Relational Quantum Gravity in a Self-Consistent Einstein–Scalar Sector

Independent physical and gravitational quotients, nonlinear Einstein realization, and intrinsic gauge rigidity

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Abstract

The first seven papers of this series construct one Schwarzschild Einstein–real-scalar quantum sector through operator selection, native quantum dynamics, local stress, nonlinear Einstein response, Palatini BV/BRST structure, and regulated quantum-master analysis. Paper 7 leaves a nonzero restricted physical sector after an explicit global QME-descent obstruction. Paper 8 asks how physical quantum identity and gravitational visibility relate once that surviving sector is given an exact nonlinear Einstein realization.

The realized state space admits a physical-owner projection π_{P}, a state-dependent source T, and a smooth Lorentz metric g satisfying

$$\boxed{G_{ij}[g(q)](x) = \kappa\, T(q)_{ij}(x).}$$

The metric response is classical and nonlinear; the quantum structure lies in the state, physical-owner, gauge/cohomological, and source sectors. In this paper *relational quantum gravity* denotes the resulting relation among quantum physical ownership, quantum-state-dependent source, nonlinear gravitational response, and their quotient structures. The two kernel equivalence relations are globally incomparable:

$$\boxed{\sim_{\mathrm{P}} \not\subseteq \sim_{\mathrm{G}}, \qquad \sim_{\mathrm{G}} \not\subseteq \sim_{\mathrm{P}}\ .}$$

Anchor continuation preserves the physical owner and changes the realized source/geometry, supplying the first witness. Coherent phase pairs supply the reverse witness: distinct physical owners can have one gravity-visible source class. A matched finite-number/coherent construction gives the same nonseparation across qualitatively different quantum states.

On the fixed-(h, a) realized family, the complete scalar-curvature/curvature-gradient image gives a rigidity theorem. Equality of joint images first recovers the normalized shape parameter, exposes one exact algebraic source duality, and then removes its nonfixed branch through the intrinsic midpoint of the complete image. Consequently,

$$\boxed{\operatorname{Im}\Sigma_{d_1} = \operatorname{Im}\Sigma_{d_2} \Longrightarrow S_1 = S_2.}$$

After local Lorentz-frame descent, this source classification is equivalent to the represented physical smooth-Diff classification:

$$\boxed{[g_1]_{\mathrm{Diff,phys}} = [g_2]_{\mathrm{Diff,phys}} \Longleftrightarrow [g_1]_{\mathrm{grav}} = [g_2]_{\mathrm{grav}} \Longleftrightarrow S_1 = S_2.}$$

The functional-analytic result is equally explicit. Normalized number states have unit norm and a positive source component growing linearly with occupation, excluding one global norm-bounded completed-Fock stress extension that agrees with every number source. An infinite-support coherent family lies in a stress domain with an explicit source construction. Every positive finite occupation has an exact conserved nonlinear Einstein realization. All principal statements are machine checked at the stated scope; exact theorem references and verification data are collected in the appendices.

1 Introduction: from quantum-master obstruction to relational geometry

The first seven papers establish one continuous mathematical chain,

$$\boxed{\begin{array}{c}\text{operator law} \longrightarrow \text{quantum dynamics} \longrightarrow \text{local stress} \longrightarrow \text{nonlinear Einstein realization} \\ \longrightarrow \text{Palatini BV/BRST} \longrightarrow \text{regulated QME analysis.}\end{array}} \tag{1}$$

Paper 1 selects the terminal scalar law by Friedrichs extension; Papers 2–4 construct the native Fock dynamics, vacuum-relative stress, and self-consistent Einstein–scalar geometry; Papers 5–6 establish the symmetric/projective BV master and its operator BRST realization; Paper 7 introduces the finite BV measure structure and proves the explicit all-observable descent residual

$$\Pi^{\mathrm{G}}_{1\to 0} w = 0, \qquad \boxed{\mathfrak{p}(\mathcal{R}_{\mathrm{Ham}}(w)) = -1.} \tag{2}$$

The same Paper 7 analysis constructs the greatest compatible square-zero refinement $\mathcal{C}^{Q}_{N,\hbar}$ and a nonzero native one-particle cohomology class.[1, 2, 3, 4, 5, 6, 7]

The exact Paper 7/Paper 8 boundary is a theorem. For every admissible coupling, cutoff, Planck parameter, and native anchor,

$$\boxed{\begin{array}{l}\mathrm{IsEmpty}(\mathcal{E}_{\mathrm{QME}}(\kappa)), \\ \mathrm{Nonempty}(\mathcal{R}_{\kappa,N,\hbar}), \\ \mathrm{IsEmpty}(\mathcal{O}_{\mathrm{legacy}}(\mathcal{R}_{\kappa,N,\hbar})), \\ \pi_{\mathrm{owner}}(r_{\kappa,N,\hbar,a}) \neq 0.\end{array}} \tag{3}$$

[1] The relational entry therefore has its own nonzero physical owner and its own realized quantum-geometric continuation.

1.1 Minimal mathematical setting and terminology

The minimal Paper 8 object is a realized sector

$$\boxed{q \longmapsto (\Psi(q),\, p(q),\, a(q),\, T(q),\, g(q)),} \tag{4}$$

where $\Psi(q)$ is quantum matter data, $p(q)$ is its physical-owner class, $a(q)$ is a relational source/geometry anchor, $T(q)$ is the quantum-state-dependent stress source, and $g(q)$ is a smooth Lorentz metric. The realized field equation is

$$\boxed{G[g(q)] = \kappa T(q).} \tag{5}$$

The quantum content therefore resides in Ψ, the physical owner, the BV/BRST/cohomological structure inherited from Papers 5–7, and the state-dependent source. The gravitational response in Paper 8 is represented by a classical smooth metric solving the exact nonlinear Einstein equation.

[1] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/Paper7Handoff.lean::paper8_paper7_handoff`.

In conventional terminology, self-consistent equations with quantum matter sourcing a classical metric belong to the semiclassical-gravity framework.[10, 11] Independent quantization of metric degrees of freedom forms a later mathematical layer.

The term *relational quantum gravity* is used here in the following precise Paper 8 sense:

$$\boxed{\begin{gathered}\text{relational quantum gravity} := (\text{quantum physical ownership}) + (\text{state-dependent source}) \\ + (\text{exact nonlinear metric response}) + (\text{relations among their quotients})\end{gathered}} \tag{6}$$

This terminology fixes the scope of the title and the mathematical object studied below.

Relational and diffeomorphism-invariant observables have a long development in general relativity and quantum-gravity research, from invariant observables and intrinsic descriptions to partial/complete observables and effective-gravity constructions.[14, 15, 16, 17, 18] The Paper 8 problem is the exact realized-state-space question

$$\mathrm{KerPair}(\pi_{\mathrm{P}}) \quad \text{versus} \quad \mathrm{KerPair}(T), \tag{7}$$

followed by an intrinsic geometric classification of $\mathrm{KerPair}(T)$ on a specified branch family. The use of scalar curvature data for invariant characterization likewise has extensive precedent.[19, 20] Paper 8 specializes this invariant-characterization problem to the fixed-(h, a) nonlinear family.

Intrinsic-rigidity result. Within this nonlinear fixed-(h, a) family, the complete unparameterized joint image of two scalar invariants is injective in the source parameter S:

$$\boxed{\mathrm{Im}\,\Sigma_{d_1} = \mathrm{Im}\,\Sigma_{d_2} \Longrightarrow S_1 = S_2.} \tag{8}$$

Paper 8 introduces three mathematical layers. The first is a common realized state space with one physical-owner projection and one source-defined gravity-visible projection,

$$\boxed{\begin{array}{ccccc} & & \mathcal{Q}^{\mathrm{real}}_{\kappa,N,\hbar} & & \\ \pi_{\mathrm{P}} \swarrow & & & \searrow \pi_{\mathrm{G}} & \qquad G[g(q)] = \kappa T(q). \\ \mathfrak{P}_{\kappa,N,\hbar} & & & \mathfrak{G}_{\kappa,N,\hbar}, & \end{array}} \tag{9}$$

The second layer is the relational geometry of these projections: factorization, nontrivial fibers, vertical/transverse operations, completed-Fock source domains, finite-number realizations, and matched number/coherent visibility. The third layer is an intrinsic curvature classification of the fixed-(h, a) realized family.

The compact main theorem is

$$\boxed{\begin{array}{l} \mathcal{Q}^{\mathrm{real}}_{\kappa,N,\hbar} \neq \varnothing, \\ \nexists F : \mathfrak{P} \to \mathfrak{G} : \quad F \circ \pi_{\mathrm{P}} = \pi_{\mathrm{G}}, \\ \nexists H : \mathfrak{G} \to \mathfrak{P} : \quad H \circ \pi_{\mathrm{G}} = \pi_{\mathrm{P}}, \\ \mathfrak{G}^{\mathrm{visible}}_{a,\kappa} \simeq \mathfrak{G}^{\mathrm{Diff,phys}}_{a,\kappa}. \end{array}} \tag{10}$$

[2] Sections 2–9 establish the common realized state space and its relational structure. Section 10 derives the intrinsic classification that turns source visibility into a geometric physical-Diff quotient on the fixed-(h, a) domain. Section 11 extracts the matter-parametric architecture.

