A Self-Consistent Einstein–Scalar Quantum Sector

Friedrichs selection, native quantum evolution, and exact gravitational matter

Zed James

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Abstract

The first three papers in this series derive a selected real-scalar law on Schwarzschild geometry, construct its physical quantum evolution, and promote that evolution to local conserved quantum matter. The present fourth paper closes that physical-sector cycle on an independently constructed nonlinear Einstein–real-scalar geometry and supplies the completed sector carried forward into Paper 5.

The nonlinear geometry determines its own terminal coordinate, real scalar action, and full radial potential. Its curvature endpoint is Hardy-critical,

$$x^2 V_{\mathrm{NL}}(x) \longrightarrow -\frac{1}{4},$$

with inner limit-circle and outer limit-point classification, deficiency indices $(1,1)$, physical endpoint coefficients (A, B), and the complete self-adjoint projective family. The native ground-state form obeys

$$\mathcal{Q}_{\mathrm{NL}}[u] < \infty \quad \Longleftrightarrow \quad B_u = 0, \qquad \boxed{K_{\mathrm{F}}^{\mathrm{NL}} = K_{\text{no-log}}^{\mathrm{NL}}}.$$

The same action yields global signed Cauchy modes, action-derived Bogoliubov transport, and an all-mode real Fock implementation. On each owned positive compact interval,

$$|\beta_k(x)| \le C\Omega_k^{-2}, \qquad N_k(x) := |\beta_k(x)|^2 \le C^2 \Omega_k^{-4},$$

so the anchor-vacuum occupation spectrum has finite total particle number. An independently constructed self-adjoint full-action Hamiltonian generates a two-time physical unitary satisfying the strong Schrödinger equation on the original H_0 domain.

Every signed mode has C^2 spacetime regularity and satisfies the native covariant wave equation. Directed observable-bank removal gives a conserved vacuum-relative all-mode core stress with complete normal and anomalous sectors. For every admissible coherent amplitude z with nonzero real mean-field flux F_z, the quantum quadratic pairing equals the classical product of the real mean field $\phi_z = 2\,\mathrm{Re}(z\phi_0)$, and the corresponding branch satisfies

$$\boxed{G_{\mu\nu}[g_z] = \kappa\, \Delta T^{z}_{\mu\nu}.}$$

The canonical choice $z = i$ defines the distinguished branch $g_{\mathrm{NL}} = g_i$. The independently constructed one-occupation source supplies a second exact branch realization. The vacuum-relative stress also has a precise comparison target in the common-prescription difference of locally covariant Hadamard-renormalized stresses.

Finally, $R[g_{\mathrm{NL}}] \to -\infty$ along the shrinking-radius endpoint, excluding regular C^2 endpoint extensions in the formalized isometric-open-embedding class. On the canonical coherent branch, the positive-time physical state also has no strong endpoint vector in the original Fock carrier. Papers 1–4 therefore close one self-consistent Einstein–scalar physical sector. Paper 5 begins from that sector and turns upstream to the functional gauge/master law of the same Einstein–real-scalar system and its compatibility with the quantum Hamiltonian, unitary evolution, source states, and stress constructed here.

1 Introduction

This series follows one real scalar field through a growing chain of ownership. Paper 1 derives the singular Schwarzschild radial operator and shows that finite full-potential action selects its no-log Friedrichs realization.[1] Paper 2 constructs the associated continuum and physical Cauchy quantum theories, including the complete signed real-field carrier, self-adjoint all-mode Hamiltonian, Bogoliubov transport, and physical clock.[2] Paper 3 differentiates that field into local vacuum-relative matter, proves all-mode conservation on transported regular fibers, and shows that the exact nonlinear source belongs to a changed geometric law.[3] The present fourth paper completes the self-consistent physical sector generated by that sequence. Paper 5 begins from this completed sector and moves upstream to the functional gauge/master ownership of the same Einstein–real-scalar action—the variational and gauge structure whose classical projection yields that action—and its quantum realization.

The present paper asks the fourth question:

$$\boxed{\begin{array}{c}\text{Can the geometry generated by quantum matter own a quantum theory}\\ \text{that contains an exact source for that same geometry?}\end{array}} \tag{1}$$

Let g_{NL} denote the independently constructed nonlinear stationary Einstein–real-scalar geometry. The central theorem realizes the chain

$$\boxed{g_{\mathrm{NL}} \longrightarrow S_{\mathbb{R}}^{\mathrm{NL}} \longrightarrow K_{\mathrm{F}}^{\mathrm{NL}} \longrightarrow \mathcal{F}_{\mathrm{NL}} \longrightarrow H_{\mathrm{NL}}(x) \longrightarrow U_{\mathrm{NL}}(x_2, x_1) \longrightarrow \Delta T_{\mu\nu}^{\mathrm{NL}} \longrightarrow G_{\mu\nu}[g_{\mathrm{NL}}].} \tag{2}$$

The geometry supplies the action and radial potential. The native endpoint form selects the Friedrichs realization. The same action supplies the global Cauchy family, all-mode quantum Hamiltonian, and physical unitary. The unitary transports the quantum field and its reference vacuum. The resulting local stress is conserved by the same metric connection. The resulting carrier contains exact native source states, including a one-occupation realization and a genuine coherent realization whose real mean field supplies the classical scalar source.

This is the sense in which the construction is self-consistent. A geometry–matter sector (g, Ψ) closes when

$$\Psi \in \mathcal{Q}(g), \qquad G_{\mu\nu}[g] = \kappa\, \Delta T_{\mu\nu}[g, \Psi], \tag{3}$$

where $\mathcal{Q}(g)$ includes the action-selected operator domains, quantum carrier, Hamiltonian, physical evolution, and local matter law derived from g. The gravitational metric remains a classical Einstein field and the scalar matter is quantum. The result therefore occupies the self-consistent semiclassical interface where quantum matter and classical geometry determine one another within one closed sector.

Within the architecture of the series, this is a physical-sector closure. It supplies the right-hand construction in the next ownership chain,

$$\boxed{S_{\mathrm{master}} \longrightarrow S_{\text{Einstein-scalar}} \longrightarrow \mathfrak{S}_{\mathrm{NL}}} \tag{4}$$

The present paper constructs $S_{\text{Einstein-scalar}} \longrightarrow \mathfrak{S}_{\mathrm{NL}}$ through operator selection, quantum evolution, matter, and exact source closure. Paper 5 takes up the master/gauge ownership on the left and the compatibility of its generated differential with the Hamiltonian, physical unitary, source states, stress, and self-consistency residual of the sector constructed here.

