak{H}_V = 0. \tag{60}$$

[8]

7.2 Flat-fiber BV–Stokes moment defect

Let $\Pi : \mathcal{A}_1 \to \mathcal{A}_0$ be a normalized finite BV fiber pushforward. Its defining finite Stokes conditions are

$$\boxed{\Pi\iota_{0\to 1} = I, \qquad \Pi\Delta_1 = \Delta_0\Pi.} \tag{61}$$

Applying the second identity to the same eliminated-pair quadratic q_+ gives

$$\Delta_0\Pi(q_+) = \Pi(\Delta_1 q_+) = -\varepsilon(p_+)\Pi(1) = -\varepsilon(p_+)\,1. \tag{62}$$

A base-independent scalar moment would have the form

$$\Pi(q_+) = \lambda\,1. \tag{63}$$

Since $\Delta_0(\lambda 1) = 0$, Eqs. (62)–(63) imply

$$0 = -\varepsilon(p_+)\,1, \tag{64}$$

contradicting $\varepsilon(p_+) \in \{\pm 1\}$. Therefore every normalized flat BV–Stokes pushforward satisfies

$$\boxed{\Pi(q_+) \neq \lambda 1 \quad \text{for every } \lambda \in \mathbb{Q}.} \tag{65}$$

The centered Gaussian density supplies the required nonflat integration-by-parts structure in the next subsection. [9]

[8] *Formal counterpart:* lean/Tower/BV/Quantum/RegulatorRefinement.lean;lean/Tower/BV/Quantum/CompensatedRefinement.lean;lean/QGC12/BV/QME/RegulatorRefinement.lean;lean/QGC12/BV/QME/CompensatedRefinement.lean.

[9] *Formal counterpart:* lean/Tower/BV/Quantum/FiberPushforward.lean;lean/Tower/BV/Quantum/WeightedFiberPushforward.lean;lean/QGC12/BV/QME/FiberPushforward.lean;lean/QGC12/BV/QME/WeightedFiberPushforward.lean.

7.3 Finite Gaussian/Berezin fiber pushforward

The eliminated even coordinates carry a centered Gaussian expectation. The eliminated odd coordinates carry the finite exterior/Berezin factor. Their tensor product defines a normalized finite BV fiber. For the Palatini complement the resulting weighted pushforward

$$\Pi^{\mathrm{G}}_{N+1\to N} : \mathcal{A}_{N+1} \longrightarrow \mathcal{A}_N \tag{66}$$

satisfies

$$\boxed{\Pi^{\mathrm{G}}_{N+1\to N}(1) = 1, \qquad \Pi^{\mathrm{G}}_{N+1\to N}\iota_{N\to N+1} = I,} \tag{67}$$

and, for the recursively matched densities,

$$\boxed{\Pi^{\mathrm{G}}_{N+1\to N}\Delta_{\rho_{N+1}} = \Delta_{\rho_N}\Pi^{\mathrm{G}}_{N+1\to N}.} \tag{68}$$

This is the finite BV–Stokes theorem used in the rest of the paper. [10]

7.4 Arbitrary finite cutoff chains

The density at cutoff $N+1$ retains the cutoff-N density and adds one centered Gaussian shell:

$$\boxed{\log\rho_{N+1} = \iota\log\rho_N + \log\rho_{\mathrm{shell},N+1}.} \tag{69}$$

The successor pushforwards compose associatively. For example,

$$\Pi^{\mathrm{G}}_{2\to 0} = \Pi^{\mathrm{G}}_{1\to 0}\Pi^{\mathrm{G}}_{2\to 1}, \tag{70}$$

and arbitrary finite prefixes define coherent transitive pushforwards. Each prefix retracts the corresponding transitive embedding and transports the density odd Laplacian. [11]

8 Same-order transport obstruction and Wick regrading

A coherent BV shell map and physical-master transport carry independent perturbative obligations. Let

$$\mathcal{R}^{(0)}_N := \Pi^{\mathrm{G}}_{N+1\to N}S_{N+1} - S_N \tag{71}$$

be the direct same-order residual.

[10] *Formal counterpart:* `lean/Tower/BV/Quantum/GaussianFiberKernel.lean;lean/Tower/BV/Quantum/FiniteGaussianExpectation.lean;lean/Tower/BV/Quantum/FiniteGaussianBVFiber.lean;lean/QGC12/BV/QME/FiniteGaussianComplement.lean;lean/QGC12/BV/QME/FiniteGaussianBVComplement.lean;lean/QGC12/BV/QME/RetainedGaussianPushforward.lean`.

[11] *Formal counterpart:* `lean/QGC12/BV/QME/SuccessiveGaussianPushforward.lean;lean/QGC12/BV/QME/PalatiniGaussianCutoffSystem.lean`.

8.1 Explicit first-shell trace calculation

Let h^{00} denote the retained cutoff-zero inverse-density coordinate and let ε_0 be the symmetric-algebra augmentation that sends every positive-degree monomial to zero. Define

$$\boxed{\mathfrak{t} := \varepsilon_0 \circ \partial_{h^{00}} \circ \mathrm{pr}_{\mathrm{even}}\,.} \tag{72}$$

Its real scalar extension acts on the physical master residual. Write the cutoff-0 → 1 shell residuals of the curvature, scalar-kinetic, and cotangent sectors as R_{curv}, R_{kin}, and R_{cot}. The projected real master gives

$$\mathcal{R}_0^{(0)} = \frac{1}{2\kappa} R_{\mathrm{curv}} - \frac{1}{2} R_{\mathrm{kin}} + R_{\mathrm{cot}}. \tag{73}$$

Proposition (first-shell same-order trace). Write the eliminated scalar first jets as $x_\mu := \partial_\mu \phi$. The cutoff-one scalar kinetic polynomial entering the projected master is

$$S^{(1)}_{\mathrm{kin}} = \sum_{\mu,\nu=0}^{3} h^{\mu\nu} x_\mu x_\nu, \qquad P_0 S^{(1)}_{\mathrm{kin}} = 0. \tag{74}$$

The centered unit-covariance Gaussian shell satisfies

$$\Pi^{\mathrm{G}}_{1\to 0}(x_\mu x_\nu) = \delta_{\mu\nu}, \tag{75}$$

so its kinetic residual is the retained inverse-density trace,

$$R_{\mathrm{kin}} = \Pi^{\mathrm{G}}_{1\to 0} S^{(1)}_{\mathrm{kin}} = \sum_{\mu,\nu} h^{\mu\nu}\delta_{\mu\nu} = \sum_{\mu=0}^{3} h^{\mu\mu}, \tag{76}$$

$$\mathfrak{t}(R_{\mathrm{kin}}) = \varepsilon_0\left(\partial_{h^{00}} \sum_{\mu=0}^{3} h^{\mu\mu}\right) = 1. \tag{77}$$

Together with the curvature and cotangent evaluations,

$$\boxed{\mathfrak{t}(R_{\mathrm{curv}}) = 0, \qquad \mathfrak{t}(R_{\mathrm{kin}}) = 1, \qquad \mathfrak{t}(R_{\mathrm{cot}}) = 0,} \tag{78}$$

one obtains

$$\boxed{\mathfrak{t}(\mathcal{R}_0^{(0)}) = \frac{1}{2\kappa}(0) - \frac{1}{2}(1) + 0 = -\frac{1}{2}.} \tag{79}$$

Proof sketch. Equation (75) is the finite Gaussian second-moment identity used by the BV fiber. The cutoff-zero projection removes both scalar first jets in Eq. (74), so the contraction displayed in Eq. (77) is the full kinetic shell residual. The selected curvature coefficient is zero, and the cotangent residual has zero pure-even projection under $\mathfrak{t}$. Substitution into Eq. (73) gives the stated value and certifies $\mathcal{R}_0^{(0)} \neq 0$. [12]

[12] *Formal counterpart:* `lean/QGC12/BV/QME/FirstShellEvaluation.lean`.

8.2 One contraction raises loop order

The obstruction identifies a grading error in the same-order ansatz. Raw restriction is the order-preserving map. A positive Gaussian contraction contributes one additional loop order. Write

$$\Pi^{\mathrm{G}} = \sum_{k\geq 0} \Pi^{\mathrm{G},(k)}, \tag{80}$$

where $\Pi^{\mathrm{G},(k)}$ keeps exactly k Wick contractions. Coefficient transport is the finite Cauchy convolution

$$\boxed{(\Pi^{\mathrm{G}} S)_n = \sum_{k=0}^{n} \Pi^{\mathrm{G},(k)} S_{n-k}.} \tag{81}$$

At the first shell the classical master transports exactly at grade zero. The earlier $-1/2$ term appears in the grade-one counterterm equation.

The Gaussian grading is literal contraction number. Quadratic moments occur at grade one. The quartic control satisfies

$$\boxed{\mathbb{E}^{(1)}[x^4] = 0, \qquad \mathbb{E}^{(2)}[x^4] = 3.} \tag{82}$$

Each finite Gaussian monomial has at most one nonzero contraction grade, and the graded pieces recombine to the full finite expectation. [13]

8.3 Counterterm recurrence across shells

Write $S_{N,n}$ for the perturbative coefficient of order n at cutoff N, and let $\Pi^{(k)}_{N+1\to N}$ denote the k-contraction part of the successor Gaussian transport. The coefficient identity in Eq. (81) is equivalent to the exact successor recurrence

$$\boxed{S_{N,n} - P^{\mathrm{raw}}_{N+1\to N} S_{N+1,n} = \sum_{k=1}^{n} \Pi^{(k)}_{N+1\to N} S_{N+1,n-k}.} \tag{83}$$

The right-hand side contains only lower input orders. At first order,

$$\boxed{S_{N,1} - P^{\mathrm{raw}}_{N+1\to N} S_{N+1,1} = \Pi^{(1)}_{N+1\to N} S_{N+1,0}.} \tag{84}$$

For the first physical shell $N = 0$, the real component equals the master transport residual:

$$(S_{0,1} - P^{\mathrm{raw}}_{1\to 0} S_{1,1})_{\mathbb{R}} = \mathcal{R}^{(0)}_0, \qquad \mathfrak{t}(S_{0,1} - P^{\mathrm{raw}}_{1\to 0} S_{1,1}) = -\frac{1}{2}. \tag{85}$$

The local QME equation and the inter-cutoff transport equation are simultaneous conditions:

$$\boxed{\mathfrak{Q}^{\mathrm{flat}}_{N,[n]}(S_N) = 0, \qquad S_{N,n} - P^{\mathrm{raw}}_{N+1\to N} S_{N+1,n} = \sum_{k=1}^{n} \Pi^{(k)}_{N+1\to N} S_{N+1,n-k}.} \tag{86}$$

A regulator-independent perturbative family is therefore a solution of both finite equations at every declared cutoff and order. [14]

[13] *Formal counterpart:* `lean/QGC12/BV/QME/FirstLoopWickTransport.lean;lean/QGC12/BV/QME/ContractionGradedWickTransport.lean`.