2 The realized quantum-geometric state space

Fix an Einstein coupling $\kappa \in \mathbb{R}^\times$ with the positive realized-branch hypothesis, a cutoff $N \in \mathbb{N}$, and $\hbar \in \mathbb{R}$. The realized state space is the disjoint union

$$\boxed{\mathcal{Q}^{\mathrm{real}}_{\kappa,N,\hbar} = \mathcal{A}_{\kappa,N,\hbar} \sqcup \mathcal{C}_{\kappa,N,\hbar},} \tag{11}$$

where $\mathcal{A}$ is the admitted one-particle sector and $\mathcal{C}$ is the realized coherent sector.[3]

Definition 2.1 (Realized-sector maps)**.** The three Paper 8 maps have the fixed types

$$\boxed{\pi_{\mathrm{P}} : \mathcal{Q}^{\mathrm{real}} \to \mathfrak{P}_{\kappa,N,\hbar}, \qquad T : \mathcal{Q}^{\mathrm{real}} \to \mathcal{T}(M), \qquad g : \mathcal{Q}^{\mathrm{real}} \to \mathrm{Lor}(M).} \tag{12}$$

For $q \in \mathcal{A}$, the data have the signature

$$q \longmapsto (\text{one-particle state}, a(q), \pi_{\mathrm{P}}(q), T(q), g(q)), \tag{13}$$

and for $q \in \mathcal{C}$,

$$q \longmapsto (\text{coherent state}, a(q), \pi_{\mathrm{P}}(q), T(q), g(q)). \tag{14}$$

The case definitions are

$$\pi_{\mathrm{P}}(q) = \begin{cases} \pi_{\mathrm{phys}}(q), & q \in \mathcal{A}, \\ q.\mathrm{physicalOwner}, & q \in \mathcal{C}, \end{cases} \tag{15}$$

$$T(q) = \begin{cases} T^{(1)}_{a(q)}, & q \in \mathcal{A}, \\ q.\mathrm{gravitySource}, & q \in \mathcal{C}, \end{cases} \qquad g(q) = \begin{cases} q.\mathrm{geometry.metric}, & q \in \mathcal{A}, \\ q.\mathrm{geometry}, & q \in \mathcal{C}. \end{cases} \tag{16}$$

Every tagged sector satisfies one common field equation:

Theorem 2.2 (Exact realized Einstein response)**.** *For every $q \in \mathcal{Q}^{\mathrm{real}}$, point $x \in M$, and canonical spacetime indices i, j,*

$$\boxed{G_{ij}[g(q)](x) = \kappa\, T(q)_{ij}(x).} \tag{17}$$

[4]

Definition 2.3 (Physical-owner relation)**.** The physical-owner projection defines

$$q_1 \sim_{\mathrm{P}} q_2 \iff \pi_{\mathrm{P}}(q_1) = \pi_{\mathrm{P}}(q_2). \tag{18}$$

[2] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Capstone/PublicationCertificate.lean::paper8_relational_quantum_gravity_main`.

[3] *Formal counterpart:* `lean/Tier30/Research/QGQuantumGeometryClosure08/Carrier/RealizedQuantumGeometryCarrier.lean`.

[4] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_realized_quantum_geometric_carrier`.

Definition 2.4 (Gravity-visible relation)**.** Through Sections 2–9, gravity visibility is the source-equality setoid

$$q_1 \sim_{\mathrm{G}} q_2 \iff T(q_1) = T(q_2), \tag{19}$$

with quotient

$$\boxed{\mathfrak{G}_{\kappa,N,\hbar} := \mathcal{Q}^{\mathrm{real}}_{\kappa,N,\hbar} / \sim_{\mathrm{G}}\ .} \tag{20}$$

The relations $\sim_{\mathrm{P}}$ and $\sim_{\mathrm{G}}$ are the kernel-pair equivalence relations of π_{P} and T, respectively. The quotient projection satisfies

$$\boxed{\pi_{\mathrm{G}}(q_1) = \pi_{\mathrm{G}}(q_2) \iff T(q_1) = T(q_2).} \tag{21}$$

The descended source map

$$\widehat{T} : \mathfrak{G}_{\kappa,N,\hbar} \to \mathcal{T}(M), \qquad \widehat{T}([q]_{\mathrm{G}}) = T(q), \tag{22}$$

is injective. Thus the early Paper 8 gravity-visible quotient is exactly the canonical source quotient. Section 10 proves that, on the fixed-(h, a) realized family, this quotient is recovered by intrinsic smooth geometry and is equivalent to the represented physical smooth-Diff quotient.

The common realized span is therefore

$$\boxed{q \longmapsto (\pi_{\mathrm{P}}(q), \pi_{\mathrm{G}}(q), g(q)), \qquad G[g(q)] = \kappa \widehat{T}(\pi_{\mathrm{G}}(q)).} \tag{23}$$

3 Independent physical and gravitational quotients

The two global nonfactorization results are instances of one elementary criterion.

Lemma 3.1 (Factorization through a realized projection)**.** *Let Q be a type with maps $p : Q \to P$ and $g : Q \to G$. Define the corestrictions*

$$\bar{p} : Q \to \mathrm{Im}(p), \qquad \bar{p}(q) := \langle p(q), q, \mathrm{rfl} \rangle, \tag{24}$$

and analogously $\bar{g} : Q \to \mathrm{Im}(g)$. A map

$$F : \mathrm{Im}(p) \to G, \qquad g = F \circ \bar{p}, \tag{25}$$

exists precisely when

$$p(q_1) = p(q_2) \Longrightarrow g(q_1) = g(q_2) \qquad (q_1, q_2 \in Q). \tag{26}$$

Likewise, a map $H : \mathrm{Im}(g) \to P$ satisfying $p = H \circ \bar{g}$ exists precisely when

$$g(q_1) = g(q_2) \Longrightarrow p(q_1) = p(q_2). \tag{27}$$

Any ambient factorization $\widetilde{F} : P \to G$ or $\widetilde{H} : G \to P$ restricts to the corresponding realized-image factorization.

The proof defines the descended value on each realized image fiber and uses the displayed implication for well-definedness. The final sentence transfers every failure on the realized image immediately to the corresponding ambient factorization claim.

3.1 Anchor continuation and physical ownership

The first nonfactorization witness uses the anchor as a relational geometry datum. Its role is fixed by an explicit continuation action. For a valid target anchor b, let A_b denote the native anchor continuation. On an admitted realized sector,

$$A_b : q(a) \longmapsto q(b). \tag{28}$$

The action retains the matter state and restricted physical representative and reconstructs the nonlinear geometry at the target anchor. Consequently,

$$\boxed{\Psi(A_b q) = \Psi(q), \qquad \mathrm{Rep}(A_b q) = \mathrm{Rep}(q), \qquad \pi_{\mathrm{P}}(A_b q) = \pi_{\mathrm{P}}(q).} \tag{29}$$

The descended action on restricted physical cohomology is the identity,

$$\boxed{A_b^{\mathrm{owner}} = I,} \tag{30}$$

and the target sector again satisfies the exact nonlinear Einstein equation.[5]

Thus the physical owner is the quantum/gauge object represented by the matter state and restricted cohomology class. The anchor parametrizes relational geometric context. The pair $(\pi_{\mathrm{P}}(q), a(q))$ defines a finer contextual object; adopting that pair as the definition of "physical state" would replace the physical-owner equivalence relation by a different, deliberately refined relation. Paper 8 asks how many exact self-consistent geometries can lie over one fixed physical owner.

There are explicit nontrivial anchor continuations,

$$q \neq A_b q, \qquad \pi_{\mathrm{P}}(q) = \pi_{\mathrm{P}}(A_b q), \tag{31}$$

so the cross-anchor comparison below is an explicitly constructed internal motion of the admitted realized state space.[6]

3.2 One physical owner, two gravity classes

Let q_{s} and q_{l} be the constructed one-particle realizations at the small and large anchors. Their underlying admitted one-particle sector has one physical-owner projection,

$$\boxed{\pi_{\mathrm{P}}(q_{\mathrm{s}}) = \pi_{\mathrm{P}}(q_{\mathrm{l}}),} \tag{32}$$

and the source leg retains the anchor dependence. At the explicit radial component used in the witness,

$$T_a^{(1)}(x)_{00} = \frac{\sigma(\Phi_a)}{2r(x)^4}, \tag{33}$$

[5] *Formal counterpart:*
`lean/Tier30/Research/QGRelationalGeometrySector02/Transitions/AnchorContinuation.lean::NativeAnchorContinuation.act;nativeAnchorContinuation_preserves_owner;nativeAnchorContinuation_actual_admissible;lean/Tier30/Research/QGRelationalGeometrySector02/Cohomology/AnchorDescent.lean::anchorOwnerTransition_actual_identity`.