Two additional questions become available once the operator and source theorems are assembled. The first is particle content. The native Bogoliubov transformation supplies the anchor-vacuum occupation spectrum

$$N_k(x) = |\beta_k(x)|^2, \tag{5}$$

together with an ultraviolet bound and finite total occupation. This makes comparison with a Hawking–Planck spectrum a concrete spectral calculation.[7] The second is stress-tensor interpretation. The theorem proved here is vacuum-relative. Standard locally covariant QFT supplies a precise comparison target: for two Hadamard states on the same geometry and one renormalization prescription, the state-independent geometric ambiguities cancel in the difference of renormalized stress tensors.[11, 8, 9] The corresponding Hadamard/microlocal identification is therefore a clearly defined bridge from the present relative observable to conventional $\langle T_{\mu\nu}\rangle_{\mathrm{ren}}$ language.

The selected radial frequency

$$\omega_{\mathrm{F}}^{\mathrm{NL}} = (K_{\mathrm{F}}^{\mathrm{NL}})^{1/2} \tag{6}$$

and the timelike Cauchy frequencies retain distinct mathematical roles. Their common provenance is the nonlinear real action. This distinction remains explicit throughout the paper.

Conventions. The manuscript retains the normalized conventions of Papers 1–3. The native variables x, Ω_k, V_{NL}, κ, and ϕ are reported in that inherited normalization. Detector proper time, detector energy, and SI-unit restoration enter through a separate geometric/calibration map outside the theorem package; this is the same conversion required for comparison with a Hawking temperature or another observer-based energy scale.

The geometry also supplies its own invariant endpoint statement. Its scalar curvature tends to $-\infty$ at shrinking areal radius. A regular C^2 endpoint extension with an open isometric embedding would yield a finite scalar-curvature limit along the approaching history. The contradiction identifies the endpoint at which regular classical-metric continuation is exhausted.

Sections 2–5 derive the native radial law and Friedrichs selection. Sections 6 and 7 construct the global signed modes, particle spectrum, all-mode Fock implementation, independent action, and physical unitary. Sections 8–10 develop vacuum-relative quantum matter and the exact Einstein source. Section 11 extracts direct comparison observables against a matched Schwarzschild reference. Section 12 gives invariant endpoint exhaustion, and Section 13 collects the physical-sector closure theorem and the explicit handoff to Paper 5.

2 Nonlinear geometry, native real action, and terminal structure

The input is a global stationary Einstein–real-scalar branch with negative radial transport G, positive scale E, areal radius $r = e^z$, and smooth Lorentz metric g_{NL}. Its stationary scalar field obeys the covariant wave equation and its stress satisfies the complete Einstein system. This geometric branch is established before the quantum theory developed below.

The nonlinear geometry determines a terminal coordinate from its actual radial transport:

$$x(r) = \int_0^r -\frac{s}{G(\log s)}\, ds. \tag{7}$$

The integrand is positive on the owned radial interval. The formal theory proves positivity, strict monotonicity, range $(0, \infty)$, a continuous inverse radius $r = r(x)$, and the inverse derivative

$$\frac{dr}{dx} = -\frac{G(\log r)}{r}. \tag{8}$$

The curvature end is $x = 0$, and the Cauchy anchor has the computed value

$$X_a = x(r_a) > 0. \tag{9}$$

The physical clock below retains this actual anchor. The coordinate x is the geometry-derived terminal coordinate and the evolution parameter of the native Cauchy Hamiltonian; $X_a = x(r_a)$ is its normalization slice. The action frequency $\Omega_k = \sqrt{\lambda_k - V_{\mathrm{NL}}(X_a)}$ is conjugate to this native evolution parameter at the anchor. A detector prediction adds a geometric conversion from detector proper time or an asymptotic time convention to x. The theorem package keeps that conversion explicit, so x functions as the internally selected quantum clock while detector energy remains an observable-specific map.

The four-dimensional single-real-scalar action reduces on g_{NL} to its own native radial action. After the area-field substitution, the boundary completion has connection coefficient

$$b = \frac{G}{r^2} \tag{10}$$

and full potential

$$\boxed{V_{\mathrm{NL}}(x) = \frac{GE}{r^3} - \frac{G^2}{r^4} = \frac{r''(x)}{r(x)}.} \tag{11}$$

The equality with r''/r gives an exact positive ground-state solution of the zero-energy equation. The canonical first-order variables satisfy

$$q' = p, \qquad p' = V_{\mathrm{NL}} q, \tag{12}$$

and the real action determines the native Hamiltonian density.

The curvature endpoint independently reproduces the Hardy-critical coefficient:

$$\boxed{x^2 V_{\mathrm{NL}}(x) \longrightarrow -\frac{1}{4} \qquad (x \downarrow 0).} \tag{13}$$

This result is derived from the nonlinear metric and its inverse native radius. The resulting endpoint problem therefore belongs to the new geometry itself.[1]

The actual native ground state is positive. Its normalized form will later identify the action-selected radial law. The full metric potential also has controlled first and second derivatives on positive compact intervals, providing the regularity required for the global Cauchy and ultraviolet estimates.

3 Native radial operator and endpoint classification

Let

$$\tau_{\mathrm{NL}} = -\frac{d^2}{dx^2} + V_{\mathrm{NL}}(x) \tag{14}$$

[1] *Formal counterpart:* `QGC12.Geometry.NonlinearFlux.NativeCriticalEndpoint`.

act on $L^2((0,\infty), dx; \mathbb{C})$. The compact C^2 field domain is densely embedded into this Hilbert space, and Green integration proves symmetry and closability of the associated core operator.

The graph closure gives a densely defined closed minimal operator

$$K^{\mathrm{NL}}_{\mathrm{min}}. \tag{15}$$

Independently, the maximal domain is constructed from genuine local-AC representatives whose field and full differential image belong to L^2. Pointwise recovery, derivative uniqueness, and almost-everywhere second-derivative equality make the maximal action representative-independent.