[14] *Formal counterpart:* `lean/QGC12/BV/QME/ContractionGradedCountertermRecurrence.lean;lean/QGC12/BV/QME/CoefficientwiseStabilization.lean`.

9 Locality and regulator adjudication

The regulator-refinement problem also tests which finite carriers preserve the locality structure required by the functional BV theory. Several candidate architectures are decided explicitly.

9.1 Rectangular locality obstruction

The locality transport requirement can be stated as existence of a retained successor map

$$\sigma_\mu : \mathfrak{R}_N \longrightarrow \mathfrak{R}_N, \qquad \iota_N(\sigma_\mu p) = \mathrm{shift}_\mu(\iota_N p) \quad (\mu = 0, \dots, 3). \tag{87}$$

The top-face computation in Eqs. (26)–(27) gives

$$\boxed{\{\sigma_\mu \text{ satisfying Eq. (87)}\} = \varnothing \quad \text{for every } N.} \tag{88}$$

Thus the continuum local-functional counterterm laws require a different finite derivative geometry.

9.2 Cyclic derivative closure and primitive collapse

For the cyclic bank, every retained pair p has a predecessor $\mathrm{pred}_\mu p$ satisfying

$$\mathrm{succ}_\mu(\mathrm{pred}_\mu p) = p. \tag{89}$$

If e_p denotes either the even or odd coordinate generator, then

$$D_\mu e_{\mathrm{pred}_\mu p} = e_p. \tag{90}$$

Hence every primitive generator vanishes in the cyclic local-functional quotient:

$$\boxed{[e_p]_{\mathrm{cyc}} = 0.} \tag{91}$$

For a continuum zero-jet field generator $e_{(q,0)}$, the Euler probe gives

$$\frac{\delta}{\delta q}[e_{(q,0)}]_\infty = 1, \qquad [e_{(q,0)}]_\infty \neq 0, \tag{92}$$

Its cyclic image is zero by Eq. (91). Therefore

$$\boxed{\ker(\Phi_N^{\mathrm{cyc}}) \neq \{0\},} \tag{93}$$

so cyclic derivative closure comes with primitive-class collapse.

9.3 Nilpotent derivative bank and separated bracket

At cutoff one, write x_0 for a zero-jet coordinate and $x_1 = D_0 x_0$ for its first jet. The nilpotent derivative satisfies

$$D_0 x_0 = x_1, \qquad D_0 x_1 = 0. \tag{94}$$

Take the explicit boundary witness

$$W := x_1 x_0^2. \tag{95}$$

Then

$$D_0 W = 2x_1^2 x_0. \tag{96}$$

For the standard truncated first-order Euler candidate

$$E^{(1)} := \partial_{x_0} - D_0 \partial_{x_1}, \tag{97}$$

one obtains

$$E^{(1)}(D_0 W) = 2x_1^2 - D_0(4x_1 x_0) \tag{98}$$

$$= 2x_1^2 - 4x_1^2 \tag{99}$$

$$= \boxed{-2x_1^2 \neq 0}. \tag{100}$$

Equation (100) fixes the exact nilpotent quotient to the proved descent kernel; the truncated Euler candidate carries the displayed boundary remainder.

The continuum cone

$$\rho : \mathcal{F}_\infty \longrightarrow \prod_N \mathcal{F}_N^{\mathrm{nil}} \tag{101}$$

has joint kernel

$$K_{\mathrm{joint}} := \ker \rho. \tag{102}$$

The divergence-reflection theorem identifies the exact faithfulness condition

$$\boxed{\text{regulated divergence reflection} \iff K_{\mathrm{joint}} = \{0\}.} \tag{103}$$

At the proved Palatini scope $K_{\mathrm{joint}} = \{0\}$, so the induced quotient bracket is representative-free:

$$[F], [G] \longmapsto [(F, G)]. \tag{104}$$

[15]

9.4 Regulator decision map

The three finite locality branches and the Gaussian refinement operation have distinct mathematical roles. Table 1 records the exact role of each construction before the global descent theorem is applied.

These locality decisions are retained as part of the Paper-7 result. They classify which regulator modifications preserve derivative closure, primitive coordinate faithfulness, and the BV bracket simultaneously.

[15] *Formal counterpart:* `lean/QGC12/BV/QME/RectangularLocalityNoGo.lean;lean/QGC12/BV/QME/CyclicJetRegulator.lean;lean/QGC12/BV/QME/CyclicLocalityObstruction.lean;lean/QGC12/BV/QME/NilpotentJetRegulator.lean;lean/QGC12/BV/QME/NilpotentEulerBoundary.lean;lean/QGC12/BV/QME/NilpotentFunctionalTower.lean;lean/QGC12/BV/QME/NilpotentJointKernel.lean;lean/QGC12/BV/QME/NilpotentDivergenceReflection.lean;lean/QGC12/BV/QME/NilpotentDivergenceReflectionClosure.lean`.

Construction	Earned structure	Exact obstruction or restriction	Role retained later
Rectangular finite bank	Conjugate BV pairs, finite odd Laplacian, finite master/QME carrier	Total derivatives leave the bank; local-functional descent has a nonzero boundary defect	Provides the initial finite BV/QME and finite shell banks
Cyclic jet bank	Internal derivative closure and migrated physical master/grading	Primitive local-functional coordinate classes collapse	Records the exact primitive-collapse boundary
Nilpotent jet bank	Commuting top-zero derivatives and zero-jet primitive survival	Naive global Euler descent has a nonzero top-boundary remainder	Provides the separated descent kernel and representative-free bracket at the stated scope
Gaussian/Berezin fiber pushforward	Normalized finite fiber functional, retained retraction, density-odd-Laplacian intertwining	Full physical Hamiltonian descent has a nonzero explicit first-shell residual	Provides finite shell transport and the restricted differential comparison

Table 1: Decision map for the finite regulator and refinement constructions used in Paper 7.

10 The global Hamiltonian descent obstruction

The Wick grading supplies the correct perturbative placement of Gaussian shell moments. The next compatibility condition asks for full density-aware quantum-differential intertwining with Gaussian pushforward on every observable.

For a real master family S_N define the Hamiltonian residual

$$\mathcal{R}_{\mathrm{Ham},N}(F) := \Pi^{\mathrm{G}}_{N+1\to N}(S_{N+1}, F)_{N+1} - (S_N, \Pi^{\mathrm{G}}_{N+1\to N}F)_N. \tag{105}$$

Because the weighted Gaussian pushforward already transports the density odd Laplacian, the full quantum-differential comparison reduces exactly to this Hamiltonian residual.

10.1 Explicit first-shell Hamiltonian witness

Let

$$x_0 := \partial_0\phi, \qquad x_0^* := \text{the odd BV coordinate conjugate to } x_0, \qquad \boxed{w := x_0 x_0^*.} \tag{106}$$

This is the real scalar extension of the canonical quadratic coordinate pair used by the finite BV bracket. The eliminated odd coordinate has zero Berezin expectation, giving

$$\boxed{\Pi^{\mathrm{G}}_{1\to 0} w = 0.} \tag{107}$$

Let $\mathfrak{p}$ denote the real scalar extension of the probe in Eq. (72).

Proposition (first-shell Hamiltonian obstruction). For the cutoff-one projected physical master,

$$S_1 = \frac{1}{2\kappa} S_{\mathrm{curv}} - \frac{1}{2} S_{\mathrm{kin}} + S_{\mathrm{cot}}, \tag{108}$$

and the Gaussian-bracket evaluations are

$$\boxed{\mathfrak{p}\left(\Pi^{\mathrm{G}}_{1\to 0}(S_{\mathrm{curv}}, w)_1\right) = 0, \quad \Pi^{\mathrm{G}}_{1\to 0}(S_{\mathrm{kin}}, w)_1 = 2h^{00}, \quad \mathfrak{p}\left(\Pi^{\mathrm{G}}_{1\to 0}(S_{\mathrm{cot}}, w)_1\right) = 0.} \tag{109}$$

Since $\mathfrak{p}(h^{00}) = 1$ and $\Pi^{\mathrm{G}}_{1\to 0}w = 0$,

$$\mathfrak{p}(\mathcal{R}_{\mathrm{Ham},0}(w)) = \frac{1}{2\kappa}(0) - \frac{1}{2}(2) + 0 \tag{110}$$

$$= \boxed{-1}. \tag{111}$$

Thus

$$\boxed{\mathcal{R}_{\mathrm{Ham},0}(w) \neq 0.} \tag{112}$$

Proof sketch. Using Eq. (74) and $w = x_0 x_0^*$, the canonical-pair bracket differentiates the selected x_0 factor and restores the even coordinate x_0. The two symmetric kinetic slots give

$$(S^{(1)}_{\mathrm{kin}}, w)_1 = \sum_{\nu=0}^{3} h^{0\nu} x_\nu x_0 + \sum_{\mu=0}^{3} h^{\mu 0} x_\mu x_0, \tag{113}$$

$$\Pi^{\mathrm{G}}_{1\to 0}(S^{(1)}_{\mathrm{kin}}, w)_1 = \sum_{\nu} h^{0\nu}\delta_{\nu 0} + \sum_{\mu} h^{\mu 0}\delta_{\mu 0} = h^{00} + h^{00} = 2h^{00}. \tag{114}$$

The curvature and cotangent sectors have zero selected h^{00} trace, and $\mathfrak{p}(h^{00}) = 1$. Multiplication by the kinetic master coefficient $-1/2$ therefore gives -1. The coupling hypothesis $\kappa \neq 0$ enters through the gravitational coefficient $1/(2\kappa)$. [16]

The Hamiltonian residual occurs at classical order. Hence for every real $\hbar$,

$$\boxed{\Pi^{\mathrm{G}}_{1\to 0} s^{\rho}_{\hbar,1} \neq s^{\rho}_{\hbar,0} \Pi^{\mathrm{G}}_{1\to 0} \quad \text{on the full observable carrier.}} \tag{115}$$

Every contraction-graded effective-master candidate used here has the physical Palatini master as its $\hbar = 0$ coefficient. The same witness therefore supplies the classical residual for every member of that candidate class.