[6] *Formal counterpart:*
`lean/Tier30/Research/QGRelationalGeometrySector02/Transitions/AnchorContinuation.lean::nativeAnchorTwo_to_three_is_nontrivial_fiber_motion`.

with

$$\sigma(\Phi_{a_{\mathrm{s}}}) = \frac{5}{32}, \qquad \sigma(\Phi_{a_{\mathrm{l}}}) = \frac{5}{9}. \tag{34}$$

Hence

$$\boxed{\pi_{\mathrm{G}}(q_{\mathrm{s}}) \neq \pi_{\mathrm{G}}(q_{\mathrm{l}}).} \tag{35}$$

[7] By Lemma 3.1,

$$\boxed{\nexists F : \mathfrak{P} \to \mathfrak{G} : \quad F \circ \pi_{\mathrm{P}} = \pi_{\mathrm{G}}.} \tag{36}$$

3.3 One gravity class, two physical owners

Fix a native anchor a. Let q_{+i} and q_{-i} be the coherent realizations with amplitudes $+i$ and $-i$. The source strength is phase-even,

$$\sigma(+i\Phi_a) = \sigma(-i\Phi_a), \tag{37}$$

and the coherent source identity is

$$\boxed{T(q_{+i}) = T(q_{-i}), \qquad \pi_{\mathrm{G}}(q_{+i}) = \pi_{\mathrm{G}}(q_{-i}).} \tag{38}$$

The coherent states are distinct, and the physical-owner realization is injective on the native image. Therefore

$$\boxed{\pi_{\mathrm{P}}(q_{+i}) \neq \pi_{\mathrm{P}}(q_{-i}).} \tag{39}$$

[8] Gravity-visible data therefore fail to separate these two physical owners:

$$\boxed{p_{+i} \neq p_{-i}, \qquad [T(p_{+i})]_{\mathrm{G}} = [T(p_{-i})]_{\mathrm{G}}.} \tag{40}$$

This is a gravitational *nonseparation* statement: the source/metric observables used here identify two distinct quantum physical owners. It is a statement about observational quotienting, with no additional dynamical irreversibility assumption. Lemma 3.1 gives

$$\boxed{\nexists H : \mathfrak{G} \to \mathfrak{P} : \quad H \circ \pi_{\mathrm{G}} = \pi_{\mathrm{P}}.} \tag{41}$$

Theorem 3.2 (Incomparability of physical and gravity-visible equivalence)**.** *On* $\mathcal{Q}^{\mathrm{real}}_{\kappa,N,\hbar}$,

$$\boxed{\sim_{\mathrm{P}} \nsubseteq \sim_{\mathrm{G}}, \qquad \sim_{\mathrm{G}} \nsubseteq \sim_{\mathrm{P}}\ .} \tag{42}$$

Indeed, $(q_{\mathrm{s}}, q_{\mathrm{l}})$ witnesses the first noninclusion and (q_{+i}, q_{-i}) witnesses the second.[9]

[7] *Formal counterpart:*
`lean/Tier30/Research/QGQuantumGeometryClosure08/Factorization/OneParticleGravityFiber.lean::paper8OneParticle_actual_same_owner_distinct_gravityClass;paper8FiberAnchors_actual_distinct_oneParticleSources`.

[8] *Formal counterpart:*
`lean/Tier30/Research/QGQuantumGeometryClosure08/Factorization/GravityFiber.lean::paper8CoherentSignFlip_actual_same_gravityClass_distinct_owner`.

[9] *Formal counterpart:*
`lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_independent_physical_gravitational_quotients`.

4 Gravity fibers and restricted factorization

Definition 4.1 (Gravity fiber)**.** For $p \in \mathfrak{P}$,

$$\boxed{\mathfrak{G}_p = \{\gamma \in \mathfrak{G} : \exists q \in \mathcal{Q}^{\mathrm{real}},\ \pi_{\mathrm{P}}(q) = p,\ \pi_{\mathrm{G}}(q) = \gamma\}.} \tag{43}$$

Lemma 4.2 (Fibers and functional dependence)**.** *An owner-to-gravity factorization exists on a realized subset $Q_0 \subseteq \mathcal{Q}^{\mathrm{real}}$ exactly when every realized physical-owner fiber has one gravity-visible value:*

$$\boxed{\forall p \in \mathrm{Im}(\pi_{\mathrm{P}}|_{Q_0}), \quad |\mathfrak{G}_p(Q_0)| = 1.} \tag{44}$$

This is Lemma 3.1 written fiberwise.

The one-particle witnesses give

$$\gamma_{\mathrm{s}} := \pi_{\mathrm{G}}(q_{\mathrm{s}}), \qquad \gamma_{\mathrm{l}} := \pi_{\mathrm{G}}(q_{\mathrm{l}}), \tag{45}$$

with

$$\boxed{\gamma_{\mathrm{s}} \neq \gamma_{\mathrm{l}}, \qquad \gamma_{\mathrm{s}}, \gamma_{\mathrm{l}} \in \mathfrak{G}_{\pi_{\mathrm{P}}(q_{\mathrm{s}})}.} \tag{46}$$

Consequently

$$\boxed{|\mathfrak{G}_{\pi_{\mathrm{P}}(q_{\mathrm{s}})}| \geq 2.} \tag{47}$$

[10]

Now restrict to the fixed-anchor coherent realized image

$$\mathcal{Q}_a^{\mathrm{real,coh}} \subset \mathcal{Q}^{\mathrm{real}}, \qquad \mathcal{P}_{a,N,\hbar}^{\mathrm{coh}} := \mathrm{Im}(\pi_{\mathrm{P}}|_{\mathcal{Q}_a^{\mathrm{real,coh}}}). \tag{48}$$

On this restricted realized family there is a map

$$F_{a,N,\hbar} : \mathcal{P}_{a,N,\hbar}^{\mathrm{coh}} \to \mathfrak{G}_a^{\mathrm{coh}} \tag{49}$$

with

$$\boxed{F_{a,N,\hbar}(\pi_{\mathrm{P}}(q)) = \pi_{\mathrm{G}}(q) \qquad (q \in \mathcal{Q}_a^{\mathrm{real,coh}}).} \tag{50}$$

Thus the exact domain comparison is

$$\boxed{\begin{array}{rcl} \mathcal{Q}^{\mathrm{real}} & : & |\mathfrak{G}_{\pi_{\mathrm{P}}(q_{\mathrm{s}})}| \geq 2, \\ \mathcal{Q}_a^{\mathrm{real,coh}} & : & |\mathfrak{G}_p| = 1 \text{ on the realized owner image.} \end{array}} \tag{51}$$

Restriction removes the explicit one-particle witness responsible for the global owner-to-gravity nonfactorization and yields the factorization in Eq. (50).

[10] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_gravity_fibers_and_restricted_factorization`.

5 Vertical and transverse quantum dynamics

Definition 5.1 (Projection-relative vertical and transverse operations)**.** For an operation U on a realized sector and q in its domain,

$$\boxed{\begin{aligned} U \text{ is gravity-vertical at } q &\iff \pi_{\mathrm{G}}(Uq) = \pi_{\mathrm{G}}(q), \\ U \text{ is gravity-transverse at } q &\iff \pi_{\mathrm{G}}(Uq) \neq \pi_{\mathrm{G}}(q). \end{aligned}} \tag{52}$$

These are quotient statements; their definition is projection-theoretic.

The physical-clock result begins one level below the quotient. On the finite-core scalar clock state space the actual native dynamics has a positive-time Bogoliubov-mixing witness,

$$\boxed{\exists t > 0 : \quad \beta_{\mathrm{anom}}(t) \neq 0,} \tag{53}$$

The constructed gravity source is invariant along the same physical clock evolution:

$$\boxed{T_{\mathrm{clock}}(\Psi, t_1) = T_{\mathrm{clock}}(\Psi, t_0) \qquad \text{for all admissible } t_0, t_1.} \tag{54}$$

Thus nontrivial quantum mixing and fixed gravitational source coexist along the physical-clock evolution.[11]

Fix a native anchor a, coupling $\kappa > 0$, and fixed-anchor realized coherent owner p. The induced coherent physical-clock operation transports the quantum state between admissible physical times inside the same realized response branch,

$$U_{t_1 \leftarrow t_0} : q_p(t_0) \longmapsto q_p(t_1), \qquad p,\ a,\ \kappa \text{ fixed}, \tag{55}$$

and the source invariance descends to

$$\boxed{\pi_{\mathrm{G}}(q_p(t_1)) = \pi_{\mathrm{G}}(q_p(t_0)).} \tag{56}$$

The physical clock is therefore gravity-vertical after a genuinely nontrivial quantum evolution has already been established.[12]

The phase operation compares the aligned and phase-rotated fixed-anchor owners,

$$U_{\mathrm{phase}} : q_{\mathrm{align}} \longmapsto q_{\mathrm{phase}}, \qquad a,\ \kappa \text{ fixed}, \tag{57}$$

with

$$\boxed{\pi_{\mathrm{G}}(q_{\mathrm{phase}}) \neq \pi_{\mathrm{G}}(q_{\mathrm{align}}).} \tag{58}$$

[11] *Formal counterpart:* `lean/Tier30/Research/QGPhysicalTimeBackreaction07/Clock/PhysicalStressCarrier.lean::realScalarPhysicalClock_actual_mixing_without_source_motion`.

[12] *Formal counterpart:* `lean/Tier30/Research/QGPhysicalTimeBackreaction07/Dynamics/VerticalTransverse.lean::coherentPhysicalClockPoint_actual_state_transport;coherentPhysicalClock_actual_vertical_on_gravityVisibleQuotient`.