Constructive weak regularity identifies the adjoint domain with the maximal differential domain. Hence

$$\boxed{(K^{\mathrm{NL}}_{\mathrm{min}})^* = K^{\mathrm{NL}}_{\mathrm{max}}.} \tag{16}$$

The same equality holds for the adjoint of the original compact core after graph closure.[2]

The outer endpoint is limit-point. The proof uses the native potential's bounded continuous tail, tail Sobolev control, Wronskian decay, and Cauchy uniqueness. The inner endpoint is limit-circle. Global full-potential Cauchy solutions lie in L^2 near the inner endpoint, and two independently constructed columns retain nonzero Wronskian. These endpoint laws yield the genuine deficiency decision

$$\boxed{(n_+, n_-) = (1, 1).} \tag{17}$$

The deficiency subspaces are actual Hilbert-operator kernels. Real conjugation exchanges them, and their one-dimensionality yields a full circle of unitary deficiency identifications. The von Neumann graph construction then gives a genuine S^1 family of self-adjoint radial realizations

$$\{K^{\mathrm{NL}}_{\theta}\}_{\theta \in S^1}. \tag{18}$$

The minimal operator is nonnegative, inherited from the actual positive-ground-state energy identity and graph closure. The outer maximal Green boundary term tends to zero for every maximal-domain field.

4 Physical endpoint traces and self-adjoint boundary laws

The abstract deficiency family acquires direct physical coordinates through an independently normalized native ground/log pair. Let $g(x)$ denote the normalized positive ground column and $h(x)$ the normalized logarithmic column. Their Wronskian is

$$gh' - g'h = 1. \tag{19}$$

The metric-derived endpoint rates give

$$\frac{g(x)}{\sqrt{x}} \longrightarrow 1, \qquad \frac{h(x)}{\sqrt{x}} - \log x \longrightarrow 0, \tag{20}$$

and

$$\sqrt{x}\, g'(x) \longrightarrow \frac{1}{2}, \qquad \sqrt{x}\, h'(x) - \frac{\log x + 2}{2} \longrightarrow 0. \tag{21}$$

[2] *Formal counterpart:* `QGC12.Operator.NonlinearFlux.NativeMaximalOperator`.

For a maximal representative u, define the pointwise coefficients

$$A_u(x) = u(x)h'(x) - u'(x)h(x), \qquad B_u(x) = g(x)u'(x) - g'(x)u(x). \tag{22}$$

Finite Green identities and the actual L^2 conditions prove finite limits

$$A_u = \lim_{x\downarrow 0} A_u(x), \qquad B_u = \lim_{x\downarrow 0} B_u(x). \tag{23}$$

The resulting physical asymptotics are

$$\frac{u(x)}{\sqrt{x}} - A_u - B_u \log x \longrightarrow 0 \tag{24}$$

and

$$\sqrt{x}\, u'(x) - \frac{A_u + B_u \log x + 2B_u}{2} \longrightarrow 0. \tag{25}$$

These coefficients descend to the genuine maximal Hilbert domain and are onto: explicit maximal probes realize $(A, B) = (1, 0)$ and $(0, 1)$.

The physical Green form is therefore

$$\mathfrak{G}(u, v) = \overline{B_u} A_v - \overline{A_u} B_v, \tag{26}$$

with the orientation fixed by the Hilbert pairing convention. The trace-zero kernel is exactly the closed-minimal domain.

Every Hermitian projective law

$$aA + bB = 0, \qquad \overline{a}b = \overline{b}a, \tag{27}$$

defines an actual self-adjoint partial operator, and every member of the S^1 deficiency family is represented by one such physical line. The distinguished native no-log law is

$$\boxed{B = 0.} \tag{28}$$

At this stage it is an actual self-adjoint operator on the native geometry. The next section derives its variational selection.

5 Native finite action and Friedrichs selection

The nonlinear geometry's own positive ground state determines the literal quadratic form

$$\mathcal{Q}_{\mathrm{NL}}[u] = \int_0^\infty g(x)^2 \left| \left(\frac{u}{g}\right)'(x) \right|^2 dx. \tag{29}$$

On the genuine maximal domain, its density is expressed through the actual boundary coefficient $B_u(x)$. The endpoint calculation gives

$$x\, g(x)^2 \left| \left(\frac{u}{g}\right)'(x) \right|^2 \longrightarrow |B_u|^2. \tag{30}$$

Consequently

$$\boxed{\mathcal{Q}_{\rm NL}[u] < \infty \quad \Longleftrightarrow \quad B_u = 0.} \tag{31}$$

Local-AC integration by parts, the full ground-state equation, the two defining L^2 classes, and the earned endpoint flux limits give the exact same-operator identity

$$\mathcal{Q}_{\rm NL}[u] = \mathrm{Re}\langle u, K^{\rm NL}_{\text{no-log}} u\rangle \tag{32}$$

on the selected maximal domain.

The compact-core closure is then constructed directly. Logarithmic cutoffs transition across (e^{-2s}, e^{-s}), and the native ground weight supplies an endpoint capacity bound of order $O(s^{-1})$. Multiplication by the actual outer-cutoff ground jet produces compact C^2 tests whose L^2 and form error tend to zero. The physical trace kernel identifies the closed-minimal component, and the explicit ground cutoff handles the surviving A direction.

The resulting form core is dense in the selected form domain. The represented closed form has the same unit resolvent as the native no-log realization, which gives equality of full partial operators:

$$\boxed{K^{\rm NL}_{\rm F} = K^{\rm NL}_{\text{no-log}}.} \tag{33}$$

This is the nonlinear geometry's independently earned finite-action selector.[3]

Functional calculus now supplies the positive self-adjoint continuum frequency

$$\omega^{\rm NL}_{\rm F} = (K^{\rm NL}_{\rm F})^{1/2}. \tag{34}$$

Its squared domain and action recover the selected radial operator, and its norm square equals the ground form. The kernel is trivial, giving injectivity of the native continuum frequency. The theorem surface records strict positivity on each nonzero selected vector and retains the full continuum behavior of the spectrum.

6 Global native Cauchy modes, Bogoliubov transport, and particle content

The timelike theory begins from the nonlinear real action and the complete signed spatial carrier used as a neutral Fourier enumeration. For each signed label k, the native anchor fixes the initial action frequency

$$\Omega_k = \sqrt{\lambda_k - V_{\rm NL}(X_a)}. \tag{35}$$

The initial field and momentum carry the real-action normalization with amplitude square 1/2.

The full-potential Cauchy equation is solved globally on positive native coordinate by overlapping Dyson constructions whose equality on common intervals follows from Cauchy uniqueness. The resulting mode $q_k(x)$ has continuous second derivative and satisfies

$$q_k''(x) + (\lambda_k - V_{\rm NL}(x))q_k(x) = 0 \qquad (x > 0). \tag{36}$$

[3] *Formal counterpart:* `QGC12.Operator.NonlinearFlux.nativeFriedrichs_eq_noLog`.