10.2 Empty global completion branch

Let $\mathfrak{C}_{\mathrm{glob}}$ denote the declared global Gaussian completion class: a contraction-graded candidate together with global classical Hamiltonian descent. Then

$$\boxed{\mathfrak{C}_{\mathrm{glob}} = \varnothing.} \tag{116}$$

The Paper-7 obstruction certificate therefore carries the witness w, its zero pushforward, the residual probe -1, the all-$\hbar$ global descent obstruction, and the classical-coefficient obstruction for every declared contraction-graded candidate. [17]

The finite Paper-6 common-sector intertwiner remains available. Equation (115) fixes the all-observable boundary, and finite observables with vanishing comparison residual populate the compatible sector.

[16] *Formal counterpart:* `lean/QGC12/BV/QME/FirstShellHamiltonianObstruction.lean`.
[17] *Formal counterpart:* `lean/QGC12/BV/QME/Paper7ObstructionCapstone.lean`.

11 Canonical greatest compatible quantum subcomplex

The global obstruction determines a canonical compatible restriction. Let

$$\mathfrak{R}_{N,\hbar} := \Pi^{\mathrm{G}}_{N+1\to N} s^{\rho}_{\hbar,N+1} - s^{\rho}_{\hbar,N} \Pi^{\mathrm{G}}_{N+1\to N} \tag{117}$$

be the adjacent-cutoff differential-comparison residual. Define

$$\boxed{\mathcal{C}_{N,\hbar} := \ker \mathfrak{R}_{N,\hbar}.} \tag{118}$$

Its universal property is

$$\boxed{U \le \mathcal{A}_{N+1}, \quad \mathfrak{R}_{N,\hbar}|_U = 0 \Longrightarrow U \le \mathcal{C}_{N,\hbar}.} \tag{119}$$

Thus $\mathcal{C}_{N,\hbar}$ is the unique greatest linear submodule on which finite Gaussian pushforward intertwines the source and target density-aware quantum differentials.

The unit belongs to $\mathcal{C}_{N,\hbar}$ for every cutoff and every $\hbar$, so

$$\mathcal{C}_{N,\hbar} \neq \{0\}. \tag{120}$$

The first-shell witness w has nonzero comparison residual and therefore lies in the complementary carrier to $\mathcal{C}_{0,\hbar}$, hence

$$\boxed{\{0\} \subsetneq \mathcal{C}_{0,\hbar} \subsetneq \mathcal{A}_1.} \tag{121}$$

[18]

11.1 Square-zero invariant refinement

Cohomology requires intertwining, invariance, and square-zero closure. The chain-compatible sector is defined explicitly by

$$\boxed{\mathcal{C}^{Q}_{N,\hbar} := \Big(\mathcal{C}_{N,\hbar} \cap (s^{\rho}_{\hbar,N+1})^{-1}(\mathcal{C}_{N,\hbar})\Big) \cap \ker\big((s^{\rho}_{\hbar,N+1})^2\big).} \tag{122}$$

Its corresponding universal property is

$$\boxed{U \le \mathcal{C}_{N,\hbar}, \quad s^{\rho}_{\hbar,N+1}(U) \subseteq U, \quad (s^{\rho}_{\hbar,N+1})^2|_U = 0 \Longrightarrow U \le \mathcal{C}^{Q}_{N,\hbar}.} \tag{123}$$

Thus $\mathcal{C}^{Q}_{N,\hbar}$ is the canonical greatest source submodule carrying comparison intertwining, source-differential invariance, and square-zero restriction. The restricted differential

$$d^{Q}_{N,\hbar} := s^{\rho}_{\hbar,N+1}|_{\mathcal{C}^{Q}_{N,\hbar}} \tag{124}$$

satisfies

$$\boxed{(d^{Q}_{N,\hbar})^2 = 0.} \tag{125}$$

Moreover Gaussian pushforward is a chain map on this sector, and the target differential squares to zero on the image.

[18] *Formal counterpart:* `lean/QGC12/BV/QME/CompatibleQuantumSubcomplex.lean`.

The associated restricted cohomology is therefore a genuine quotient

$$\boxed{H_{Q,\mathrm{res}}(N,\hbar) := \ker d^{Q}_{N,\hbar} / \operatorname{im} d^{Q}_{N,\hbar}.} \tag{126}$$

The current formal object is an ungraded square-zero complex over the real carrier. The manuscript therefore uses H at this stage. A graded H^k refinement requires a separate theorem that the restricted chain submodule and restricted differential preserve the integer ghost-number decomposition. The unit is a cycle. Its possible boundary status is expressed by an exact preimage criterion. [19]

12 Restricted physical sector and Paper-6 comparison

The Gaussian density modifies the odd Laplacian at positive cutoff, so the Paper-6 comparison sector is defined with the density-aware operator. Let

$$\mathcal{P}_{N,\hbar} := \ker \mathcal{D}_{\mathrm{cl},N} \cap \ker(\hbar\Delta_{\rho_N}) \tag{127}$$

be the common sector in the real component. On $\mathcal{P}_{N,\hbar}$,

$$\boxed{s^{\rho}_{\hbar,N} = Q^{(6)}_{N}.} \tag{128}$$

Let

$$\mathcal{P}^{Q}_{N,\hbar} := \mathcal{C}^{Q}_{N,\hbar} \cap \operatorname{im}_{\mathbb{R}}(\mathcal{P}_{N+1,\hbar} \hookrightarrow \mathcal{A}^{\mathrm{dbl}}_{N+1}) \tag{129}$$

be the restricted quantum sector satisfying the density-aware Paper-6 comparison conditions. Its differential-invariant core is

$$\boxed{\mathcal{C}^{\mathrm{phys}}_{N,\hbar} := \mathcal{P}^{Q}_{N,\hbar} \cap \left(s^{\rho}_{\hbar,N+1}\right)^{-1}(\mathcal{P}^{Q}_{N,\hbar}).} \tag{130}$$

The inherited square-zero law restricts to this invariant core, and its cohomology is

$$H_{\mathrm{phys,res}}(N,\hbar) := \ker d^{\mathrm{phys}}_{N,\hbar} / \operatorname{im} d^{\mathrm{phys}}_{N,\hbar}. \tag{131}$$

Let ϵ_N be the transported Paper-6 vacuum augmentation on the invariant physical complex. It satisfies

$$\boxed{\epsilon_N \circ d^{\mathrm{phys}}_{N,\hbar} = 0, \qquad \epsilon_N(1) = 1.} \tag{132}$$

If $1 = d^{\mathrm{phys}}_{N,\hbar} F$, applying ϵ_N gives $1 = 0$. Hence

$$\boxed{[1]_{\mathrm{phys}} \neq 0} \tag{133}$$

for every cutoff and real $\hbar$. [20]

[19] *Formal counterpart:* `lean/QGC12/BV/QME/RestrictedQuantumCohomology.lean`.

[20] *Formal counterpart:* `lean/QGC12/BV/QME/RestrictedPhysicalSector.lean;lean/QGC12/BV/QME/RestrictedPhysicalCohomology.lean`.

12.1 Multiplicative scalar core

The minimal multiplication-stable subcomplex is the real line generated by the doubled unit,

$$\boxed{\mathcal{S}_N := \mathbb{R} \cdot 1 = \mathrm{span}_{\mathbb{R}}\{(1,0)\}.} \tag{134}$$

For $a, b \in \mathbb{R}$,

$$(a1) \cdot (b1) = (ab)1 \in \mathcal{S}_N, \qquad d^{\mathrm{phys}}_{N,\hbar}(a1) = 0, \tag{135}$$

so

$$\boxed{\mathcal{S}_N \cdot \mathcal{S}_N \subseteq \mathcal{S}_N, \qquad H(\mathcal{S}_N, 0) \cong \mathcal{S}_N \cong \mathbb{R}.} \tag{136}$$

The existence of a non-scalar restricted physical density is therefore equivalent to the strict inclusion

$$\boxed{\exists F \in \mathcal{C}^{\mathrm{phys}}_{N,\hbar} \setminus \mathcal{S}_N \Longleftrightarrow \mathcal{S}_N < \mathcal{C}^{\mathrm{phys}}_{N,\hbar}.} \tag{137}$$

[21]

13 Native matter-state cohomology survives

The restricted physical density complex is now tensored with the already constructed native scalar Fock space,

$$\mathcal{K}_{N,\hbar} := \mathcal{C}^{\mathrm{phys}}_{N,\hbar} \otimes_{\mathbb{R}} \mathcal{F}_{\mathrm{NL}}. \tag{138}$$

The restricted quantum differential acts on the BV-density factor:

$$D_{N,\hbar} = d^{\mathrm{phys}}_{N,\hbar} \otimes I_{\mathcal{F}_{\mathrm{NL}}}. \tag{139}$$

This is a spectator representation of the established matter carrier. The BV bracket remains carried entirely by the density factor.

Tensor the physical augmentation with the identity on the Fock factor:

$$r_{N,\hbar} := \epsilon_N \otimes I_{\mathcal{F}_{\mathrm{NL}}} : \mathcal{K}_{N,\hbar} \longrightarrow \mathcal{F}_{\mathrm{NL}}. \tag{140}$$

The two decisive identities are

$$\boxed{r_{N,\hbar} \circ D_{N,\hbar} = 0, \qquad r_{N,\hbar}(1 \otimes \psi) = \psi.} \tag{141}$$

Thus a boundary representation $1 \otimes \psi = D_{N,\hbar}\Xi$ would imply $\psi = 0$. Consequently

$$\boxed{\psi \neq 0 \Longrightarrow [1 \otimes \psi] \neq 0 \quad \text{in } H(\mathcal{K}_{N,\hbar}, D_{N,\hbar}).} \tag{142}$$

An explicit one-particle occupation in mode zero defines a nonzero non-vacuum class separated from the scalar-vacuum state line. [22]

Thus the global descent obstruction selects a nontrivial restricted state cohomology with a faithful native matter sector. The physical unit is inherited from ghost number zero, and the spectator Fock factor retains zero BV ghost action. The restricted state-cohomology quotient is stated in its currently constructed ungraded form. An H^0 identification therefore requires a dedicated graded quotient theorem.