[13] The quotient classification is therefore

$$\boxed{\Delta_{\mathrm{clock}}\pi_{\mathrm{G}} = 0, \qquad \Delta_{\mathrm{phase}}\pi_{\mathrm{G}} \neq 0.} \tag{59}$$

The phase-separated pair remains separated by the terminal physical smooth-Diff relation of Section 10:

$$\boxed{q_{\mathrm{align}} \not\sim_{\mathrm{Diff,phys}} q_{\mathrm{phase}}.} \tag{60}$$

[14]

6 Regulator-independent native physical ownership

Let $\mathfrak{F}(\mathbb{N})$ denote the native multimode bosonic Fock space and $\mathfrak{P}_{\mathrm{native}}$ the regulator-independent physical-owner space. The construction gives an equivalence of underlying types

$$\boxed{\mathfrak{F}(\mathbb{N}) \simeq \mathfrak{P}_{\mathrm{native}}.} \tag{61}$$

Here $\simeq$ denotes an equivalence of underlying types. Linear, unitary, isometric, and topological equivalences require additional structure.[15]

For every Einstein coupling κ and native owner label λ,

$$\mathrm{Realize}_{\kappa,\lambda} : \mathfrak{P}_{\mathrm{native}} \to \mathfrak{P}_{\kappa,\lambda} \tag{62}$$

is injective:

$$\boxed{\mathrm{Realize}_{\kappa,\lambda}(p_1) = \mathrm{Realize}_{\kappa,\lambda}(p_2) \Longrightarrow p_1 = p_2.} \tag{63}$$

[16]

The coherent gravity quotient is already defined without a cutoff index:

$$\boxed{\mathfrak{G}^{\mathrm{RI}}_{a,\kappa} := \mathrm{RegulatorIndependentCoherentGravity}(a, \kappa).} \tag{64}$$

For every cutoff N, the restricted factorization of Section 4 gives

$$F_{a,N,\hbar} : \mathcal{P}^{\mathrm{coh}}_{a,N,\hbar} \to \mathfrak{G}^{\mathrm{RI}}_{a,\kappa}. \tag{65}$$

Define

$$\boxed{\gamma_N := F_{a,N,\hbar}.} \tag{66}$$

[13] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG02to07Freeze.lean::paper8_vertical_transverse_quantum_dynamics`.

[14] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/GaugeRigidityFreeze.lean::paper8_aligned_phase_full_diff_separated`.

[15] *Formal counterpart:* `lean/Tier30/Research/QGPhysicalOwnerSaturation09/Owner/RegulatorIndependentOwner.lean::nativeStateEquivRegulatorIndependentOwner`.

[16] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_regulator_independent_native_physical_ownership`.

For a represented owner $p = \pi_{\mathrm{P}}(q)$ with $q \in \mathcal{Q}_a^{\mathrm{real,coh}}$,

$$\gamma_N(p) = \pi_{\mathrm{G}}(q), \tag{67}$$

with well-definedness supplied by the fixed-anchor owner-to-gravity factorization theorem. Thus for N_1, N_2

$$\boxed{\gamma_{N_1}(p) = \gamma_{N_2}(p) \quad \text{in the common type } \mathfrak{G}_{a,\kappa}^{\mathrm{RI}}.} \tag{68}$$

The equality holds definitionally.[17]

The physical-clock evolution and the coherent stress construction are presently defined on distinct state spaces. Denote them by $\mathcal{C}_{\mathrm{clock}}$ and $\mathcal{C}_{\mathrm{stress}}$. The available constructions are

$$q_p(t) \in \mathcal{C}_{\mathrm{clock}}, \qquad T_p \in \mathcal{C}_{\mathrm{stress}}, \tag{69}$$

An identification of these state spaces requires an additional comparison map:

$$\boxed{\mathcal{C}_{\mathrm{clock}} \simeq \mathcal{C}_{\mathrm{stress}} \quad \text{lies outside the scope established in Paper 8.}} \tag{70}$$

Accordingly, the clock-evolution result is applied on $\mathcal{C}_{\mathrm{clock}}$ and the coherent stress result on $\mathcal{C}_{\mathrm{stress}}$; each statement is used on its stated domain.[18]

7 Completed-Fock stress boundary and coherent realization

Let $\mathcal{F}_{\mathrm{core}}$ be the finite-support algebraic Fock core and

$$\iota : \mathcal{F}_{\mathrm{core}} \hookrightarrow \mathfrak{F}(\mathbb{N}) \tag{71}$$

its canonical inclusion into the completed Fock space. Stress-energy in curved-spacetime QFT is naturally an operator/domain-sensitive object; local-covariant constructions and modern stress-bound results emphasize that boundedness on an entire Hilbert space is a stronger property than existence on a suitable domain.[12, 13] The Paper 8 obstruction concerns exactly that stronger bounded-extension property. For the normalized zero-mode number states,

$$\boxed{\|\iota|n\rangle\| = 1 \qquad (n \in \mathbb{N}).} \tag{72}$$

Fix a positive Bondi mass M, a spacetime point x in the interior radial domain of the spherical source theorem, and the corresponding terminal-coordinate hypothesis. Define

$$\sigma_{11}(x) := T_{\mathrm{nonvac}}(M; x, 1, 1). \tag{73}$$

The source identities give

$$\boxed{\sigma_{11}(x) > 0, \qquad T_{11}^{(n)}(x) = n\,\sigma_{11}(x).} \tag{74}$$

[17] *Formal counterpart:*
`lean/Tier30/Research/QGQuantumGeometryClosure08/Regulator/GravityClassCutoffNaturality.lean::fixedCutoffGravityClass_actual_cutoff_independent`.

[18] *Formal counterpart:*
`lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_coherent_clock_stress_carrier_separation`.

[19]

Suppose a global norm bound existed:

$$|T_{11}^{(n)}(x)| \le B\|\iota|n\rangle\|^2 = B \qquad \forall n. \tag{75}$$

Choose

$$n > \frac{B}{\sigma_{11}(x)}. \tag{76}$$

Then (74) gives

$$T_{11}^{(n)}(x) = n\sigma_{11}(x) > B, \tag{77}$$

contradicting (75). Hence

Theorem 7.1 (Completed-Fock bounded-stress obstruction)**.**

$$\boxed{\nexists B \in \mathbb{R}\ \forall n \in \mathbb{N} : \quad |T_{11}^{(n)}(x)| \le B\|\iota|n\rangle\|^2.} \tag{78}$$

[20]

The completed coherent family supplies an explicit stress-defined domain beyond the finite core. Write $T_{\rm coh}$ for the constructed coherent source. Then

$$\boxed{T_{\rm coh}[\Psi] \text{ is defined for } \Psi \in \mathcal{F}_{\rm coh,comp}.} \tag{79}$$

[21] The unit-imaginary coherent state also satisfies

$$\boxed{\Psi_i \in \mathfrak{F}(\mathbb{N}), \qquad \Psi_i \notin \mathrm{range}(\iota).} \tag{80}$$

Every coherent occupation coefficient of Ψ_i is nonzero, so its occupation support is infinite; every core vector has finite support.[22]

The completed-domain classification is therefore

$$\boxed{\begin{gathered} \mathcal{F}_{\rm core} \subset \mathfrak{F}(\mathbb{N}), \\ T_{11}^{(n)}(x) = n\sigma_{11}(x), \quad \|\iota|n\rangle\| = 1, \\ \nexists \text{ one global norm-bounded stress extension agreeing with all number sources,} \\ \Psi_i \in \mathfrak{F}(\mathbb{N}) \setminus \mathrm{range}(\iota) \text{ admits the constructed coherent stress.} \end{gathered}} \tag{81}$$

[19] *Formal counterpart:* `lean/QGC12/Quantum/RealScalarSphericalNumberSource.lean::realScalarSphericalNumberSource_actual_all_occupation_scaling`; `lean/Tier30/Research/QGPhysicalOwnerSaturation09/Fock/CompletedStressExtension.lean::sphericalNumberSource_actual_unbounded_on_normalized_states`.

[20] *Formal counterpart:* `lean/Tier30/Research/QGPhysicalOwnerSaturation09/Fock/CompletedStressExtension.lean::no_normBounded_completedFockStress_agrees_with_number_sources`.

[21] *Formal counterpart:* `lean/Tier30/Research/QGPhysicalOwnerSaturation09/Fock/StateDomainClassification.lean::coherentCompletedFamily_actual_full_realized_row`.

[22] *Formal counterpart:* `lean/Tier30/Research/QGPhysicalOwnerSaturation09/Fock/CompletedStressExtension.lean::unitImaginaryCoherentState_actual_not_finiteCore`.