The canonical charge is conserved with the real-field half-unit normalization.

The action-derived Bogoliubov coefficients obey their coupled differential equations,

$$\alpha_k(X_a) = 1, \qquad \beta_k(X_a) = 0, \qquad |\alpha_k(x)|^2 - |\beta_k(x)|^2 = 1. \tag{37}$$

Conjugate signed partners share the corresponding native coefficients; the zero label remains the single real self-conjugate oscillator.

Actual first and second derivatives of V_{NL} give compact oscillator-energy bounds and corrected pair transport. For every owned compact interval $I \Subset (0, X_a]$ there is a finite constant C_I such that the ultraviolet mixing obeys

$$|\beta_k(x)| \le \frac{C_I}{\Omega_k^2}, \qquad x \in I. \tag{38}$$

The complete signed enumeration therefore satisfies

$$\sum_k |\beta_k(x)|^2 < \infty \tag{39}$$

together with the weighted summability required for the original energy domain. The zero oscillator is implemented by the real one-mode squeeze and the nonzero labels by disjoint opposite-sign pairs. Their product gives the strongly continuous all-mode real canonical frame

$$W_{\mathrm{NL}}(x) : \mathcal{F} \longrightarrow \mathcal{F}, \tag{40}$$

with forward and inverse H_0 standing.

6.1 The native particle spectrum

Relative to the anchor vacuum, the instantaneous number expectation in mode k is

$$\boxed{N_k(x) = \langle \Omega_a, a_k^\dagger(x) a_k(x) \Omega_a \rangle = |\beta_k(x)|^2.} \tag{41}$$

Equation (38) immediately yields

$$N_k(x) \le \frac{C_I^2}{\Omega_k^4}, \qquad \sum_k N_k(x) < \infty, \tag{42}$$

on each owned compact interval. The formal Fock implementability theorem therefore also supplies a concrete ultraviolet statement about particle production: the anchor-vacuum occupation spectrum has quartic suppression in the native initial frequency.

Hawking's black-hole calculation provides a natural physical benchmark because its Bogoliubov coefficients generate a thermal occupation law with temperature fixed by the horizon surface gravity.[7] The present construction supplies an exact native spectrum $k \mapsto N_k(x)$ whose thermality can be tested directly. For $N_k(x) > 0$, define the modewise Planck diagnostic

$$T_{\mathrm{eff}}(k, x) = \frac{\Omega_k}{\log\bigl(1 + N_k(x)^{-1}\bigr)}. \tag{43}$$

Approximate k-independence of T_{eff} over a physical spectral window is the corresponding thermal signature. A comparison with the usual Hawking temperature additionally requires the map from the native compact-mode energy Ω_k to the detector or horizon-energy convention used in the thermal measurement. The theorem package now isolates every quantity entering that calculation.

The particle spectrum is relative to the polarization fixed at the native anchor through the chain $(r_a, g_{\mathrm{NL}}) \mapsto X_a \mapsto \Omega_k \mapsto |\Omega_a\rangle \mapsto N_k(x)$. Each admissible anchor therefore defines an internally complete particle convention for its sector. Comparison of distinct anchors, unitary equivalence between their polarizations, and an anchor-independent detector spectrum are separate comparison theorems. The exact Einstein source statements below use the same declared anchor data throughout.

7 Independent all-mode action and physical quantum evolution

The physical Hamiltonian is constructed independently from the nonlinear real action. The instantaneous positive-frequency diagonalization has the same maximal H_0 domain as the original reference free graph. The reference-normal-ordered vacuum shift is absolutely summable, and the literal Fourier action equals the corresponding maximal mode sum.

The independent full-action Hamiltonian

$$H_{\mathrm{NL}}(x) \tag{44}$$

is defined on the original energy domain using the free image together with the exact zero and opposite-sign relative perturbations. Eigenvector graph identities and modal sums identify this operator with the instantaneous realization in full domain and action. Compact occupation approximation converges simultaneously in vector and action image, so the original algebraic action core graph-closes to $H_{\mathrm{NL}}(x)$. Thus

$$\boxed{H_{\mathrm{NL}}(x) = H_{\mathrm{NL}}(x)^*, \qquad \mathcal{D}(H_{\mathrm{NL}}(x)) = \mathcal{D}(H_0)} \tag{45}$$

for every owned positive physical time.

The derivative of the all-mode canonical frame is computed on the whole H_0 domain. Its action identity has the form

$$\frac{d}{dx} W_{\mathrm{NL}}(x)\Psi = -i(H_{\mathrm{NL}}(x) - c_{\mathrm{NL}}(x)I)W_{\mathrm{NL}}(x)\Psi, \tag{46}$$

where $c_{\mathrm{NL}}(x)$ is the finite scalar ledger obtained from zero once plus the complete nonzero signed-pair vacuum contraction.

The same Cauchy coefficients determine an all-signed-mode scalar phase

$$\Gamma_{\mathrm{NL}}(x) = \exp\left(-i \int_{X_a}^{x} c_{\mathrm{NL}}(s)\, ds\right), \qquad |\Gamma_{\mathrm{NL}}(x)| = 1. \tag{47}$$

Define the physical unitary

$$U_{\mathrm{NL}}(x) = \Gamma_{\mathrm{NL}}(x) W_{\mathrm{NL}}(x). \tag{48}$$

The scalar phase cancels the exact defect in (46), giving the same-action Schrödinger law

$$\boxed{\frac{d}{dx} U_{\mathrm{NL}}(x)\Psi = -iH_{\mathrm{NL}}(x)U_{\mathrm{NL}}(x)\Psi} \tag{49}$$

for every $\Psi \in \mathcal{D}(H_0)$ and every interior owned positive time.[4]

The two-time transport is

$$U_{\mathrm{NL}}(x_2, x_1) = U_{\mathrm{NL}}(x_2) U_{\mathrm{NL}}(x_1)^{-1}. \tag{50}$$

It is norm preserving, jointly strongly continuous, normalized at X_a, preserves $\mathcal{D}(H_0)$ in both directions, and satisfies

$$U_{\mathrm{NL}}(x_3, x_2) U_{\mathrm{NL}}(x_2, x_1) = U_{\mathrm{NL}}(x_3, x_1). \tag{51}$$

This closes the full native physical clock on the positive Cauchy interval $(0, X_a]$.