[21] *Formal counterpart:* `lean/QGC12/BV/QME/RestrictedMultiplicativeCore.lean`.
[22] *Formal counterpart:* `lean/QGC12/BV/QME/RestrictedPhysicalStateCohomology.lean`.

14 Native scalar density boundary

The state theorem in Eq. (142) leaves the density/observable factor as its own classification problem. The remainder of the paper classifies a natural native scalar-generated density algebra under the two simultaneous physical conditions:

$$\boxed{s_{\text{fin}}^2 F = 0, \qquad \mathcal{D}_{\text{cl}}(F) = 0.} \tag{143}$$

The first is the source-square gate. The second is Paper-6 comparison compatibility.

14.1 Retained scalar zero jet

The retained scalar zero jet ϕ is a genuine nonconstant density in the finite Paper-6 comparison sector at $\hbar = 0$. Its first finite-master image is the ghost–scalar descendant

$$D := s_{\text{fin}}\phi. \tag{144}$$

The second image is the explicit two-ghost transport polynomial

$$\boxed{O := s_{\text{fin}} D = \sum_{\gamma,\tau=0}^{3} c^{\tau} (\partial_\tau c^{\gamma})\, \phi_\gamma, \qquad \phi_\gamma := \partial_\gamma \phi.} \tag{145}$$

Let $\pi_{0,1}$ extract the coefficient of the monomial $c^1(\partial_1 c^0)\phi_0$. Then

$$\boxed{\pi_{0,1}(O) = 1, \qquad O \neq 0.} \tag{146}$$

Thus $s_{\text{fin}}^2 \phi = O$ is a literal nonzero source-square obstruction.

14.2 Scalar first jets

Let

$$F_a := \sum_{\mu=0}^{3} a_\mu \phi_\mu, \qquad \phi_\mu := \partial_\mu \phi, \qquad a = (a_0, \dots, a_3) \in \mathbb{R}^4. \tag{147}$$

At cutoff one the finite master annihilates every retained scalar first jet,

$$s_{\text{fin}} \phi_\mu = 0, \qquad s_{\text{fin}}^2 F_a = 0. \tag{148}$$

For each direction ν, let ℓ_ν be the dual mixed Taylor probe on the one-ghost Paper-6 response. The response matrix is the identity:

$$\boxed{\ell_\nu\Big(Q^{(6)}\phi_\mu\Big) = \delta_{\nu\mu}, \qquad \ell_\nu\Big(Q^{(6)} F_a\Big) = a_\nu.} \tag{149}$$

Therefore

$$\boxed{\mathcal{D}_{\text{cl}}(F_a) = 0 \Longleftrightarrow a = 0.} \tag{150}$$

Combining F_a with the zero jet ϕ and the descendant $D = s_{\text{fin}}\phi$ gives the finite-dimensional simultaneous kernel used below. [23]

[23] *Formal counterpart:* `lean/QGC12/BV/QME/RestrictedNonconstantChainDecision.lean;lean/QGC12/BV/QME/DerivativeClosedNonconstantChain.lean;lean/QGC12/BV/QME/RestrictedPhysicalNonconstantDecision.lean;lean/QGC12/BV/QME/GhostScalarDescendantClosure.lean;lean/QGC12/BV/QME/JointScalarJetDescendantDecision.lean`.

14.3 All retained-scalar polynomials

The finite master satisfies the graded Leibniz law on the even scalar coordinate. For every $n \geq 0$,

$$\boxed{s_{\text{fin}}(\phi^{n+1}) = (n+1)\phi^n D,} \tag{151}$$

and

$$\boxed{s_{\text{fin}}^2(\phi^{n+1}) = (n+1)\phi^n O.} \tag{152}$$

A faithful family of degree-indexed probes proves

$$\phi^n O \neq 0 \qquad (n \geq 0). \tag{153}$$

Degree-indexed probes separate distinct powers in every finite real sum. Therefore for every finite polynomial P,

$$\boxed{s_{\text{fin}}^2 P(\phi) = 0 \iff P \text{ is constant.}} \tag{154}$$

The same classification holds after intersecting with the Paper-6 common sector. [24]

14.4 Complete first-jet polynomial algebra

Let X_μ denote the four first-jet coordinates. On the finite polynomial algebra $\mathbb{Q}[X_0, X_1, X_2, X_3]$, the diagonal ghost coefficient of the Paper-6 response is exactly the Euler operator

$$\boxed{\mathrm{Coeff}_\mu(Q^{(6)}P) = X_\mu \, \partial_{X_\mu} P.} \tag{155}$$

Multi-index Taylor probes separate arbitrary finite monomials and rule out cross-degree and cross-direction cancellation. The finite master annihilates this top-shell first-jet algebra, so its source-square condition is automatic. Paper-6 comparison then gives

$$\boxed{P(X_0, X_1, X_2, X_3) \in \mathcal{P}_{1,0} \iff P \text{ is constant.}} \tag{156}$$

[25]

14.5 The scalar zero/first-jet/descendant superalgebra

Introduce the finite polynomial algebra

$$A := \mathbb{Q}[\phi, X_0, X_1, X_2, X_3] \tag{157}$$

and extend it by the actual odd descendant D with

$$\boxed{D^2 = 0.} \tag{158}$$

[24] *Formal counterpart:* `lean/QGC12/BV/QME/RetainedScalarPolynomialBoundary.lean;lean/QGC12/BV/QME/RetainedScalarPolynomialCancellation.lean`.

[25] *Formal counterpart:* `lean/QGC12/BV/QME/ScalarJetPolynomialBoundary.lean;lean/QGC12/BV/QME/MixedScalarJetCompositeBoundary.lean;lean/QGC12/BV/QME/ScalarJetMultivariatePolynomialBoundary.lean`.

The resulting native scalar-generated superalgebra is

$$\boxed{\mathcal{A}_{\mathrm{sc}} = A \oplus AD.} \tag{159}$$

The finite-master action is exact. For $a, b \in A$,

$$\boxed{s_{\mathrm{fin}}(a) = (\partial_\phi a)D,} \tag{160}$$

and

$$\boxed{s_{\mathrm{fin}}(bD) = bO.} \tag{161}$$

Thus

$$s_{\mathrm{fin}}^2 a = (\partial_\phi a)O. \tag{162}$$

The compressed Paper-6 differential closes D and differentiates its polynomial coefficient. [26]

14.6 Exact odd-kernel separation

The odd comparison term is

$$bO - Q^{(6)}(b)D. \tag{163}$$

A selected two-ghost coefficient map reduces this expression to

$$\boxed{\mathrm{Coeff}\left(bO - Q^{(6)}(b)D\right) = X_0 X_1 \left(1 + X_1 \partial_{X_1}\right) b} \tag{164}$$

up to the fixed overall sign and the faithful coordinate substitution. The coefficientwise monomial calculation proves

$$\boxed{(1 + X_1 \partial_{X_1})b = 0 \Longleftrightarrow b = 0.} \tag{165}$$

Therefore Eq. (163) has polynomial kernel $\{0\}$. [27]

14.7 Complete simultaneous kernel

The two-ghost projector annihilates the complete even comparison sector and separates the odd coefficient equation exactly. Faithful scalar extension transports the rational calculation to the actual real carrier. Combining the comparison equation with the source-square equation yields the complete classification:

$$\boxed{\left.\begin{array}{l} s_{\mathrm{fin}}^2 a = 0, \\ \mathcal{D}_{\mathrm{cl}}(a + bD) = 0 \end{array}\right\} \Longleftrightarrow \exists c \in \mathbb{Q} : \ a = c, \ b = 0.} \tag{166}$$

This is the final native scalar-density boundary of the present paper. [28]

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[26] *Formal counterpart:* `lean/QGC12/BV/QME/ScalarJetDescendantSuperalgebra.lean`.

[27] *Formal counterpart:* `lean/QGC12/BV/QME/ScalarJetDescendantKernel.lean`.

[28] *Formal counterpart:* `lean/QGC12/BV/QME/ScalarJetDescendantSimultaneousKernel.lean;lean/QGC12/BV/QME/RestrictedPublicationCapstone.lean`.

14.8 State and density results

The state and density conclusions occupy distinct carrier-specific result classes. The spectator state construction satisfies

$$\boxed{\psi \neq 0 \Longrightarrow [1 \otimes \psi] \neq 0,} \tag{167}$$

The complete native scalar-generated density superalgebra satisfies

$$\boxed{\big(s_{\mathrm{fin}}^2 a = 0 \;\wedge\; \mathcal{D}_{\mathrm{cl}}(a + bD) = 0\big) \Longleftrightarrow \exists c \in \mathbb{Q} :\; a = c,\; b = 0.} \tag{168}$$

The first statement is survival of the pre-existing matter-state sector under spectator tensoring. The second classifies the native scalar-generated density kernel under the simultaneous finite-master and Paper-6 comparison gates. Each statement retains its declared carrier and exact scope.

15 Complete cutoff-one generator-linear adequacy boundary

The scalar-generated superalgebra is a native nonlinear test family. The final adequacy search returns to the original cutoff-one Palatini generator bank and classifies every generator-linear coefficient simultaneously across all families. Let $\mathcal{I}_1$ denote the complete retained generator index, including the even and odd coordinates associated with physical fields, diffeomorphism ghosts, projective ghosts, and their BV conjugates. A real coefficient family

$$c : \mathcal{I}_1 \longrightarrow \mathbb{R} \tag{169}$$

defines the corresponding full generator-linear ansatz.