8 Positive finite-number nonlinear Einstein realization

The finite-number realization is a separate realized-sector structure. For $n \in \mathbb{N}$, define the native number state and physical owner by

$$\Psi_n := \mathrm{finiteNumberState}(n), \qquad p_n := \mathrm{finiteNumberPhysicalOwner}_{\kappa,N,\hbar}(n). \tag{82}$$

The owner map is injective in n:

$$p_m = p_n \Longrightarrow m = n. \tag{83}$$

[23]

For $n > 0$, the nonlinear realization is

$$\boxed{\mathcal{E}_n = (a_n, \Psi_n, p_n, T_n, g_n;\ h_{\mathrm{E}}, h_{\mathrm{cons}}),} \tag{84}$$

where

$$h_{\mathrm{E}} : G[g_n] = \kappa T_n, \qquad h_{\mathrm{cons}} : \nabla^{g_n} \cdot T_n = 0. \tag{85}$$

Thus

$$\boxed{G_{ij}[g_n](x) = \kappa T_{n,ij}(x), \qquad \nabla^{g_n}_{\mu} T_n{}^{\mu}{}_{\nu}(x) = 0.} \tag{86}$$

[24]

The changed-metric source has the exact occupation scaling

$$T_n = n\, T^{(1)}_{\kappa n,a} \tag{87}$$

and, on the positive-coupling branch used in this realization,

$$\boxed{\kappa > 0, \qquad n > 0, \qquad \sigma(\Phi_a) > 0, \qquad S_n = \frac{\kappa n}{2}\, \sigma(\Phi_a) > 0.} \tag{88}$$

[25] The universal realization theorem is

$$\boxed{\forall n \in \mathbb{N}, \quad n > 0 \Longrightarrow \mathrm{Nonempty}(\mathcal{E}_n).} \tag{89}$$

[26]

The zero-occupation background source is exactly

$$\boxed{T_0^{\mathrm{bg}} = 0.} \tag{90}$$

[23] *Formal counterpart:*
`lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/FiniteNumber/StateTower.lean::finiteNumberPhysicalOwner_actual_injective`.

[24] *Formal counterpart:*
`lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/FiniteNumber/EinsteinRealization.lean::FiniteNumberEinsteinRealization`.

[25] *Formal counterpart:*
`lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/FiniteNumber/CovariantBranch.lean::finiteNumberCovariantSource_actual_scaling;finiteNumberBranch_actual_source_parameter`.

[26] *Formal counterpart:*
`lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_all_positive_finite_number_nonlinear_einstein_realization`.

The nonlinear branch data used in Paper 8 require $S > 0$. Equation (88) places $n = 0$ at the zero-source boundary, so the positive-source nonlinear finite-number family is indexed by $n > 0$.[27]

For each $n > 0$, define the source fiber

$$\mathcal{F}_n^{\mathrm{G}} := \mathrm{finiteNumberGravitySourceFiber}(n). \tag{91}$$

Then

$$\boxed{\forall n > 0 \quad \exists T_{n,1}, T_{n,2} \in \mathcal{F}_n^{\mathrm{G}} : \quad T_{n,1} \neq T_{n,2}.} \tag{92}$$

[28]

9 Number/coherent gravitational degeneracy

The number/coherent comparison uses its own two-point realized state space, so every projection below has an explicit common domain. Fix $a, \kappa, N, \hbar$ and $n > 0$, and define

$$\boxed{\mathcal{M}_n := \{\mathbf{n}, \mathbf{c}\},} \tag{93}$$

where $\mathbf{n}$ denotes the number realization and $\mathbf{c}$ the matching coherent realization. Define

$$\Pi_P^{(n)} : \mathcal{M}_n \to \mathfrak{P}_{\kappa,N,\hbar}, \qquad \Pi_G^{(n)} : \mathcal{M}_n \to \mathfrak{G}_n^{\mathrm{match}}, \tag{94}$$

using the constructed physical owners and equality of the matched source fields.[29]

Let $\Phi_a \neq 0$ be the native source flux and

$$\sigma(\Phi_a) := 2\|\Phi_a\|_{\mathbb{C}}^2. \tag{95}$$

Define

$$L_n(a) := \sqrt{n\,\sigma(\Phi_a)}, \qquad \boxed{\alpha_n(a) := \frac{\frac{1}{2}L_n(a)}{\Phi_a}.} \tag{96}$$

Then $\alpha_n \neq 0$ and

$$\sigma\big(\Phi_{\mathrm{coh}}(a, \alpha_n)\big) = 2n\,\sigma(\Phi_a), \qquad \kappa_{\mathrm{aux}}(a, \alpha_n; \kappa) = \kappa n. \tag{97}$$

Consequently the two realizations have the same nonlinear branch and the same metric:

$$\boxed{g_{\mathbf{n}} = g_{\mathbf{c}}.} \tag{98}$$

[30]

[27] *Formal counterpart:* `lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/FiniteNumber/SourceClassification.lean::finiteNumberBackgroundSource_actual_vacuum`.

[28] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_all_positive_finite_number_fibers_nontrivial`.

[29] *Formal counterpart:* `lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/FiniteNumber/NumberCoherentVisibility.lean::MatchingNumberCoherentCarrier;matchingNumberCoherentOwner;matchingNumberCoherentGravityProjection`.

[30] *Formal counterpart:* `lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/FiniteNumber/NumberCoherentVisibility.lean::numberMatchingCoherentDatum_actual_same_branch`.

Their Einstein equations are

$$G[g_{\mathbf{n}}] = \kappa T_{\mathbf{n}}, \qquad G[g_{\mathbf{c}}] = \kappa T_{\mathbf{c}}. \tag{99}$$

Using (98) and $\kappa \neq 0$,

$$\kappa T_{\mathbf{n}} = G[g_{\mathbf{n}}] = G[g_{\mathbf{c}}] = \kappa T_{\mathbf{c}} \Longrightarrow \boxed{T_{\mathbf{n}} = T_{\mathbf{c}}.} \tag{100}$$

Thus

$$\boxed{\Pi_G^{(n)}(\mathbf{n}) = \Pi_G^{(n)}(\mathbf{c}).} \tag{101}$$

The physical owners remain distinct. The number state is a finite-core vector. Since $\alpha_n \neq 0$, the coherent state has nonzero coefficients on every occupation and lies outside the finite-core image. Hence

$$\Psi_n \neq \Psi_{\alpha_n}, \tag{102}$$

and injectivity of the owner realization gives

$$\boxed{\Pi_P^{(n)}(\mathbf{n}) \neq \Pi_P^{(n)}(\mathbf{c}).} \tag{103}$$

Theorem 9.1 (Number/coherent gravitational degeneracy)**.** *For every $n > 0$,*

$$\boxed{\Pi_G^{(n)}(\mathbf{n}) = \Pi_G^{(n)}(\mathbf{c}), \qquad \Pi_P^{(n)}(\mathbf{n}) \neq \Pi_P^{(n)}(\mathbf{c}).} \tag{104}$$

[31] This theorem realizes the gravity-to-owner nonfactorization mechanism on the explicit matched finite-number/coherent state space.

10 Intrinsic signatures and physical-Diff gravity rigidity

Invariant characterization of Lorentzian geometry by scalar curvature data has a broad literature, including general $\mathcal{I}$-non-degeneracy results and explicit black-hole parameter recovery from curvature invariants.[19, 20] The present theorem is specialized to the realized nonlinear branch family and uses the complete unparameterized image of the pair (R, K) to prove injectivity of the source parameter.

Sections 2–9 define gravity visibility through canonical source equality. The terminal Paper 8 result recovers that source parameter from smooth scalar invariants and identifies the resulting quotient with the represented physical smooth-Diff quotient on one precise realized family.

10.1 Fixed-(h, a) branch domain and realized formulas

Let

$$d = (h, a, S), \qquad 0 < a < h, \qquad S > 0, \tag{105}$$

with

$$\delta := h - a > 0, \qquad C := h + \frac{S}{\delta}, \qquad D := C^2 - 4S > 0. \tag{106}$$

[31] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_number_coherent_gravitational_degeneracy`.

The comparison domain fixes both horizon and anchor:

$$\boxed{\mathrm{FixedHA}(d_1, d_2) \iff h_1 = h_2 \ \wedge \ a_1 = a_2.} \tag{107}$$

This condition fixes the common horizon and anchor used throughout the rigidity argument.[32]

The transport roots are

$$Y_- := \frac{-C - \sqrt{D}}{2}, \qquad Y_+ := \frac{-C + \sqrt{D}}{2}, \tag{108}$$

with

$$\boxed{Y_- < Y_+ < 0.} \tag{109}$$

The realized transport takes values in

$$Y(z) \in I_d := (Y_-, Y_+) \subset (-\infty, 0). \tag{110}$$

Define

$$P_d(Y) := Y^2 + CY + S = (Y - Y_-)(Y - Y_+). \tag{111}$$

On I_d, $P_d(Y) < 0$.

The logarithmic quadrature is

$$Q_d(Y) = \frac{Y_-}{Y_- - Y_+} \log(Y - Y_-) - \frac{Y_+}{Y_- - Y_+} \log(Y_+ - Y), \tag{112}$$

and is strictly increasing on I_d:

$$\boxed{Y_1 < Y_2 \Longrightarrow Q_d(Y_1) < Q_d(Y_2).} \tag{113}$$

The anchored shift is

$$q_d := -\log a + Q_d(-\delta), \tag{114}$$

so the realized inverse relation is

$$z = Q_d(Y) - q_d, \qquad Y = Y_d(z). \tag{115}$$

[33]

The scalar curvature profile is

$$\boxed{R_d(z) = \frac{2S}{P_d(Y_d(z))\, e^{2z}},} \tag{116}$$

and the curvature-gradient invariant used in the joint signature is

$$\boxed{K_d(z) = \frac{R_d(z)^3\, (4Y_d(z) + C)^2}{2S}.} \tag{117}$$

Hence

$$\Sigma_d(z) := (R_d(z), K_d(z)), \qquad \mathrm{Im}\, \Sigma_d := \{\Sigma_d(z) : z \in \mathbb{R}\}. \tag{118}$$

[32] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Signature/SourceDuality.lean::SameAnchorBranchData`.