8 Native physical quantum matter and the renormalized-stress bridge

The local matter theory acts on the complete physical Fock carrier. Maximal annihilation and creation graphs are available on the original H_0 standing, including arbitrary occupations and spectator modes. The real all-signed-mode Bogoliubov law therefore acts on genuine ladder observables,

$$a_k(x) = \alpha_k(x) a_k(X_a) + \beta_k(x) a_{\bar{k}}^{\dagger}(X_a). \tag{52}$$

For a finite signed observable bank B, the physical field is

$$\widehat{\Phi}_B(p) = \sum_{k \in B} \left(\Phi_k(p) a_k + \overline{\Phi_k(p)} a_k^{\dagger} \right). \tag{53}$$

The native Cauchy field and momentum reconstruct the spacetime gradient weights, and literal core-field differentiation identifies the Fock-valued derivative with the geometric mode gradient entering the metric stress.

The quantum observable used throughout the paper is vacuum-relative. For an owned state Ψ and reference vacuum Ω transported by the same physical unitary,

$$\Delta T^B_{\mu\nu}(x, p; \Psi, \Omega) = T^B_{\mu\nu}(x, p; \Psi) - \|\Psi\|^2 T^B_{\mu\nu}(x, p; \Omega). \tag{54}$$

The joint transport theorem gives exact state–field covariance. The transported core class is dense, remains in the original H_0 domain, and is preserved by two-time evolution.

The full relative stress retains both normal and anomalous sectors,

$$\Delta T^B_{\mu\nu} = \Delta T^{B,\mathrm{N}}_{\mu\nu} + \Delta T^{B,\mathrm{A}}_{\mu\nu}, \tag{55}$$

with same-family anomalous coherence generated by the native β coefficients. The finite bank regulates the local observable; the state and physical unitary remain on the complete carrier.

[4] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativePhysicalOwnedTransport_actual_same_independent_action_hasDerivAt`.

8.1 Relation to locally covariant renormalized stress

Standard curved-spacetime QFT defines local Wick observables and renormalized stress tensors on Hadamard state classes, with renormalization freedom represented by local state-independent geometric terms.[11, 8, 9] For two Hadamard states Ψ and Ω on one geometry and one locally covariant prescription, these common geometric terms cancel in the difference

$$\Delta\langle T_{\mu\nu}\rangle_{\mathrm{ren}} = \langle T_{\mu\nu}\rangle_{\mathrm{ren},\Psi} - \langle T_{\mu\nu}\rangle_{\mathrm{ren},\Omega}. \tag{56}$$

Equation (56) is the precise conventional comparison target for (54). The present formal theory proves the weak vacuum-relative observable, its physical-clock covariance, its complete normal/anomalous polynomial on the controlled core, and its native covariant conservation. The remaining bridge has a clearly defined mathematical form: establish common Hadamard/microlocal standing for the transported source/reference pair and identify the weak relative observable with the corresponding locally covariant Wick-polynomial difference. This separates the exact Einstein source theorem proved here from the additional microlocal theorem required for a conventional absolute-renormalization interpretation.

9 Independent native quantum source and exact Einstein closure

The exact source sector is constructed from native geometric data. A source anchor fixes the horizon scale and a positive interior radius; the native anchor potential fixes the initial frequency, while canonical initial field and momentum determine an offset and a nonzero canonical flux $F_\star$. With $C(F)$ denoting the stationary flux-strength functional, the branch parameter is $\sigma_\star = \kappa C(F_\star)/2$. Interior Cauchy uniqueness identifies the resulting geometric zero mode with the native Cauchy solution carried by the quantum theory. For the transported unit zero-mode occupation $\Psi_1(x)$,

$$\boxed{G_{\mu\nu}[g_1](p) = \kappa\,\Delta T^{\mathrm{NL}}_{\mu\nu}[\Psi_1(x), \Omega(x)](p).} \tag{57}$$

[5]

9.1 Classical-to-quantum source binding

A genuine coherent state makes the normalization mechanism explicit. On the completed zero-mode carrier, $|z\rangle$ has normalized coefficients $e^{-|z|^2/2}z^n/\sqrt{n!}$. Call $z \in \mathbb{C}$ *admissible* when its real mean-field flux

$$F_z := 2\,\mathrm{Re}(zF_\star) \tag{58}$$

is nonzero. Together with $\kappa > 0$, this is the branch-construction condition used below. For zero-mode coefficients $u, v \in \mathbb{C}$,

$$\boxed{\langle z|\Phi(u)\Phi(v)|z\rangle - \langle 0|\Phi(u)\Phi(v)|0\rangle = 4\ \mathrm{Re}(uz)\ \mathrm{Re}(vz).} \tag{59}$$

[5] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeQuantumSourcePhysicalRelativeStress_actual_nonlinear_Einstein`.

[6] The corresponding real mean field is $\phi_z = 2\,\mathrm{Re}(z\phi_0)$, so the complete normal/anomalous quantum pairing is exactly the quadratic gradient product entering its classical real-scalar stress:

$$\boxed{\nabla\phi_z = 2\,\mathrm{Re}(z\nabla\phi_0), \qquad \Delta T^z_{\mu\nu} = T_{\mu\nu}[\phi_z].} \tag{60}$$

For every admissible z, the exact branch normalization and Einstein identity are

$$\boxed{\frac{\kappa}{2}C(F_z) = 2\sigma_z, \qquad G_{\mu\nu}[g_z] = \kappa\,\Delta T^z_{\mu\nu}.} \tag{61}$$

[7] The canonical amplitude $z = i$ has proved nonzero real flux.[8] The formal development carries g_1 and g_z as separate branch constructors. In this manuscript we define $g_{\mathrm{NL}} := g_i$. The one-occupation theorem therefore supplies an exact source theorem for its own branch member g_1, while the canonical coherent theorem supplies the distinguished branch $g_i = g_{\mathrm{NL}}$ used for the physical-sector closure theorem. The coherent realization is structurally stronger than a numerical stress match: its complete normal/anomalous quantum quadratic expectation *is* the classical quadratic stress of the real mean field ϕ_z, and the branch normalization converts that same field flux into the Einstein source coefficient. Classification of all source states producing fixed macroscopic data, local uniqueness, and stability remain separate problems.

For the sector theorem below, let $\Psi_\star = |i\rangle_{\mathrm{coh}}$ and let $g_{\mathrm{NL}} = g_i$ denote its corresponding self-consistent branch. Then

$$\boxed{\Psi_\star \in \mathcal{Q}(g_{\mathrm{NL}}), \qquad \mathcal{E}(\Delta T[g_{\mathrm{NL}}, \Psi_\star]) = g_{\mathrm{NL}}.} \tag{62}$$

10 All-mode spacetime closure and conserved quantum matter

The global Cauchy modes are promoted to actual spacetime fields on g_{NL}. The native terminal Jacobian and its first derivative determine the logarithmic second jet, and the scale-corrected reconstruction gives genuine C^2 spacetime regularity for every signed mode.