The two physical conditions remain the same source-square and Paper-6 comparison gates used above. The full calculation first proves that parity and exact integer ghost number are preserved strongly enough to prohibit cross-block cancellation. The generator carrier decomposes into input ghost degrees

$$\boxed{\mathcal{G}_1 = \mathcal{G}_{-2} \oplus \mathcal{G}_{-1} \oplus \mathcal{G}_0 \oplus \mathcal{G}_1,} \tag{170}$$

corresponding respectively to ghost antifields, physical antifields, physical fields, and ghosts. The finite master bracket and compressed Paper-6 operator raise the declared ghost number by one. The master square raises it by two. Hence the two simultaneous equations split into four independent finite systems.

15.1 Physical field block and the origin of the diagonal weights

The degree-zero block is the complete antifield-free physical field/jet carrier. The master-square gate acts on the zero shell, and the Paper-6 comparison gate acts on the nonzero shells. With the normalized scalar, inverse-density, and connection Taylor probes, the zero-shell source-square response is

$$\boxed{R_0^{\mathrm{phys}} = I_\phi \oplus I_h \oplus I_\Gamma = I_1 \oplus I_{10} \oplus I_{64}.} \tag{171}$$

Equivalently, every diagonal probe reads coefficient 1 and every cross-family entry is zero, so the complete physical zero-jet kernel is trivial.

For a physical field q and retained jet $D^I q$, let $\Theta_{q,I}$ denote the mixed Taylor coefficient that extracts the selected $D^I q$ coordinate together with the diagonal first-ghost trace $\sum_\rho \partial_\rho c^\rho$. The relevant gauge assignments are

$$s\phi = c^\rho \partial_\rho \phi, \tag{172}$$
$$sh^{ab} = c^\rho \partial_\rho h^{ab} + (\partial_\rho c^\rho) h^{ab} - (\partial_\rho c^a) h^{\rho b} - (\partial_\rho c^b) h^{a\rho}, \tag{173}$$
$$s\Gamma^\lambda{}_{\mu\nu} = c^\rho \partial_\rho \Gamma^\lambda{}_{\mu\nu} - (\partial_\rho c^\lambda)\Gamma^\rho{}_{\mu\nu} + (\partial_\mu c^\rho)\Gamma^\lambda{}_{\rho\nu} + (\partial_\nu c^\rho)\Gamma^\lambda{}_{\mu\rho} + \partial_\mu \partial_\nu c^\lambda + \delta^\lambda_\nu \eta_\mu. \tag{174}$$

For the scalar the representation weight is zero. For the inverse-density field the diagonal representation trace is

$$4 - 1 - 1 = 2, \tag{175}$$

coming from the density weight and the two contravariant slots. For the connection it is

$$-1 + 1 + 1 = 1, \tag{176}$$

from the upper and two lower tensor slots. The affine second-ghost derivative and projective-ghost term have zero selected even-field trace.

One total derivative contributes one additional diagonal transport term. Hence the first-shell weights are

$$\boxed{1 \quad \text{for } \partial\phi, \qquad 1 + 2 = 3 \quad \text{for } \partial h, \qquad 1 + 1 = 2 \quad \text{for } \partial\Gamma,} \tag{177}$$

which gives

$$\boxed{I_\phi \oplus 3I_h \oplus 2I_\Gamma.} \tag{178}$$

For rectangular jet order r, prolongation contributes r transport copies and preserves the representation weights in Eqs. (175)–(176). Therefore

$$\boxed{rI_\phi \oplus (r+2)I_h \oplus (r+1)I_\Gamma, \qquad r = 2, 3, 4.} \tag{179}$$

Field-support separation eliminates the cross-family blocks, and the carry-free directional grading separates distinct retained multiindices. Every diagonal weight is positive, giving

$$\boxed{\ker(\text{simultaneous gates on } \mathcal{G}_0) = \{0\}.} \tag{180}$$

[29]

15.2 Ghost block: complete trivial-kernel theorem

Write the degree-one coefficient family as

$$c_1 = (g_0, \eta_0;\ g_1, \eta_1;\ g_{\geq 2}, \eta_{\geq 2}), \tag{181}$$

[29] *Formal counterpart:* `lean/QGC12/BV/QME/PhysicalNonzeroJetTraceResponse.lean;lean/QGC12/BV/QME/PhysicalNonzeroJetTensorTraceResponse.lean;lean/QGC12/BV/QME/PhysicalFirstShellResponseMatrix.lean;lean/QGC12/BV/QME/PhysicalHigherJetTensorResponseMatrix.lean;lean/QGC12/BV/QME/PhysicalHigherJetOffDiagonalResponse.lean`.

where $g_0, \eta_0 : \mathrm{Fin}(4) \to \mathbb{R}$ are the eight zero-jet coefficients, g_1, η_1 are the 32 first-shell coefficients, and $g_{\geq 2}, \eta_{\geq 2}$ are the 88 higher-shell coefficients. The exact shell classification is

$$s_{\mathrm{fin}}^2(c_1) = 0 \iff g_0 = 0 \ \wedge \ \eta_0 = 0, \tag{182}$$

$$\mathcal{D}_{\mathrm{cl}}(c_1) = 0 \iff g_1 = \eta_1 = g_{\geq 2} = \eta_{\geq 2} = 0. \tag{183}$$

The first/higher directional grading separates the two nonzero shells, and the ghost/projective-ghost probes separate their two field families. Hence

$$\boxed{\mathrm{SimGates}(c_1) \iff g_0 = \eta_0 = g_1 = \eta_1 = g_{\geq 2} = \eta_{\geq 2} = 0.} \tag{184}$$

Equivalently,

$$\boxed{\ker(\text{simultaneous gates on } \mathcal{G}_1) = \{0\}.} \tag{185}$$

[30]

15.3 Antifield blocks: exact surviving algebraic kernel

The degree--1 physical-antifield response has the low-shell matrices

$$\boxed{R_{-1}^{(0)} = 0_{\phi^*} \oplus 2I_{h^*} \oplus I_{\Gamma^*}, \qquad R_{-1}^{(1)} = I.} \tag{186}$$

All retained physical-antifield jets of order at least two lie in the algebraic kernel. Thus

$$\boxed{K_{-1} = \{c_{-1} : c_{h^*,0} = 0,\ c_{\Gamma^*,0} = 0,\ c_{\mathrm{first}} = 0\},} \tag{187}$$

with the scalar zero-antifield coefficient $c_{\phi^*,0}$ and every order-≥ 2 physical-antifield coefficient free.

For degree -2, the exact low-shell matrices are

$$\boxed{R_{-2}^{(0)} = I_{c^*} \oplus I_{\eta^*}, \qquad R_{-2}^{(1)} = -I_{c^*} \oplus (-I_{\eta^*}), \qquad R_{c^*,\mathrm{mixed}\,2} = I.} \tag{188}$$

The higher shell splits into the six mixed-second jets and the five triple/top jets. The exact kernel is

$$\boxed{K_{-2} = \{c_{-2} : c_{c^*,0} = c_{\eta^*,0} = 0,\ c_{c^*,1} = c_{\eta^*,1} = 0,\ c_{c^*,\mathrm{mixed}\,2} = 0\}.} \tag{189}$$

Accordingly, every projective-ghost antifield jet of order at least two is free, and every diffeomorphism-ghost antifield triple/top jet is free.

The complete generator-linear theorem is therefore

$$\boxed{\mathrm{SimGates}(c) \iff (c_{-2} \in K_{-2}) \wedge (c_{-1} \in K_{-1}) \wedge c_0 = 0 \wedge c_1 = 0.} \tag{190}$$

Every nonzero generator-linear survivor is supported entirely in the antifield sectors.

Imposing antifield-freedom sets $c_{-2} = c_{-1} = 0$, and therefore

$$\boxed{\mathrm{SimGates}(c) \wedge \mathrm{AntifieldFree}(c) \iff c = 0.} \tag{191}$$

This exhausts the complete cutoff-one antifield-free generator-linear search and fixes the linear boundary used by the strict nonlinear enlargement in the next section. [31]

[30] *Formal counterpart:* `lean/QGC12/BV/QME/GhostZeroGeneratorResponseMatrix.lean;lean/QGC12/BV/QME/GhostFirstShellFamilyDecision.lean;lean/QGC12/BV/QME/GhostHigherShellFamilyDecision.lean;lean/QGC12/BV/QME/GhostOddCompleteShellDecision.lean`.

[31] *Formal counterpart:* `lean/QGC12/BV/QME/PhysicalAntifieldFirstShellResponseMatrix.lean;lean/QGC12/BV`

16 Complete cutoff-one physical polynomial boundary

The generator-linear theorem fixes the complete linear boundary on the original cutoff-one Palatini generator bank. The next canonical enlargement retains the antifield-free physical coordinates and closes them under all finite commutative polynomials. The physical component bank is

$$\boxed{\mathcal{I}_{\mathrm{phys}} = \{\phi\} \sqcup \mathrm{Sym}^2(\mathrm{Fin}(4)) \sqcup \mathcal{I}_\Gamma, \qquad |\mathcal{I}_{\mathrm{phys}}| = 1 + 10 + 64 = 75.} \tag{192}$$

The cutoff-one rectangular jet shell has sixteen multiindices,

$$\mathcal{J}_1 = \{0,1\}^4, \qquad |\mathcal{J}_1| = 16, \qquad |\mathcal{J}_1 \setminus \{0\}| = 15. \tag{193}$$

Hence the complete physical polynomial index has

$$\boxed{\mathcal{P}_1 := \mathcal{I}_{\mathrm{phys}} \times \mathcal{J}_1, \qquad |\mathcal{P}_1| = 1200, \qquad |\mathcal{P}_1^{\neq 0}| = 1125.} \tag{194}$$

Define

$$\boxed{\mathcal{R}_1^{\mathrm{phys}} := \mathbb{Q}[X_i \mid i \in \mathcal{P}_1], \qquad \Phi_1 : \mathcal{R}_1^{\mathrm{phys}} \hookrightarrow \mathcal{A}_1^{\mathrm{even}} \hookrightarrow \mathcal{A}_{1,\mathbb{R}}.} \tag{195}$$

The first arrow is faithful evaluation along the injective physical coordinate map; the second is faithful real scalar extension.