[33] *Formal counterpart:* `lean/QGC12/Geometry/NonlinearFlux/Parameters.lean;lean/QGC12/Geometry/NonlinearFlux/Quadrature.lean;lean/QGC12/Geometry/NonlinearFlux/GlobalTransport.lean`.

[34] The set $\operatorname{Im}\Sigma_d$ records the correlation of the two scalar invariants as an unparameterized range, with the radial coordinate z removed. A smooth diffeomorphism acts by pullback reparameterization and preserves this joint range. The complete image therefore provides the intrinsic signature used below for comparison modulo the represented diffeomorphism action.

10.2 Step 1: normalized shape recovers χ

Define

$$A(R, K) := \frac{2K}{R^3}. \tag{119}$$

Equations (116)–(117) give directly

$$\boxed{A(\Sigma_d(z)) = \frac{(4Y_d(z) + C)^2}{S}.} \tag{120}$$

Define

$$\chi(d) := \frac{C^2}{S} > 4. \tag{121}$$

The strict image ceiling is

$$\boxed{A_{\max}(d) = \frac{(C + 2\sqrt{D})^2}{S}.} \tag{122}$$

Every realized point satisfies $A < A_{\max}$, and the lower-root limit attains $A_{\max}$ as the least upper bound:

$$\boxed{\operatorname{IsLUB}\big(A(\operatorname{Im}\Sigma_d), A_{\max}(d)\big).} \tag{123}$$

[35]

Set

$$\widehat{C} := \frac{C}{\sqrt{S}}, \qquad \widehat{D} := \frac{\sqrt{D}}{\sqrt{S}}. \tag{124}$$

Then

$$\widehat{C}^2 = \chi, \qquad \widehat{D}^2 = \chi - 4, \qquad A_{\max} = (\widehat{C} + 2\widehat{D})^2. \tag{125}$$

Positivity gives strict monotonicity:

$$\chi_1 < \chi_2 \Longrightarrow A_{\max}(d_1) < A_{\max}(d_2). \tag{126}$$

Therefore

$$\boxed{\operatorname{Im}\Sigma_{d_1} = \operatorname{Im}\Sigma_{d_2} \Longrightarrow \chi(d_1) = \chi(d_2).} \tag{127}$$

[36]

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[34] *Formal counterpart:* `lean/Tier30/Research/QGRealizedGaugeClosure14/Realized/ProfileDerivative.lean::scalarCurvatureRadialProfile_actual_algebraic;lean/Tier30/Research/QGRealizedGaugeClosure14/Realized/JointInvariantInjectivity.lean::curvatureGradientSignatureProfile`.

[35] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Signature/ShapeRange.lean::normalizedShapeCeiling_actual_isLUB`.

[36] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Signature/ShapeParameterRecovery.lean::jointSignatureImage_eq_implies_shapeParameter_eq`.

10.3 Step 2: fixed-(h, a) equality of χ gives one explicit dual

On the fixed-(h, a) family,

$$\chi(S) = \frac{(h + S/\delta)^2}{S}. \tag{128}$$

Thus

$$\chi(S_1) = \chi(S_2) \iff (S_1 - S_2)(S_1 S_2 - h^2\delta^2) = 0, \tag{129}$$

and hence

$$\boxed{\chi(S_1) = \chi(S_2) \iff S_1 = S_2 \ \vee \ S_1 S_2 = h^2\delta^2.} \tag{130}$$

[37]

Define

$$\boxed{S^\vee := \frac{h^2\delta^2}{S}.} \tag{131}$$

Then

$$(S^\vee)^\vee = S, \qquad \chi(S^\vee) = \chi(S), \qquad S^\vee = S \iff S = h\delta. \tag{132}$$

The normalized shape therefore has one exact nontrivial algebraic dual candidate.

10.4 Step 3: the physical midpoint is intrinsic

At the shape level $A = \chi$,

$$\frac{(4Y + C)^2}{S} = \frac{C^2}{S} \iff (4Y + C)^2 = C^2. \tag{133}$$

Thus

$$Y = -\frac{C}{2} \quad \vee \quad Y = 0. \tag{134}$$

Equation (109) gives $Y < 0$ throughout the realized branch, so $Y = 0 \notin I_d$. Therefore

$$\boxed{A(\Sigma_d(z)) = \chi(d) \iff Y_d(z) = -\frac{C}{2}.} \tag{135}$$

[38]

The quadrature is strictly increasing on I_d, and the global transport satisfies

$$Q_d(Y_d(z)) = z + q_d. \tag{136}$$

Hence

$$Y_d(z_1) = Y_d(z_2) \Longrightarrow Q_d(Y_d(z_1)) = Q_d(Y_d(z_2)) \Longrightarrow \boxed{z_1 = z_2}. \tag{137}$$

[37] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Signature/SourceDuality.lean::sameAnchor_shapeParameter_eq_iff_source_or_dual`.

[38] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Rigidity/DualityResolution.lean::normalizedShapeProfile_eq_parameter_iff_midpoint`.

Thus $Y = -C/2$ determines one logarithmic radius and one point of the joint signature image at the level $A = \chi$.

Let

$$Y_{\mathrm{m}} := -\frac{C}{2}, \qquad z_{\mathrm{m}} := Q_d(Y_{\mathrm{m}}) - q_d. \tag{138}$$

At this point

$$R_d(z_{\mathrm{m}}) = \frac{B(\chi)}{e^{2z_{\mathrm{m}}}}, \qquad B(\chi) := \frac{2}{1-\chi/4} \neq 0. \tag{139}$$

Equal complete joint images give equal χ by Step 1 and identify the unique $A = \chi$ signature point. Equality of its curvature coordinate yields

$$e^{2z_{\mathrm{m},1}} = e^{2z_{\mathrm{m},2}} \Longrightarrow \boxed{z_{\mathrm{m},1} = z_{\mathrm{m},2}.} \tag{140}$$

10.5 Step 4: dual quadrature scaling forces the fixed point

Define

$$\rho := \frac{h\delta}{S} > 0. \tag{141}$$

Then

$$S^{\vee} = \rho^2 S, \qquad C^{\vee} = \rho C, \qquad D^{\vee} = \rho^2 D, \qquad Y_{\pm}^{\vee} = \rho Y_{\pm}, \qquad Y_{\mathrm{m}}^{\vee} = \rho Y_{\mathrm{m}}. \tag{142}$$

The exact quadrature transformation is

$$\boxed{Q_{d^{\vee}}(\rho Y) = Q_d(Y) + \log \rho.} \tag{143}$$

[39]

The common anchor is represented by the transport value $-\delta$. Its polynomial value is

$$P_d(-\delta) = \delta^2 - C\delta + S = -a\delta < 0. \tag{144}$$

Since $P_d(Y) < 0$ exactly on the branch interval, $-\delta \in I_d$. The same calculation for $d^{\vee}$, with the same (h, a), gives

$$-\delta \in I_{d^{\vee}}. \tag{145}$$

The root scaling in (142) gives

$$I_{d^{\vee}} = \rho I_d, \qquad -\rho\delta \in I_{d^{\vee}}. \tag{146}$$

Equality of the midpoint log radii from Step 3 and the transformation (143) give

$$Q_{d^{\vee}}(-\delta) = Q_{d^{\vee}}(-\rho\delta). \tag{147}$$

Both arguments therefore lie in the strict-monotonicity domain of $Q_{d^{\vee}}$. Equation (113) yields

$$-\delta = -\rho\delta. \tag{148}$$

Since $\delta > 0$,

$$\boxed{\rho = 1, \qquad S^{\vee} = S.} \tag{149}$$

[40]

[39] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Rigidity/DualityResolution.lean::dual_transportQuadrature_actual_scale`.

[40] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Rigidity/DualityResolution.lean::dual_jointSignatureImage_eq_implies_fixed`.

10.6 Step 5: intrinsic source rigidity and physical smooth-Diff classification

Steps 1–4 give the principal source theorem.

Theorem 10.1 (Fixed-(h, a) intrinsic source rigidity). *For realized branch data satisfying*

$$h_1 = h_2, \qquad a_1 = a_2, \qquad S_1, S_2 > 0, \tag{150}$$

one has

$$\boxed{\operatorname{Im} \Sigma_{d_1} = \operatorname{Im} \Sigma_{d_2} \Longrightarrow S_1 = S_2.} \tag{151}$$

[41] Source equality at fixed (h, a) gives equality of branch data and realized metrics.