The full covariant wave equation is then proved mode by mode:

$$\boxed{\Box_{g_{\mathrm{NL}}}\Phi_k = 0 \qquad \text{for every signed mode } k.} \tag{63}$$

The proof uses the actual native metric connection drifts and the globally constructed Cauchy second jets.[9]

For a finite observable bank B, the differentiated field defines the full normal/anomalous vacuum-relative stress. Polarized stress identities and (63) yield

$$\nabla^\mu_{g_{\mathrm{NL}}}\Delta T^B_{\mu\nu} = 0. \tag{64}$$

[6] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeZeroCoherentField_actual_relative_pairing`.

[7] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentSourcePhysicalStress_actual_nonlinear_Einstein`.

[8] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentSourceFlux_actual_unit_imaginary_nonzero`.

[9] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeGlobalSpacetimePolarization_actual_all_mode_covariantWave`.

Occupied-mode support stabilizes the observable once the bank contains the occupied modes. Directed refinement therefore gives an intrinsic all-mode core stress

$$\Delta T^{\mathrm{core}}_{\mu\nu} = \lim_{B\uparrow \mathrm{modes}(\Psi)} \Delta T^{B}_{\mu\nu}, \tag{65}$$

with exact conservation

$$\boxed{\nabla^{\mu}_{g_{\mathrm{NL}}} \Delta T^{\mathrm{core}}_{\mu\nu} = 0.} \tag{66}$$

[10]

The geometric stress also binds directly to the physical clock. The native geometric gradient weights reconstruct the same H_0 field and momentum ladder vectors used by the physical unitary. Transporting the state and reference vacuum with U_{NL} gives

$$\Delta T^{\mathrm{geom}}(x,p) = \Delta T^{\mathrm{phys}}(x,p), \tag{67}$$

with the equality understood on the transported core class and directed conjugation-closed bank limit. This completes the dynamic local-matter package promised by the preceding paper.

11 Physical comparison observables

The formal construction now exposes several quantities that can be compared directly with a Schwarzschild reference. Choose a comparator by matching the geometric parameters relevant to the intended experiment, such as horizon scale and a reference radius. The resulting diagnostic family is

$$\boxed{\begin{aligned} \delta g &:= g_{\mathrm{NL}} - g_{\mathrm{Schw}}, \\ \delta V &:= V_{\mathrm{NL}} - V_{\mathrm{Schw}}, \\ \delta N_k(x) &:= |\beta_k^{\mathrm{NL}}(x)|^2 - |\beta_k^{\mathrm{Schw}}(x)|^2, \\ \delta T_{\mu\nu} &:= \Delta T^{\mathrm{NL}}_{\mu\nu} - \Delta T^{\mathrm{Schw}}_{\mu\nu}, \\ \delta R &:= R[g_{\mathrm{NL}}] - R[g_{\mathrm{Schw}}]. \end{aligned}} \tag{68}$$

These observables separate five physical layers: metric response, mode propagation, particle production, local matter, and invariant curvature.

A comparison is therefore accompanied by a declared matching prescription:

Quantity	Matching data for the comparator
g and R	common horizon scale, areal-radius event, and invariant chart identification
V	common radial/terminal normalization and the same matched anchor event
N_k	common signed mode label, reference slice, and detector-energy conversion
$\Delta T_{\mu\nu}$	common source/reference prescription and common local tensor frame

The matching prescription is part of the observational definition of δg, δV, δN_k, and $\delta T_{\mu\nu}$. Invariant conclusions should therefore be reported together with that prescription and tested across reasonable alternative matchings.

[10] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeSpacetimeCoreAllModeStress_actual_conserved`.

The particle-spectrum diagnostic is especially direct. Equation (41) gives the exact occupation spectrum relative to the native anchor polarization; a Schwarzschild calculation performed with the same detector and mode convention supplies the comparison spectrum. The stress diagnostic compares two vacuum-relative local matter laws after the state/reference prescription is matched. The metric and curvature diagnostics identify where the self-consistent branch departs geometrically from the chosen Schwarzschild reference.

Equation (68) therefore turns the formal closure theorem into a concrete phenomenological program. Numerical evaluation of these quantities, together with the detector-energy map entering (43), is the natural next layer for assessing thermality and observational distinction.

12 Invariant endpoint exhaustion

The distinguished coherent geometry also owns an invariant endpoint theorem. Contracting the full nonlinear Einstein equations with the inverse metric yields the actual scalar curvature of g_{NL}. Along shrinking areal radius,

$$\boxed{R[g_{\mathrm{NL}}] \longrightarrow -\infty.} \tag{69}$$

[11]

The extension class is formulated geometrically. A regular endpoint extension consists of an arbitrary target manifold with a C^2 Lorentzian metric, an open isometric embedding of the positive-radius geometry, an endpoint outside the embedded image, and an approaching history. Scalar-curvature naturality under the C^2 isometric open embedding identifies the curvature along the embedded history. Continuity of scalar curvature for the target C^2 metric yields a finite endpoint limit.

Equation (69) therefore gives

$$\boxed{\begin{array}{c} g_{\mathrm{NL}} \text{ is geometrically exhausted at the shrinking-radius endpoint} \\ \text{within the regular } C^2 \text{ extension class.} \end{array}} \tag{70}$$

Equivalently, the formal type of such endpoint extensions is empty for the native logarithmic radial history.[12]

The same result specializes to the independently constructed native quantum-source branch. The positive-radius quantum sector therefore closes on a geometry that also supplies an invariant signal of where regular classical-metric continuation is exhausted.

The canonical coherent branch has a second, logically independent endpoint decision on the quantum carrier. Along an explicit owned history $x_n \downarrow 0$, the physical coherent state has no strong limit vector in the original Fock space. The proof passes continuous Hilbert coordinates to a hypothetical limit, derives the exact spectator-fiber quadrature recurrence, and shows that square summability forces every coordinate to vanish, while the physical state retains norm one. Unitary evolution remains established at every positive native time; the endpoint theorem states that this unitary

[11] *Formal counterpart:*
`QGC12.Geometry.NonlinearFlux.globalMetric_actual_scalar_curvature_tendsto_atBot`.