16.1 Strict enlargement beyond the generator-linear search

The scalar zero-shell quadratic

$$Q_\phi := X^2_{(\phi,0)} \tag{196}$$

provides an explicit nonlinear coordinate. Let $\mathbb{E}_{\mathrm{G}}^{(1)}$ denote contraction grade one of the finite Gaussian BV expectation. The exact comparison is

$$\boxed{\mathbb{E}_{\mathrm{G}}^{(1)}[\Phi_1(Q_\phi)] = 1, \qquad \mathbb{E}_{\mathrm{G}}^{(1)}[L_c] = 0 \quad \text{for every generator-linear ansatz } L_c.} \tag{197}$$

Therefore

$$\boxed{\Phi_1(Q_\phi) \notin \mathrm{im}(\text{generator-linear evaluation}).} \tag{198}$$

The physical polynomial carrier is thus a strict nonlinear enlargement of the complete linear search. [32]

/QME/PhysicalAntifieldZeroShellResponseMatrix.lean;lean/QGC12/BV/QME/PhysicalAntifieldShellKernelBridge.lean;lean/QGC12/BV/QME/GhostFirstAntifieldResponseMatrix.lean;lean/QGC12/BV/QME/GhostZeroAntifieldResponseMatrix.lean;lean/QGC12/BV/QME/GhostMixedSecondResponseMatrix.lean;lean/QGC12/BV/QME/GhostAntifieldShellKernelBridge.lean;lean/QGC12/BV/QME/FullGeneratorLinearKernelDecision.lean;lean/QGC12/BV/QME/CompletionStatus.lean.

[32] *Formal counterpart:* `lean/QGC12/BV/QME/FullPhysicalPolynomialCarrier.lean`.

16.2 Comparison gate as a weighted Euler derivation

The diagonal weights from Section 15 extend to a weight function

$$w : \mathcal{P}_1 \longrightarrow \mathbb{Q}_{\geq 0} \tag{199}$$

with

$$\boxed{w(a,J) = \begin{cases} 0, & J = 0, \\ 1,3,2, & |J| = 1 \text{ for } a = \phi, h, \Gamma, \\ r, r+2, r+1, & |J| = r \in \{2,3,4\} \text{ for } a = \phi, h, \Gamma. \end{cases}} \tag{200}$$

Every weight on $\mathcal{P}_1^{\neq 0}$ is strictly positive. After applying the proved physical ghost-trace retraction, the actual finite-master/Paper-6 comparison derivation is exactly

$$\boxed{\mathcal{T}_{\mathrm{cmp}}(P) = -\sum_{i \in \mathcal{P}_1} w_i X_i \partial_i P.} \tag{201}$$

Consequently,

$$\mathcal{D}_{\mathrm{cl}} \Phi_1(P) = 0 \Longrightarrow \partial_i P = 0 \quad (i \in \mathcal{P}_1^{\neq 0}). \tag{202}$$

Equivalently, there is a unique zero-shell polynomial $P_0 \in \mathbb{Q}[X_a \mid a \in \mathcal{I}_{\mathrm{phys}}]$ with

$$\boxed{P = \iota_0(P_0).} \tag{203}$$

Thus the comparison gate removes dependence on all 1,125 nonzero-shell variables by an exact polynomial theorem. [33]

16.3 Source-square selector on the seventy-five zero-shell variables

Let $\mathcal{S}(P)$ denote the rational polynomial derivation whose faithful real extension equals the second finite-master action on $\Phi_1(P)$. For each physical zero-shell component $a \in \mathcal{I}_{\mathrm{phys}}$, the polynomial-valued two-ghost source selector Θ_a satisfies

$$\boxed{\Theta_a(\mathcal{S}(\iota_0 P_0)) = \iota_0(\partial_a P_0)\,.} \tag{204}$$

Faithfulness of ι_0 gives

$$s_{\mathrm{fin}}^2 \Phi_1(\iota_0 P_0) = 0 \Longrightarrow \partial_a P_0 = 0 \qquad (a \in \mathcal{I}_{\mathrm{phys}}). \tag{205}$$

Over $\mathbb{Q}$, simultaneous vanishing of all seventy-five formal partial derivatives fixes the polynomial to its scalar coefficient:

$$\boxed{\big(\forall a \in \mathcal{I}_{\mathrm{phys}},\ \partial_a P_0 = 0\big) \Longleftrightarrow \exists q \in \mathbb{Q} :\ P_0 = \mathrm{C}(q).} \tag{206}$$

[34]

[33] *Formal counterpart:* `lean/QGC12/BV/QME/FullPhysicalPolynomialTraceMatrix.lean;lean/QGC12/BV/QME/FullPhysicalPolynomialEulerKernel.lean;lean/QGC12/BV/QME/FullPhysicalPolynomialComparisonKernel.lean`.

[34] *Formal counterpart:* `lean/QGC12/BV/QME/FullPhysicalPolynomialSourceSquare.lean;lean/QGC12/BV/QME/FullPhysicalPolynomialSourceKernel.lean;lean/QGC12/BV/QME/FullPhysicalPolynomialSimultaneousKernel.lean`.

16.4 Complete full-polynomial simultaneous kernel

Combining the two independent reductions gives the exact common kernel of the actual real gates:

$$\boxed{\left.\begin{array}{l} s_{\mathrm{fin}}^2\Phi_1(P)=0, \\ \mathcal{D}_{\mathrm{cl}}\Phi_1(P)=0 \end{array}\right\} \iff \exists q\in\mathbb{Q}:\ \ P=\mathrm{C}(q).} \tag{207}$$

Hence

$$\boxed{\ker_{\mathcal{R}_1^{\mathrm{phys}}}(\mathrm{SimGates})=\mathbb{Q}\cdot 1.} \tag{208}$$

Define a nonconstant physical-polynomial QME owner as a polynomial $P\in\mathcal{R}_1^{\mathrm{phys}}\setminus\mathbb{Q}\cdot 1$ satisfying both gates. Equation (207) gives

$$\boxed{\mathrm{IsEmpty}\big(\mathrm{FullPhysicalPolynomialNonconstantQMEOwner}(\kappa)\big).} \tag{209}$$

This theorem is carrier-scoped to the faithful cutoff-one antifield-free physical polynomial algebra. It strengthens the Paper-7 density boundary and supplies the final publication certificate used below. [35]

17 Paper-level main result

For every $\kappa\in\mathbb{R}^\times$, the results combine into five carrier-specific statements:

$$\boxed{\begin{array}{c} \mathfrak{C}_{\mathrm{glob}}=\varnothing, \qquad [1\otimes|1_0\rangle]_{\mathrm{phys}}\neq 0, \\ \ker_{\mathcal{A}_{\mathrm{sc}}}(\mathrm{SimGates})=\mathbb{Q}\cdot 1, \qquad \ker_{\mathrm{AF,lin}}(\mathrm{SimGates})=\{0\}, \\ \ker_{\mathcal{R}_1^{\mathrm{phys}}}(\mathrm{SimGates})=\mathbb{Q}\cdot 1. \end{array}} \tag{210}$$

The first statement concerns global all-observable Gaussian descent, the second the restricted state complex, the third the native scalar-generated polynomial density algebra, the fourth the complete cutoff-one antifield-free generator-linear Palatini carrier, and the fifth the complete faithful cutoff-one antifield-free physical polynomial algebra. The unrestricted generator-linear kernel is the product $K_{-2}\times K_{-1}$ of the explicit ghost-antifield and physical-antifield kernels. The full-polynomial publication certificate additionally records emptiness of the nonconstant physical-polynomial QME-owner type. [36]

18 Interpretation of the obstruction calculations

The two numerical obstructions encode different information. The first-shell value $-1/2$ arises from a single Gaussian contraction in the scalar kinetic density and fixes the loop grading of shell

[35] *Formal counterpart:* `lean/QGC12/BV/QME/FullPhysicalPolynomialSimultaneousKernel.lean;lean/QGC12/BV/QME/FullPolynomialPublicationCapstone.lean`.

[36] *Formal counterpart:* `lean/QGC12/BV/QME/RestrictedPublicationCapstone.lean;lean/QGC12/BV/QME/FullGeneratorLinearKernelDecision.lean;lean/QGC12/BV/QME/FullPhysicalPolynomialSimultaneousKernel.lean;lean/QGC12/BV/QME/FullPolynomialPublicationCapstone.lean;lean/QGC12/BV/QME/CompletionStatus.lean`.

transport. The Hamiltonian value -1 arises from the explicit canonical-pair witness $w = x_0x_0^*$ and remains at $\hbar = 0$, fixing the global all-observable descent boundary for the candidate class used here.

The regulator calculations follow the same pattern. Rectangular truncation carries a nonzero top-face derivative defect. The cyclic regulator supplies derivative closure together with primitive-class collapse. The nilpotent regulator preserves primitive coordinates and carries an explicit Euler boundary remainder. Each result identifies the exact hypotheses available to the subsequent construction.

The generator-linear theorem gives the exact linear local-density boundary. The physical and ghost blocks have trivial simultaneous kernels, every nonzero unrestricted survivor lies in an explicit antifield kernel, and the restricted state complex contains the nonzero one-particle class. The full physical-polynomial theorem then supplies a strict nonlinear enlargement and classifies its common kernel as the rational constant line. These statements refer to distinct mathematical carriers and together form the complete result at the stated cutoff.

19 Relation to perturbative BV and effective-field-theory quantization

The finite construction follows the standard BV principle that a quantum measure structure contributes a second-order odd operator whose product defect generates the antibracket. [7, 8] The local obstruction classes and Wess–Zumino identities lie in the usual BRST/BV cohomological pattern. [9]

The regulator architecture is explicitly finite. The Gaussian/Berezin fiber pushforward is a finite-dimensional regulator operation with an exact covariance and Berezin factor. Regulator removal is expressed through compatible families, coefficient transport, and proved shell identities. The paper therefore uses an effective-field-theory interpretation of perturbative coefficients grounded in the constructed finite Gaussian/Berezin measures. This stance is compatible with the general perturbative renormalization viewpoint in which effective interactions are related by finite-scale integration maps. [10]

The formal development adds two features that are especially useful in this setting. First, candidate structures are distinguished by type and theorem: graph/Hodge Laplacians, odd BV Laplacians, BRST charges, Hamiltonian differentials, and Gaussian pushforwards occupy separate interfaces. Second, obstruction theorems remain executable dependencies, so every later theorem inherits the exact established boundary conditions.