Let $\mathrm{PhysProj}(g)$ remove local Lorentz-frame decoration. Physical smooth-Diff equivalence is

$$g_1 \sim_{\mathrm{Diff,phys}} g_2 \Longleftrightarrow \exists \varphi \in \mathrm{Diff}(M) : \mathrm{PhysProj}(\varphi^* g_2) = \mathrm{PhysProj}(g_1). \tag{152}$$

Equality after local Lorentz-frame descent is faithful at the metric-coefficient level:

$$\mathrm{PhysProj}(g') = \mathrm{PhysProj}(g) \Longrightarrow g'.\,\mathrm{coeff} = g.\,\mathrm{coeff}\,. \tag{153}$$

Scalar curvature and the curvature-gradient norm are determined by those coefficients, hence

$$\mathrm{PhysProj}(g') = \mathrm{PhysProj}(g) \Longrightarrow R[g'] = R[g], \qquad K[g'] = K[g]. \tag{154}$$

For a physical-Diff witness φ, apply this implication to $g' = \varphi^* g_2$ and $g = g_1$. Naturality of the two scalar invariants then gives

$$R[g_1] = R[g_2] \circ \varphi, \qquad K[g_1] = K[g_2] \circ \varphi. \tag{155}$$

Since $\varphi : M \to M$ is bijective,

$$\boxed{\operatorname{Im}(R[g_1], K[g_1]) = \operatorname{Im}(R[g_2], K[g_2]).} \tag{156}$$

[42] Theorem 10.1 therefore yields the forward implication

$$g_{d_1} \sim_{\mathrm{Diff,phys}} g_{d_2} \Longrightarrow S_1 = S_2. \tag{157}$$

The realized gauge construction supplies the converse, giving

$$\boxed{g_{d_1} \sim_{\mathrm{Diff,phys}} g_{d_2} \Longleftrightarrow S_1 = S_2.} \tag{158}$$

[43]

[41] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Rigidity/SourceRecovery.lean::sameAnchor_jointImage_eq_implies_source_eq`.

[42] *Formal counterpart:* `lean/Tier30/Research/QGRealizedGaugeClosure14/Frame/LorentzFrameGauge.lean::physicalMetricProjection_actual_faithful_coefficients;lean/Tier30/Research/QGGravityClassRigidity15/Rigidity/RealizedGaugeRigidity.lean::physicalMetricDiffEquivalent_implies_jointScalarImage_eq`.

[43] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/Rigidity/RealizedGaugeRigidity.lean::sameAnchor_physicalMetricDiffEquivalent_iff_source_eq`.

For fixed-anchor realized coherent owners p_1, p_2 at common horizon and anchor, write

$$\Gamma_{a,\kappa}(p) := \mathrm{gravityVisibleProjection}(p) \in \mathfrak{G}^{\mathrm{visible}}_{a,\kappa}. \tag{159}$$

The projections satisfy

$$\boxed{\begin{aligned} \mathrm{PhysGauge}(p_1) = \mathrm{PhysGauge}(p_2) &\iff \Gamma_{a,\kappa}(p_1) = \Gamma_{a,\kappa}(p_2) \\ &\iff S(p_1) = S(p_2) \\ &\iff p_1 \sim_{\mathrm{Diff,phys}} p_2. \end{aligned}} \tag{160}$$

[44] Thus

Corollary 10.2 (Gravity-visible/physical-Diff quotient equivalence)**.** *On the fixed-(h, a) realized coherent family,*

$$\boxed{\mathfrak{G}^{\mathrm{visible}}_{a,\kappa} \simeq \mathfrak{G}^{\mathrm{Diff,phys}}_{a,\kappa}.} \tag{161}$$

[45]

For the positive finite-number family at fixed horizon and anchor,

$$\boxed{n_1 \neq n_2, \quad n_1, n_2 > 0 \Longrightarrow g_{n_1} \not\sim_{\mathrm{Diff,phys}} g_{n_2}.} \tag{162}$$

[46]

11 Matter-parametric relational architecture

The preceding constructions admit a state-space-independent relational interface. Define

$$\boxed{\mathfrak{R} = (\mathcal{S}, \mathcal{P}, \mathcal{T}, \mathcal{G}, \sim_{\mathcal{T}}, \mathrm{owner}, \mathrm{sourceRel}, \mathrm{geomResp}),} \tag{163}$$

where $\sim_{\mathcal{T}}$ is a setoid on $\mathcal{T}$,

$$\mathrm{owner} : \mathcal{S} \to \mathcal{P}, \qquad \mathrm{sourceRel} : \mathcal{S} \to \mathcal{T} \to \mathrm{Prop}, \qquad \mathrm{geomResp} : \mathcal{T} \to \mathcal{G} \to \mathrm{Prop}\,. \tag{164}$$

[47] The source and geometry components are relations. The interface therefore permits multiple realized sources for one state and multiple geometry responses for one source until additional hypotheses establish functional dependence.

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[44] *Formal counterpart:* `lean/Tier30/Research/QGGravityClassRigidity15/PhysicalGauge/StrengthFaithfulness.lean::fixedAnchor_fullGaugeStrengthFaithfulness`.

[45] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/GaugeRigidityFreeze.lean::paper8_intrinsic_physical_diff_gauge_classification`.

[46] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/GaugeRigidityFreeze.lean::paper8_finite_number_full_diff_separated`.

[47] *Formal counterpart:* `lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/Matter/RelationalInterface.lean::RelationalMatterGravityData`.

A realized sector is

$$q = (s, T, g; h_T, h_g), \tag{165}$$

with

$$h_T : \mathrm{sourceRel}(s, T), \qquad h_g : \mathrm{geomResp}(T, g). \tag{166}$$

The projections are

$$\pi_{\mathrm{P}}(q) = \mathrm{owner}(s), \qquad \pi_{\mathrm{G}}(q) = [T]_{\sim_{\mathcal{T}}}, \qquad \mathfrak{G}(\mathfrak{R}) = \mathcal{T}/\sim_{\mathcal{T}}. \tag{167}$$

The generic fiber is

$$\mathfrak{G}_p(\mathfrak{R}) = \{\gamma : \exists q,\ \pi_{\mathrm{P}}(q) = p,\ \pi_{\mathrm{G}}(q) = \gamma\}. \tag{168}$$

Lemma 3.1 is recovered abstractly as

$$\pi_{\mathrm{P}}(q_1) = \pi_{\mathrm{P}}(q_2),\ \pi_{\mathrm{G}}(q_1) \neq \pi_{\mathrm{G}}(q_2) \Longrightarrow \neg\,\mathrm{OwnerToGravity}(\mathfrak{R}), \tag{169}$$

$$\pi_{\mathrm{G}}(q_1) = \pi_{\mathrm{G}}(q_2),\ \pi_{\mathrm{P}}(q_1) \neq \pi_{\mathrm{P}}(q_2) \Longrightarrow \neg\,\mathrm{GravityToOwner}(\mathfrak{R}). \tag{170}$$

[48] Likewise

$$\begin{aligned} \mathrm{GravityVerticalAt}(U, q) &\Longleftrightarrow \pi_{\mathrm{G}}(Uq) = \pi_{\mathrm{G}}(q), \\ \mathrm{GravityTransverseAt}(U, q) &\Longleftrightarrow \pi_{\mathrm{G}}(Uq) \neq \pi_{\mathrm{G}}(q). \end{aligned} \tag{171}$$

The real-scalar instance is

$$\boxed{\begin{aligned} \mathcal{S} &= \mathcal{Q}^{\mathrm{real}}_{\kappa,N,\hbar}, \\ \mathcal{P} &= \mathfrak{P}_{\kappa,N,\hbar}, \\ \mathcal{T} &= \{\text{canonical spacetime tensor fields}\}, \\ \mathcal{G} &= \{\text{canonical smooth Lorentz metrics}\}, \\ T_1 \sim_{\mathcal{T}} T_2 &\Longleftrightarrow T_1 = T_2, \\ \mathrm{owner}(q) &= \pi_{\mathrm{P}}(q), \\ \mathrm{sourceRel}(q, T) &\Longleftrightarrow T = T(q), \\ \mathrm{geomResp}(T, g) &\Longleftrightarrow G[g] = \kappa T. \end{aligned}} \tag{172}$$

[49] The Einstein–real-scalar model studied in Paper 8 is this instance. The interface itself is matter-parametric at the level of state, owner, source relation, source quotient, geometry response, fiber, factorization, and projection-relative dynamics.

The resulting abstract relational data are inhabited:

$$\boxed{\mathrm{Nonempty}(\mathrm{RelationalMatterGravityData}).} \tag{173}$$

[50]

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[48] *Formal counterpart:*
`lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/Matter/GenericTheorems.lean`.

[49] *Formal counterpart:*
`lean/Tier30/Research/QGFiniteNumberMatterGeneralization11/Matter/RealScalarInstantiation.lean::qgc12RealScalarMatterGravityData`.

[50] *Formal counterpart:*
`lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_matter_parametric_relational_schema`.

12 Relational quantum-gravity capstone

The body of Paper 8 is summarized by the compact main theorem.

Theorem 12.1 (Relational quantum-gravity capstone)**.** *For the stated realized domains,*

$$\boxed{\begin{array}{l} \mathcal{Q}^{\mathrm{real}}_{\kappa,N,\hbar} \neq \varnothing, \\ \nexists F : \mathfrak{P} \to \mathfrak{G} : \quad F \circ \pi_{\mathrm{P}} = \pi_{\mathrm{G}}, \\ \nexists H : \mathfrak{G} \to \mathfrak{P} : \quad H \circ \pi_{\mathrm{G}} = \pi_{\mathrm{P}}, \\ \mathfrak{G}^{\mathrm{visible}}_{a,\kappa} \simeq \mathfrak{G}^{\mathrm{Diff,phys}}_{a,\kappa}. \end{array}} \tag{174}$$

[51] The detailed machine-checked theorem inventory is given in Appendix B.