[12] *Formal counterpart:*
`QGC12.Geometry.NonlinearFlux.globalMetric_actual_no_regular_endpoint_extension`.

orbit has no strong limit in the original Fock representation as $x \downarrow 0$. Thus the positive-time unitary has no strong endpoint extension on this original carrier for the canonical coherent sector.[13]

The geometric theorem is a C^2 curvature obstruction, and the quantum theorem is a strong-Hilbert endpoint obstruction on the declared Fock carrier. Geodesic completeness, C^0 or C^1 extendibility, causal character of the endpoint, Tipler–Królak strength, horizon visibility, and cosmic-censorship classification form separate geometric questions. Alternative quantum representations or endpoint topologies form separate quantum questions.

This gives the curvature theorem a physical role beyond endpoint classification. The sector establishes a complete positive-time quantum-matter law on $r > 0$; simultaneously,

$$|R[g_{\mathrm{NL}}]| \longrightarrow \infty \tag{71}$$

marks the boundary at which a regular C^2 classical geometry leaves the theorem class. The construction thus identifies both the self-consistent semiclassical sector and the invariant geometric frontier where a theory with quantum gravitational degrees of freedom becomes the natural next object.

13 The self-consistent Einstein–scalar quantum sector

Physical-sector closure theorem. Fix an admissible native nonlinear source anchor and positive gravitational coupling. There exists a nonlinear Einstein–real-scalar geometry g_{NL}, a Friedrichs-selected radial operator $K_{\mathrm{F}}^{\mathrm{NL}}$, a real bosonic Fock carrier $\mathcal{F}_{\mathrm{NL}}$, a self-adjoint native action $H_{\mathrm{NL}}(x)$, a two-time physical unitary $U_{\mathrm{NL}}(x_2, x_1)$, a dense transported regular core, and a canonical coherent source state $\Psi_\star = |i\rangle_{\mathrm{coh}}$ such that:

(i) The nonlinear metric determines its native real scalar action and radial operator; physical endpoint traces give a $(1,1)$ self-adjoint family, and finite native form energy selects

$$K_{\mathrm{F}}^{\mathrm{NL}} = K_{\text{no-log}}^{\mathrm{NL}}.$$

(ii) The same action determines global signed Cauchy modes, ultraviolet-summable Bogoliubov transport, the all-mode real Fock implementation, and the anchor-vacuum occupation spectrum $N_k(x) = |\beta_k(x)|^2$.

(iii) The independently constructed full-action Hamiltonian generates the physical unitary,

$$\partial_x U_{\mathrm{NL}}(x, x_0) = -iH_{\mathrm{NL}}(x)U_{\mathrm{NL}}(x, x_0),$$

with identity, cocycle, joint strong continuity, and two-way H_0 standing on positive native time.

(iv) Every signed mode is a C^2 solution of the native covariant wave equation. Directed observable-bank removal gives a vacuum-relative all-mode core stress with complete normal/anomalous sectors and

$$\nabla^\mu_{g_{\mathrm{NL}}} \Delta T_{\mu\nu} = 0.$$

[13] *Formal counterpart:* `QGC12.Quantum.NonlinearFlux.nativeCoherentSourceUnitary_actual_no_original_Fock_strong_endpoint`.

(v) The canonical coherent source state belongs to this same physical quantum theory. Its vacuum-relative stress equals the classical stress of its real mean field, and the branch normalization gives

$$\boxed{G_{\mu\nu}[g_{\mathrm{NL}}] = \kappa\, \Delta T^{\mathrm{NL}}_{\mu\nu}[\Psi_\star].}$$

The independently constructed one-occupation state supplies an exact source theorem for its separately constructed branch member g_1; the distinguished geometry used here is the canonical coherent member $g_i = g_{\mathrm{NL}}$.

(vi) The scalar curvature tends to $-\infty$ at shrinking radius, excluding regular C^2 endpoint extensions in the formalized isometric-open-embedding class. On the canonical coherent branch, the physical state also has no strong endpoint vector in the original Fock carrier.

Define

$$\mathfrak{S}_{\mathrm{NL}} = (g_{\mathrm{NL}}, K_{\mathrm{F}}^{\mathrm{NL}}, \mathcal{F}_{\mathrm{NL}}, H_{\mathrm{NL}}, U_{\mathrm{NL}}, \Psi_\star, \Delta T^{\mathrm{NL}}). \tag{72}$$

Then

$$\boxed{g_{\mathrm{NL}} \longmapsto \mathcal{Q}(g_{\mathrm{NL}}) \ni \Psi_\star \longmapsto \Delta T[g_{\mathrm{NL}}, \Psi_\star] \longmapsto g_{\mathrm{NL}}} \tag{73}$$

is the completed physical-sector ownership cycle of Papers 1–4.

The sector remains within the classical-metric/quantum-matter interface familiar from QFT in curved spacetime.[10, 11] Its additional structure is closure: the classical metric that appears in Einstein's equation is also the metric from which the source state's complete quantum law is derived.

This closure supplies the physical input to Paper 5. The next paper begins from the same real Einstein–scalar action and develops its functional gauge/master ownership, with the immediate bridge problem being the compatibility of the master-generated differential with the native Hamiltonian, physical unitary, source states, vacuum-relative stress, and the self-consistency residual

$$\mathcal{E}_{\mu\nu} := G_{\mu\nu}[g_{\mathrm{NL}}] - \kappa\, \Delta T^{\mathrm{NL}}_{\mu\nu}[\Psi_\star]. \tag{74}$$

The physical-sector theorem proved here fixes the target of that master/gauge construction. Local uniqueness and stability of closed sectors, relations among distinct sectors, and quantum superpositions of geometry–matter sectors follow after this master-to-quantum bridge as the next series-level questions.

14 Conclusion

Paper 4 completes the physical-sector arc that began with the Schwarzschild endpoint problem. A singular geometry selected an admissible scalar law; that law generated a quantum dynamics; the dynamics generated local matter; and the resulting nonlinear geometry has now been shown to regenerate its own selected operator theory, physical quantum evolution, and exact quantum source. Its role in the continuing series is therefore precise: Papers 1–4 construct and close one self-consistent Einstein–scalar quantum sector.