The machine-checked implementation uses Lean 4 and mathlib together with the project-specific finite BV and operator libraries. [11, 12]

20 Scope and next mathematical boundary

Appendices A–C summarize the proved statements, their exact scopes, and their formal sources. The present paper ends with the global Gaussian obstruction, the canonical restricted cohomology, the scalar-density classification, the complete cutoff-one generator-linear classification, and the full

1,200-variable physical-polynomial classification in Eq. (210).

The strengthened publication boundary leaves the restricted physical complex and its nonzero state classes intact. The complete antifield-free physical polynomial algebra contributes exactly the rational constant line to the simultaneous physical kernel. The formal prerequisite review therefore retains an empty current Paper-8 entry type and an empty ownership edge on every inhabited current candidate. A future positive quantum-gravity program begins with a genuinely new QME owner and a newly justified ownership edge. Geometry quantization follows after that entry theorem is inhabited.

21 Conclusion

The symmetric/projective Palatini master of the self-consistent Einstein–scalar sector admits a finite quantum-BV treatment with a genuine odd Laplacian, a regulated QME defect, Wess–Zumino consistency, admissible anomaly recursion, and finite Gaussian/Berezin shell integration. The explicit first-shell trace calculation gives $-1/2$ and determines the Wick-contraction loop grading.

The explicit canonical-pair witness $w = x_0 x_0^*$ gives zero Gaussian pushforward and Hamiltonian residual probe value -1. This classical-order residual makes the stated global completion class empty. The comparison residual simultaneously selects a canonical greatest square-zero compatible subcomplex whose physical sector contains a nonzero unit class and preserves every nonzero native scalar Fock state, including the one-particle state.

The local-density analysis supplies the complementary carrier hierarchy. The native scalar-generated superalgebra contributes exactly the constant even line. The complete cutoff-one antifield-free generator-linear Palatini carrier has trivial simultaneous kernel. The faithful 1,200-coordinate antifield-free physical polynomial algebra strictly enlarges that linear carrier, and its exact simultaneous kernel is again the rational constant line. The corresponding nonconstant physical-polynomial QME-owner type is empty.

These results fix the terminal Paper-7 boundary. The subsequent positive program requires a genuinely new QME owner, a replacement ownership edge, and an inhabited entry theorem before independent geometry quantization begins.

A Reader theorem map

The main results can be read in the following order.

Result	Mathematical content
Finite regulator	Split finite Palatini conjugate-pair banks, physical-master projection, and refinement maps.
Finite odd Laplacian	$\Delta_N 1 = 0$, parity shift, and second-order product law generating the regulated bracket.
Density law	Modular-square identity for $\Delta_{\rho,N}$; flat-density square-zero.
Derivative boundary	Explicit top-face derivative lost by rectangular projection.
Finite flat QME	$(s^{\mathrm{flat}}_{\hbar,N})^2 = (\mathfrak{Q}^{\mathrm{flat}}_N, -)_N$ and $s^{\mathrm{flat}}_{\hbar,N}\mathfrak{Q}^{\mathrm{flat}}_N = 0$.

Result	Mathematical content
Finite-CME anomaly recursion	Vanishing admissible obstruction class iff an admissible next QME coefficient exists.
Raw refinement obstruction	Coordinate projection has a nonzero odd-Laplacian transport defect.
Flat fiber obstruction	Flat polynomial fiber expectation has a nonzero BV–Stokes moment defect.
Gaussian BV pushforward	Normalized retraction and density-odd-Laplacian intertwining.
Same-order shell obstruction	Physical first-shell transport residual has selected trace $-1/2$.
Wick grading	One contraction raises loop order; arbitrary finite contraction grades combine by Cauchy convolution.
Locality decisions	Rectangular locality defect, cyclic primitive collapse, and nilpotent separated-bracket resolution.
Global descent obstruction	Explicit kernel witness with Gaussian pushforward zero and Hamiltonian residual probe -1.
Global completion	Declared all-observable Gaussian renormalized-QME completion type is empty.
Compatible sector	Residual kernel is the unique greatest intertwining submodule, nonzero, and proper.
Restricted quantum complex	Explicit greatest invariant square-zero compatible submodule and genuine ungraded restricted cohomology.
Restricted physical cohomology	Exact Paper-6 comparison sector, nonzero unit class, invariant ungraded physical quotient.
Native state survival	Every nonzero native Fock state defines a nonzero restricted physical state class.
Scalar density classification	Complete zero/first-jet/descendant polynomial superalgebra has simultaneous physical kernel exactly constants.
Physical generator block	Complete cutoff-one physical field/jet generator-linear block has trivial simultaneous source-square/Paper-6 kernel.
Ghost generator block	Complete cutoff-one diffeomorphism/projective ghost generator-linear block has trivial simultaneous kernel.
Antifield kernel classification	Unrestricted generator-linear simultaneous kernel is exactly the product of the explicit ghost-antifield and physical-antifield shell kernels.
Antifield-free generator-linear boundary	Complete cutoff-one antifield-free generator-linear Palatini carrier has simultaneous kernel $\{0\}$.
Full physical polynomial carrier	Faithful 1,200-variable antifield-free polynomial algebra; scalar zero-shell quadratic certifies strict enlargement beyond the generator-linear image.
Polynomial comparison kernel	Exact positive weighted-Euler trace removes dependence on all 1,125 nonzero-shell variables.
Polynomial source-square kernel	Seventy-five two-ghost source selectors recover all zero-shell partial derivatives.
Full physical polynomial kernel	Simultaneous source-square/Paper-6 kernel is exactly $\mathbb{Q} \cdot 1$; the nonconstant physical-polynomial QME-owner type is empty.
Publication outcome	Global obstruction, nontrivial restricted physical cohomology, constant-only native scalar density kernel, trivial complete antifield-free generator-linear kernel, and constant-only complete physical-polynomial kernel.

B Statement and status map

Statement	Status	Scope
Finite Palatini BV regulator	proved	Every declared finite cutoff; current symmetric/projective Palatini master.
Finite odd Laplacian	proved	Finite regulated carrier.
Density modular square	proved	Finite density charts; flat density gives square-zero.
Total-derivative descent of rectangular regulator	disproved	Explicit top-face witness.
Flat regulated QME consistency	proved	Doubled-real finite carrier; square identity uses the flat odd Laplacian.
Projected physical-master square-zero on full cutoff-one carrier	disproved	Retained scalar has explicit nonzero second Hamiltonian action.
Paper-6/QME comparison	proved	Exact common retained sector.
One-loop obstruction criterion	proved	Finite-CME background and declared admissible counterterm profile.
Arbitrary-order extension criterion	proved	Finite-CME background, solved lower orders, and closed recursive anomaly.
Raw odd-Laplacian refinement	disproved	Strict finite refinements.
Unrestricted retained compensation	disproved at stated scope	Concrete first Palatini refinement.
Flat fiber BV–Stokes law	disproved	First scalar polynomial fiber.
Gaussian/Berezin BV pushforward	proved	Every finite successor cutoff.
Associative finite shell composition	proved	Arbitrary finite cutoff prefixes.
Same-order physical master transport	disproved	First shell; selected residual probe $-1/2$.
Wick-contraction grading	proved	Finite Gaussian monomials and coefficient families.
Rectangular local-functional covariance	disproved	Top-jet successor boundary.
Cyclic primitive faithfulness	disproved	Cyclic local-functional quotient.
Nilpotent separated bracket	proved at maximal scope	Declared descent kernel and reflected quotient.
Global all-observable Hamiltonian descent	disproved	Explicit first-shell kernel witness; residual probe -1.
Declared global Gaussian QME completion	empty	Every declared contraction-graded candidate.
Greatest compatible sector	proved	Kernel of adjacent-cutoff density-aware differential residual.
Restricted square-zero quantum complex	proved	Canonical greatest invariant chain-compatible submodule.
Restricted physical unit class	nonzero	Every cutoff and real $\hbar$.
Native Fock-state classes	nonzero	Every nonzero native matter state.

Statement	Status	Scope
Scalar-generated density superalgebra kernel	classified	Simultaneous source-square/Paper-6 kernel equals the constant even line.
Physical generator-linear block	classified / trivial kernel	Complete cutoff-one physical field and retained-jet block.
Ghost generator-linear block	classified / trivial kernel	All retained cutoff-one diffeomorphism and projective ghost generators.
Full generator-linear simultaneous kernel	classified	Exact product $K_{-2} \times K_{-1}$ of the two antifield kernel sectors, with physical and ghost blocks zero.
Antifield-free generator-linear simultaneous kernel	zero	Complete original cutoff-one generator-linear coefficient carrier subject to antifield-freedom.
Full physical polynomial carrier	proved / strict enlargement	Faithful polynomial algebra on all 1,200 cutoff-one antifield-free physical coordinates.
Full physical polynomial simultaneous kernel	classified / constants	Actual finite-master-square and Paper-6 comparison gates select exactly $\mathbb{Q} \cdot 1$.
Nonconstant full-polynomial QME owner	empty	Complete faithful cutoff-one physical polynomial carrier.
Current Paper-8 entry prerequisite	empty	Full-polynomial prerequisite review; a future positive branch requires a genuinely new QME owner and ownership edge.
Quantum geometry	next layer	Begins after an inhabited replacement entry theorem and an independent metric/connection quantum carrier.

C Formal authority map

The manuscript body uses standard BV, BRST, Palatini, regulator, and cohomological semantics. The formal implementation is organized under the following paper-scoped modules.