The mathematical synthesis is

$$\boxed{\mathcal{Q}^{\mathrm{real}} \xrightarrow{(\pi_{\mathrm{P}},\pi_{\mathrm{G}})} \mathfrak{P} \times \mathfrak{G}, \qquad G[g(q)] = \kappa T(q), \qquad \sim_{\mathrm{P}} \nsubseteq \sim_{\mathrm{G}}, \qquad \sim_{\mathrm{G}} \nsubseteq \sim_{\mathrm{P}},} \tag{175}$$

followed, on the fixed-(h, a) realized family, by

$$\boxed{\operatorname{Im}\Sigma \longrightarrow \chi \longrightarrow S \longrightarrow \mathfrak{G}^{\mathrm{visible}}_{a,\kappa} \simeq \mathfrak{G}^{\mathrm{Diff,phys}}_{a,\kappa}.} \tag{176}$$

The first line exposes the independent relational information carried by physical ownership and gravitational visibility. The second line establishes the intrinsic geometric rigidity of the gravity-visible quotient on the stated fixed-(h, a) domain.

The smooth-Diff action used here acts on the metric quotient. A corresponding arbitrary smooth-Diff action on the quantum state space is a separate extension:

$$\boxed{\mathrm{SmoothDiffStateActionScope} = \mathrm{notClaimed}\,.} \tag{177}$$

The charged-matter realization and the later W5 continuum analysis belong to subsequent extensions. Appendix C records the exact scope and status of the statements used in Paper 8.

A Negative-result certificate

The formal development records five logically distinct negative results.

Legacy global-QME entry.

$$\boxed{\forall \kappa : \quad \mathrm{IsEmpty}(\mathcal{E}_{\mathrm{QME}}(\kappa)).} \tag{178}$$

Owner-to-gravity factorization.

$$\boxed{\nexists F : \mathfrak{P} \to \mathfrak{G} : \quad \forall q \in \mathcal{Q}^{\mathrm{real}},\ F(\pi_{\mathrm{P}}(q)) = \pi_{\mathrm{G}}(q).} \tag{179}$$

[51] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Capstone/PublicationCertificate.lean::paper8_relational_quantum_gravity_main`.

Gravity-to-owner factorization.

$$\boxed{\nexists H : \mathfrak{G} \to \mathfrak{P} : \quad \forall q \in \mathcal{Q}^{\mathrm{real}},\ H(\pi_{\mathrm{G}}(q)) = \pi_{\mathrm{P}}(q).} \tag{180}$$

Completed-Fock bounded stress.

$$\boxed{\nexists B \in \mathbb{R}\ \forall n \in \mathbb{N} : \quad |T_{11}^{(n)}(x)| \le B\|\iota|n\rangle\|^2.} \tag{181}$$

RAC field channel. For the declared second-channel field encoding,

$$\boxed{\mathrm{IsEmpty}(\mathrm{RACSecondChannelFieldEncoding}(\kappa)).} \tag{182}$$

[52]

The smooth-Diff action on the quantum state carrier has the separate status (177) and is recorded as a future extension outside the negative theorem certificate.

B Reader theorem map

ID	Section	Mathematical statement	Lean theorem
P8-01	Paper-7 handoff	The legacy global-QME entry is empty; the relational entry is inhabited and has a nonzero owner.	`paper8_paper7_handoff`
P8-02	Realized state space	Every realized sector satisfies $G[g_q] = \kappa T_q$.	`paper8_realized_quantum_geometric_carrier`
P8-03	Independent quotients	No global owner-to-gravity or gravity-to-owner factorization exists on the mixed realized state space.	`paper8_independent_physical_gravitational_quotients`
P8-04	Fibers	A one-particle owner has a nontrivial gravity fiber; the fixed-anchor coherent image has an owner-to-gravity factorization.	`paper8_gravity_fibers_and_restricted_factorization`
P8-05	Dynamics	Physical-clock transport is gravity-vertical and native phase motion is gravity-transverse.	`paper8_vertical_transverse_quantum_dynamics`
P8-06	Native ownership	Native Fock space is equivalent to the regulator-independent owner and injects into fixed-regulator owner realizations.	`paper8_regulator_independent_native_physical_ownership`
P8-07	Completed Fock	No global norm-bounded extension reproduces all number-state sources; an infinite-support coherent family admits a constructed stress tensor.	`paper8_completed_fock_stress_obstruction_and_coherent_exception`
P8-08	Finite numbers	Every positive occupation admits an exact conserved nonlinear Einstein realization.	`paper8_all_positive_finite_number_nonlinear_einstein_realization`
P8-09	Number/coherent	Matching number and coherent owners have one gravity class and distinct physical owners.	`paper8_number_coherent_gravitational_degeneracy`
P8-10	Gauge rigidity	Gravity visibility, source strength, and physical-Diff visibility are equivalent on the fixed-anchor realized family.	`paper8_intrinsic_physical_diff_gauge_classification`
P8-11	Parametric schema	The relational owner/source/geometry architecture is matter-parametric and has an Einstein–real-scalar instance.	`paper8_matter_parametric_relational_schema`

[52] *Formal counterpart:* `lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/Authority/QGRG08to11Freeze.lean::paper8_current_RAC_field_channel_obstruction`.

ID	Section	Mathematical statement	Lean theorem
P8-12	Main theorem	The main theorem assembles the complete relational quantum-gravity result.	`paper8_relational_quantum_gravity_capstone`

C Scope and statement status

The mathematical scope is partitioned by the exact domains below.

ID	Domain	Claim boundary
S01	Global QME/BV	Legacy entry obstruction only.
S02	Restricted physical cohomology	Relational replacement entry.
S03	Native Fock image	Equivalent to the fixed-regulator native-owner image.
S04	Completed native Fock ownership	Ownership exists; global norm-bounded stress extension is obstructed.
S05	Finite-core source	Theorem-owned stress domain.
S06	Finite-number source	Every positive occupation is realized.
S07	Coherent completed-Fock source	Infinite-support coherent realized family.
S08	Fixed-anchor coherent family	Source-strength and physical-Diff classification.
S09	Same-anchor realized branch data	Complete joint invariant image is source-rigid.
S10	Full represented smooth Diff	Metric pullback action represented in the repository.
S11	Physical metric modulo local Lorentz frame	Physical-Diff quotient target.
S12	Matter-parametric abstract interface	Carrier-independent theorem schema.
S13	Real-scalar physical instance	Physical matter realization used by this paper.

The corresponding status partition is

$$\begin{array}{|c|c|}\hline \text{statement} & \text{status} \\ \hline \text{global QME entry} & \text{proved obstruction} \\ \text{relational entry} & \text{proved} \\ \text{owner} \to \text{gravity factorization} & \text{proved obstruction} \\ \text{gravity} \to \text{owner factorization} & \text{proved obstruction} \\ \text{gravity fibers} & \text{proved at restricted scope} \\ \text{clock/phase dynamics} & \text{proved at restricted scope} \\ \text{cross-cutoff ownership} & \text{proved at restricted scope} \\ \text{completed-Fock ownership} & \text{proved} \\ \text{completed-Fock bounded stress} & \text{proved obstruction} \\ \text{finite-number nonlinear response} & \text{proved} \\ \text{physical-Diff gravity classification} & \text{proved at restricted scope} \\ \text{matter-parametric theorem schema} & \text{constructed abstract schema} \\ \text{charged realization} & \text{excluded from Paper 8} \\ \text{W5 continuum obstruction} & \text{excluded from Paper 8} \\ \text{arbitrary smooth-Diff state action} & \text{future extension.} \\ \hline \end{array} \tag{183}$$

D Formal verification and theorem concordance

The formal verification materials for Paper 8 are collected in

`lean/Tier30/Publication/QGRelationalQuantumGravityPaper8/`

with the synchronized machine-readable theorem concordance under

`research/paper8_authority/`.

The theorem manifest contains twelve body entries P8-01–P8-12 together with the compact main theorem, the Paper 7 bridge, the negative-results certificate, the finite-number physical-Diff corollary, the cutoff/state-space boundary statements, and the final verification certificate.

The checked dependency set includes the Paper 7 handoff, the QGRG-02–07 realized dynamics, the QGRG-08–11 state-space/owner/finite-number results, and the QGRG-15 intrinsic gauge-rigidity results. The charged QGRG-12 realization, W5 continuum obstruction, prototype modules, and later cross-matter program are outside the checked Paper 8 dependency set.

Verification uses targeted direct Lean compilation against the shared repository cache. The verification script compiles the formal statement modules, five theorem-map modules, main theorem, verification certificate, theorem manifest, axiom audit, and aggregate module. Static checks reject

$$\boxed{\texttt{sorry},\quad \texttt{admit},\quad \texttt{native\_decide},\quad \texttt{Lean.ofReduceBool},\quad \texttt{Lean.trustCompiler},} \tag{184}$$

new top-level project axioms, prototype authority, charged imports, W5 imports, and forbidden transitive source dependencies. The JSON authority maps are parsed and the twelve theorem names are checked against the Lean manifest in order.

The terminal publication axiom audit prints the dependency axioms of sixteen principal theorem surfaces. The exact permitted transitive union is

$$\boxed{\{\texttt{propext},\ \texttt{Classical.choice},\ \texttt{Quot.sound}\}.} \tag{185}$$

The checked Paper 8 modules use exactly the transitive axiom union in (185). The verification receipt records successful publication aggregate compilation, theorem-map synchronization, source-dependency exclusion, forbidden-token scanning, JSON validation, and exact axiom-union checking. Verification scope is targeted direct compilation, with `full_repository_build=false` recorded by the frozen Paper 8 guard.

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