The physical calculations carried by that closure are explicit. The Bogoliubov coefficients define the anchor-vacuum occupation spectrum $N_k(x) = |\beta_k(x)|^2$ with finite total occupation and controlled ultraviolet decay. The vacuum-relative stress has a precise comparison target in the common-prescription difference of locally covariant Hadamard-renormalized stresses. The matched Schwarzschild observables $(\delta g, \delta V, \delta N_k, \delta T, \delta R)$ turn the formal theory into a concrete phenomenological program. The coherent source theorem makes the Einstein identity transparent at the level of the field pairing: the complete quantum normal/anomalous expectation reduces to the real mean-field quadratic stress, while the branch normalization supplies the gravitational source coefficient. The one-occupation and coherent constructions remain separately constructed members of the same nonlinear family, with $g_i = g_{\mathrm{NL}}$ selected as the distinguished coherent sector used here.

The endpoint results identify two distinct exhaustion statements for that branch. Scalar-curvature divergence excludes regular C^2 geometric continuation in the formalized extension class, and the physical coherent state admits no strong endpoint vector in the original Fock carrier. Positive-time unitary evolution and the self-consistent source theorem remain fully valid throughout their owned domain. These endpoint statements identify the boundary of the present physical sector and sharpen the requirements on any subsequent continuation theory.

Paper 5 begins from the object completed here. Its task is to move upstream from the Einstein–scalar physical action to the functional gauge/master law that owns that same action, and then to bind the resulting master-generated differential back to the quantum sector of Papers 2–4. The intended ownership chain is

$$\boxed{S_{\mathrm{master}} \longrightarrow S_{\text{Einstein-scalar}} \longrightarrow \mathfrak{S}_{\mathrm{NL}}, \qquad \mathcal{E}_{\mu\nu} = 0} \tag{75}$$

The present paper supplies the self-consistent physical target on the right. Paper 5 supplies the master/gauge ownership on the left and the bridge between them. Once that bridge is established, the program can move to local uniqueness and stability of closed sectors and, at a genuinely new series boundary, to quantum geometry itself: relations, transitions, and superpositions among distinct geometry–matter sectors.

A Lean theorem map

The following table records the principal formal counterparts used by the manuscript. Repository names are included as technical provenance alongside the physical notation.

Manuscript role	Formal source
Nonlinear geometry, native action, coordinate and potential	`QGC12.Geometry.NonlinearFlux.GlobalEinsteinClosure`, `QGC12.Matter.NonlinearFlux.NativeRealAction`, `QGC12.Geometry.NonlinearFlux.TerminalCoordinate`, `QGC12.Geometry.NonlinearFlux.NativeAreaPotential`
Native radial operator, endpoint laws and self-adjoint family	`QGC12.Operator.NonlinearFlux.NativeMinimalOperator`, `QGC12.Operator.NonlinearFlux.NativeMaximalOperator`, `QGC12.Operator.NonlinearFlux.NativeDeficiencyDecision`, `QGC12.Operator.NonlinearFlux.NativePhysicalTraceClassification`
Finite form and Friedrichs selection	`QGC12.Operator.NonlinearFlux.NativeGroundFormGlobalEnergy`, `QGC12.Operator.NonlinearFlux.NativeFormCore`, `QGC12.Operator.NonlinearFlux.NativeFriedrichs`
Global signed modes, Bogoliubov transport and real Fock implementation	`QGC12.Quantum.NonlinearFlux.NativeGlobalCauchyModes`, `QGC12.Quantum.NonlinearFlux.NativeBogoliubovEquations`, `QGC12.Quantum.NonlinearFlux.NativeSpectralSummability`, `QGC12.Quantum.NonlinearFlux.NativeAllModeCanonicalFrame`
Independent action and same-action physical evolution	`QGC12.Quantum.NonlinearFlux.NativeFullActionHamiltonian`, `QGC12.Quantum.NonlinearFlux.NativePhysicalClockAction`, `QGC12.Quantum.NonlinearFlux.NativePhysicalOwnedFamily`

Manuscript role	Formal source
Vacuum-relative matter and conserved spacetime stress	`QGC12.Quantum.NonlinearFlux.NativePhysicalStressCovariance`, `QGC12.Quantum.NonlinearFlux.NativeSpacetimeWave`, `QGC12.Quantum.NonlinearFlux.NativeSpacetimeStressConservation`, `QGC12.Quantum.NonlinearFlux.NativeSpacetimePhysicalStress`
One-occupation and coherent exact Einstein sources	`QGC12.Quantum.NonlinearFlux.NativeQuantumSourcePhysicalEinstein`, `QGC12.Quantum.NonlinearFlux.NativeZeroCoherentFieldPair`, `QGC12.Quantum.NonlinearFlux.NativeCoherentSourceBranch`, `QGC12.Quantum.NonlinearFlux.NativeCoherentSourceEinstein`
Geometric and strong-Fock endpoint exhaustion	`QGC12.Geometry.NonlinearFlux.InvariantInextendibility`, `QGC12.Quantum.NonlinearFlux.NativeCoherentGeometryExhaustion`, `QGC12.Quantum.NonlinearFlux.NativeCoherentQuantumExhaustion`
Paper-level status and focused audits	`QGC12.Quantum.NonlinearFlux.NativeSpacetimeClosureStatus`, `QGC12NativeFormAudit`, `QGC12NativeQuantumEvolutionAudit`, `QGC12NativeQuantumMatterAudit`, `QGC12NativeSpacetimeClosureAudit`, `QGC12NativeCoherentClosureAudit`

B Verification and scope

The Paper 4 physical-sector authority is frozen at repository revision

`345e7d0fdc32c28712c465e534db6fb1f6aefe26`,

covering M568–M1339. The principal focused audit roots are

`QGC12NativeFormAudit`, `QGC12NativeQuantumEvolutionAudit`, `QGC12NativeQuantumMatterAudit`, `QGC12NativeSpacetimeClosureAudit`, `QGC12NativeCoherentClosureAudit`.

The final M1213–M1339 verification receipt reports 1,668 standard-only axiom closures across the named shared targets, with the focused coherent audit restricted to `propext`, `Classical.choice`, and `Quot.sound`; all nineteen focused construction-lineage checks and all 39 QGC12 static checks passed at the frozen revision.

The theorem surface is positive native physical time, the full positive-radius chart for the reconstructed source, vacuum-relative stress on the transported regular core with directed observable-bank removal, normalized coherent matter on the completed native carrier, regular C^2 endpoint extensions in the formalized isometric-open-embedding class, and strong endpoint limits on the original Fock carrier for the explicit coherent branch. The Hadamard-renormalized identification in Section 8 is a precisely stated comparison bridge; the particle-thermal comparison in Section 6 is a spectral diagnostic built from the proved Bogoliubov coefficients. Radial Friedrichs frequency and timelike native Cauchy frequency remain distinct constructions.

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