Mathematical surface	Formal source
Finite regulator	`lean/QGC12/BV/QME/Regulator.lean`
Odd Laplacian and density law	`lean/QGC12/BV/QME/OddLaplacian.lean`, `DensityChange.lean`
Jet-cutoff boundary	`lean/QGC12/BV/QME/JetCutoffBoundary.lean`
QME defect and consistency	`lean/QGC12/BV/QME/MasterDefect.lean`
Paper-6 comparison	`lean/QGC12/BV/QME/Paper6Comparison.lean`
Local anomaly and counterterms	`LocalAnomaly.lean`, `OneLoopObstruction.lean`, `RecursiveExtension.lean`
Wess–Zumino recursion	`TwoLoopConsistency.lean`, `AutomaticRecursiveStage.lean`
Regulator-refinement decisions	`RegulatorRefinement.lean`, `CompensatedRefinement.lean`, `FiberPushforward.lean`, `WeightedFiberPushforward.lean`
Gaussian BV fiber	`GaussianFiberKernel.lean`, `FiniteGaussianComplement.lean`, `FiniteGaussianBVComplement.lean`
Gaussian cutoff system	`RetainedGaussianPushforward.lean`, `SuccessiveGaussianPushforward.lean`, `PalatiniGaussianCutoffSystem.lean`

Mathematical surface	Formal source
Wick/contraction grading	`FirstLoopWickTransport.lean`, `ContractionGradedWickTransport.lean`
Locality/regulator decisions	`RectangularLocalityNoGo.lean`, `CyclicLocalityObstruction.lean`, `NilpotentFunctionalTower.lean`, `NilpotentJointKernel.lean`
Global Hamiltonian obstruction	`FirstShellHamiltonianObstruction.lean`
Global Paper-7 obstruction capstone	`Paper7ObstructionCapstone.lean`
Compatible/restricted complex	`CompatibleQuantumSubcomplex.lean`, `RestrictedQuantumCohomology.lean`
Restricted physical cohomology	`RestrictedPhysicalSector.lean`, `RestrictedPhysicalCohomology.lean`
Native state cohomology	`RestrictedPhysicalStateCohomology.lean`
Scalar density boundary	`RetainedScalarPolynomialBoundary.lean`, `RetainedScalarPolynomialCancellation.lean`, `ScalarJetPolynomialBoundary.lean`
Scalar descendant superalgebra	`ScalarJetDescendantSuperalgebra.lean`, `ScalarJetDescendantKernel.lean`, `ScalarJetDescendantSimultaneousKernel.lean`
Generator-linear coordinates and block decomposition	`GeneratorShellCoordinates.lean`, `FullGeneratorBlockCoordinates.lean`
Physical and ghost generator kernels	`GhostOddCompleteShellDecision.lean`, `PhysicalShellKernelBridge.lean`, `GhostShellKernelBridge.lean`
Antifield kernel classification	`PhysicalAntifieldShellKernelBridge.lean`, `GhostAntifieldShellKernelBridge.lean`
Full generator-linear decision	`FullGeneratorLinearKernel.lean`, `FullGeneratorLinearKernelDecision.lean`
Full physical polynomial carrier	`FullPhysicalPolynomialCarrier.lean`, `FullPhysicalPolynomialRetraction.lean`
Polynomial comparison and Euler kernel	`FullPhysicalPolynomialTraceMatrix.lean`, `FullPhysicalPolynomialEulerKernel.lean`, `FullPhysicalPolynomialComparisonKernel.lean`
Polynomial source-square kernel	`FullPhysicalPolynomialSourceSquare.lean`, `FullPhysicalPolynomialSourceKernel.lean`
Full physical polynomial decision	`FullPhysicalPolynomialSimultaneousKernel.lean`, `FullPolynomialPublicationCapstone.lean`
Restricted state/density capstone	`RestrictedPublicationCapstone.lean`
Complete Paper-7 aggregate	`lean/QGC12/BV/QME/CompletionStatus.lean`
Full-polynomial prerequisite review	`lean/QGC12/QuantumGravity/FullPolynomialPaper7PrerequisiteDecision.lean`
Focused generator-linear audit	`lean/QGC12QMEM2771Audit.lean`
Focused physical-polynomial audit	`lean/QGC12QMEM2772Audit.lean`
Focused publication-boundary audit	`lean/QGC12QMEM2773Audit.lean`
Final program audit	`lean/QGC12FinalProgramAudit.lean`
Focused aggregate audit	`lean/QGC12Paper7QMEAudit.lean`

D Verification status and formal dependency boundary

The deposit manuscript is tied to the audited Paper-7/Paper-8 boundary snapshot at repository commit

$$\boxed{\texttt{3399b0d32ed42560aa215c43bc83768ed825b10f}} \tag{211}$$

(“`qgc12: audit two-paper program completion`”, 22 September 2026). This snapshot contains the complete generator-linear classification, the full 1,200-variable physical-polynomial kernel, the strengthened Paper-7 publication certificate, the full-polynomial Paper-8 prerequisite review, and the consolidated completion audit. Subsequent repository work belongs to separate research programs and leaves this manuscript receipt fixed.

D.1 Named publication declarations

The paper-level claims are exposed through named Lean declarations. Table 5 records the declarations that close the manuscript result package.

Table 5: Terminal declarations for the Paper-7 publication package.

Lean declaration	Mathematical role
`QGC12.BV.QME.no_palatiniRenormalizedQMECompletion`	Emptiness of the declared global Gaussian renormalized-QME completion branch.
`QGC12.BV.QME.palatiniPaper7_restricted_publication_actual`	Restricted state/density publication certificate: global obstruction, greatest restricted square-zero physical complex, nonzero unit and one-particle classes, and the constant-only native scalar-density kernel.
`QGC12.BV.QME.palatiniProjectiveRestrictedOneParticleClass_ne_zero`	Nonzero restricted cohomology class of the explicit native one-particle state.
`QGC12.BV.QME.palatiniProjectiveOneScalarJetDescendant_actualSimultaneousGates_iff_constant`	Exact constant-only kernel for the native scalar zero/first-jet/descendant superalgebra.
`QGC12.BV.QME.palatiniProjectiveGeneratorLinearSimultaneousGates_iff_kernel`	Exact four-block classification of the complete cutoff-one generator-linear simultaneous kernel.
`QGC12.BV.QME.palatiniProjectiveGeneratorLinearSimultaneousGates_and_antifieldFree_iff_zero`	Triviality of the complete antifield-free generator-linear simultaneous kernel.
`QGC12.qgc12_M2771_live_authority`	Aggregate proposition binding the complete generator-linear capstone on the original coefficient carrier.
`QGC12.BV.QME.palatiniProjectiveOnePhysicalPolynomialActualGates_iff_constant`	Exact constant-only common kernel of the two actual real gates on the complete cutoff-one physical polynomial algebra.
`QGC12.BV.QME.no_palatiniFullPhysicalPolynomialNonconstantQMEOwner`	Emptiness of the nonconstant QME-owner type on the faithful full physical polynomial carrier.

Lean declaration	Mathematical role
`QGC12.BV.QME.palatiniPaper7_fullPolynomial_publication_actual`	Strengthened Paper-7 publication certificate carrying the restricted result, the full-polynomial kernel, owner emptiness, and global completion emptiness.
`QGC12.qgc12_M2772_live_authority`	Aggregate full physical-polynomial kernel authority.
`QGC12.qgc12_M2773_live_authority`	Full-polynomial publication-boundary and Paper-8 prerequisite review authority.

The restricted state/density certificate, generator-linear aggregate, and full-polynomial publication certificate retain distinct formal roles. The restricted certificate packages the positive physical/cohomological result. The generator-linear authority fixes the complete linear boundary. The full-polynomial authority supplies the strict nonlinear enlargement and constant-kernel classification. Together they supply the five-part manuscript result in Eq. (210).

D.2 Audit roots and axiom receipt

The primary focused audit is `lean/QGC12Paper7QMEAudit.lean`. At the snapshot in Eq. (211) it imports 51 Paper-7 modules and contains 913 explicit `#check` commands together with 672 `#print axioms` commands. The generator-linear audit `lean/QGC12QMEM2771Audit.lean` contains 513 type checks and 332 axiom receipts. The full physical-polynomial audit `lean/QGC12QMEM2772Audit.lean` contains 71 type checks and 43 axiom receipts. The strengthened publication-boundary audit `lean/QGC12QMEM2773Audit.lean` contains 12 type checks and 4 axiom receipts. The consolidated boundary audit `lean/QGC12QMEM2774Audit.lean` contains 7 type checks and 3 axiom receipts, and `lean/QGC12FinalProgramAudit.lean` contains 30 type checks and 25 axiom receipts.

In Lean, `#check` forces elaboration of the named declaration and verifies its complete type. The command `#print axioms` computes the axiomatic dependencies of the elaborated theorem. The terminal Paper-7 publication declarations and the consolidated boundary audit report the exact standard axiom union

$$\boxed{\{\texttt{propext},\ \texttt{Classical.choice},\ \texttt{Quot.sound}\}.} \tag{212}$$

Here `propext` is proposition extensionality, `Classical.choice` is classical choice, and `Quot.sound` is quotient soundness. The reported union consists exclusively of these standard Lean/mathlib logical principles; the publication declarations introduce zero additional project-specific axioms.

The principal Paper-7 and terminal polynomial receipts are compiled directly by

```
lake env lean lean/QGC12Paper7QMEAudit.lean,
lake env lean lean/QGC12QMEM2771Audit.lean,
lake env lean lean/QGC12QMEM2772Audit.lean,
lake env lean lean/QGC12QMEM2773Audit.lean.
```

The consolidated cross-paper boundary is checked independently by

`lake env lean lean/QGC12QMEM2774Audit.lean`,
`lake env lean lean/QGC12FinalProgramAudit.lean`.

D.3 Static proof-integrity guard

The terminal completion script `scripts/check_qgc12_qme_qg_program_completion.sh` audits the full M2705–M2774 implementation cone. It requires every sequential live authority, checks the terminal roadmap state, enforces the proof policy across the QME and quantum-gravity modules, and invokes six focused/final Lean audits in its `-lean` mode. The proof policy excludes `sorry`, `admit`, new top-level `axiom` declarations, `native_decide`, and prototype imports from the publication cone.

The formal development therefore records three verification layers: theorem construction in the production modules, elaborated type and axiom auditing in named Lean roots, and static proof-integrity enforcement over the terminal dependency graph. The Paper-7 result terminates at the classical-metric/quantum-matter BV carrier with the full physical-polynomial boundary proved. The subsequent quantum-geometric layer begins only after a new QME owner and replacement Paper-8 ownership edge are constructed.

E Formal source note

The theorem package belongs to the larger Lean 4 development underlying the series. Internal milestone labels, repository paths, and implementation vocabulary are confined to formal-reference footnotes and appendices. The mathematical body uses the corresponding standard BV, BRST, Palatini, Gaussian integration, perturbative, and cohomological semantics.

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